Simplified Equations Calculates Head Losses in Commercial Pipes

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Abstract: Direct solutions of pipe flow problems are not possible because of the implicit form of Colebrook-White equation, which is most commonly used for determining the friction factor of commercial pipes. New empirical equations for head loss, h_f, due to friction undergone by water flowing in the PVC, commercial steel, asphalted cast iron, galvanized iron, cast iron and concrete pipes which are commonly used in pipelines and water distribution systems were developed. The calculation of the parameters of the proposed models has been done through non-linear multivariable regression. Maximum relative error of each model is less than 2%. Therefore, these simple equations can offer significant advantage in optimization studies and hydraulic analysis of pipelines and water distribution systems. [The Journal of American Science. 2005;1(1):1-3].

Key Words: commercial pipes; head losses; pipelines; water distribution systems

1. Introduction

Head losses are generally the result of two mechanisms: Friction along the pipe walls.

Turbulence due to changes in streamlines is through fittings and appurtenances.

Head losses along the pipe wall are called friction losses or head losses due to friction, while losses due to turbulence within the bulk fluid are called minor losses (Walski, 2001; White, 1994). Estimation of head losses due to friction in pipes is an important task in optimization studies and hydraulic analysis of pipelines and water distribution systems (Giles, 1962; Fox, 1992).

The best equation for computing the frictional head loss in a given pipe for a given discharge is the Darcy-Weisbach equation (Giles, 1962; Fox, 1992; Walski, 2001):

$$h_f = f \frac{L}{D} \frac{v^2}{2g} \qquad (1)$$

Where h_f is the head loss due to friction that has the unit of length (L), f is a dimensionless friction factor, L is the length of the pipe (L), D is the internal diameter (L), V is the average velocity (LT^{-1}), and g is the acceleration due to gravity (LT^{-2}). For turbulent flow (R_E>4000), the friction factor f is a function of the Reynolds number R_E, and the relative roughness (ε/D), which is the internal pipe roughness (ε) divided by the pipe diameter (D). The Reynolds number for full flowing circular pipes can be found using the following equation:

$$R_E = \frac{VD}{\nu} \qquad (2)$$

Where R_E is the Reynolds number, ν is the kinematic viscosity (L^{2}T^{-1}). The kinematic viscosity is a function of temperature, but optimization studies and hydraulic analysis of pipelines and water distribution systems treats kinematic viscosity as a constant assuming a temperature of 20°C (Walski, 2001). The kinematic viscosity of water for 20°C is 10^{-6} m^2/s (Giles, 1962). Numerous formulas exist that relate the friction factor to the Reynolds number and relative roughness. One of the earliest and most popular of these formulas is the Colebrook-White equation (Giles, 1962; Fox, 1992; Shames, 1989; White 1994):

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{R_E\sqrt{f}}\right) \qquad (3)$$

The difficulty with using the Colebrook-White equation is that it is an implicit function of the friction factor (f is found on both sides of the equation). Typically, the equation is solved by iterating through assumed values of f until both sides are equal. The optimization studies and hydraulic analysis of pipelines and water distribution systems often involves the implementation of a tedious and time-consuming iterative procedure that requires extensive use of computers. Empirical head loss equations have a long and honorable history of use in pipeline problems. Their initial use preceded by decades the development of the Moody diagram, and they are still

commonly used today in professional practice. Some prefer to continue to use such an equation owing simply to force of habit, while others prefer it to avoid some of the difficulties of determining the friction factor in the Darcy-Weisbach equation (Larock, 2000). In this work, empirical equations for estimation of head losses in PVC, commercial steel, asphalted cast iron, galvanized iron, cast iron and concrete pipes which is commonly used in pipelines and water distribution systems were improved. The calculation of the parameters of the new improved empirical equations has been done through non-linear multivariable regression.

2. Methods

The development of the models which is proposed for six commercial pipes involves three basic steps. These include the generation of data required for calculation of the parameters of the models, calculation of the parameters of the proposed models, and the evaluation of the models.

2.1 Data generation

The data set for each model consists of a two dimensional grid of 625 data points, created from twenty-five pipe diameter (D) values selected in equal

increments on the interval of 0.1 m to 1.2 m and twenty-five average velocity (V) values selected in equal increments on the interval of 0.5 m/s to 3.1 m/s. The target friction values for the 625 points are values obtained using the values Colebook–White formula (Eq. 3). The friction values, calculated iteratively using (Eq. 3), were considered to have converged when the ratio of the difference in values obtained between two iterations and the value obtained from the most recent iteration was less than 10^{-8}. Later, the head loss values obtained using the Darcy-Weisbach equation (Eq. 1).

2.3 Proposed Model

The model which is proposed for calculation of head loss is:

$$h_f = \frac{aLQ^b}{D^c} \qquad (4)$$

Where, a, b and c are model parameter.

The calculation of the parameters of the proposed model has been done through non-linear multivariable regression. The optimization toolbox of the MATLAB was used to perform these steps. Obtained formulas have been given in Table (1).

Table 1. Pipe types and proposed head loss formulas

Pipe Type	Head Loss (m)
PVC ($\varepsilon = 0.0015$ mm)	$h_f = \dfrac{0.0009343 LQ^{1.8177}}{D^{4.8210}}$
Commercial Steel ($\varepsilon = 0.05$ mm)	$h_f = \dfrac{0.0010306 LQ^{1.8817}}{D^{4.9631}}$
Asphalted Cast Iron ($\varepsilon = 0.12$ mm)	$h_f = \dfrac{0.0011177 LQ^{1.9292}}{D^{5.0797}}$
Galvanized Iron ($\varepsilon = 0.15$ mm)	$h_f = \dfrac{0.0011500 LQ^{1.9392}}{D^{5.1050}}$
Cast Iron ($\varepsilon = 0.26$ mm)	$h_f = \dfrac{0.0012511 LQ^{1.9578}}{D^{5.1545}}$
Concrete ($\varepsilon = 0.5$ mm)	$h_f = \dfrac{0.0014100 LQ^{1.9740}}{D^{5.2050}}$

2.2 Performance Evaluation of the models

Several statistical measures are available for evaluating the performance of a model. These include correlation coefficient, relative error, standard error (Dowdy, 1982), among others. Correlation coefficient and relative error are the most frequently used criteria and were employed in this study. Using the obtained formulas (Table 1) and Eq. (3) and Eq. (2), for diameter of 0.1 m -1.2 m, mean velocity 0.5 m/s – 3.1

m/s and pipeline length 1000 m, head losses for each pipe type were calculated respectively. The desired head loss versus calculated head loss plotted for each pipe type separately (Figure 1 through Figure 6). Each model's correlation coefficient is $R \cong 1$. Maximum relative error of each model is less than 2 %. It can be seen from the figures that the fitness of desired and calculated head loss of each pipe type is perfect.

3. Conclusions

In this work, non-linear multivariable regression method was used to obtain simple and explicit formulas for head loss, h_f, due to friction in the PVC, commercial steel, asphalted cast iron, galvanized iron, cast iron and concrete pipes which are commonly used in pipelines and water distribution systems. Correlation coefficient of each model is $\cong 1$ and maximum relative error of each model is less than 2 %. Therefore, these simple equations can offer significant advantage in optimization studies and hydraulic analysis of pipelines and water distribution systems.

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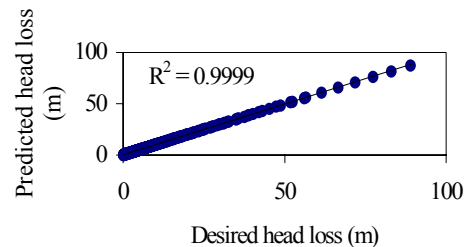[8] White FM. Fluid Mechanics, Singapore, McGraw-Hill Book Co., 1994:736.

Figure 1. Predicted versus desired values of the head loss for PVC pipes.

Figure 2. Predicted versus desired values of the head loss for commercial steel pipes.
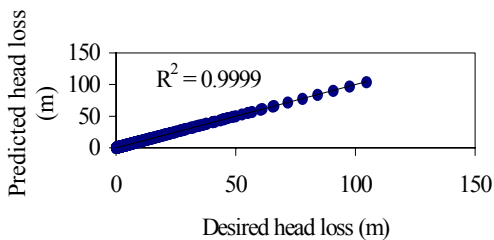

Figure 3. Predicted versus desired values of the head loss for asphalted cast iron pipes.
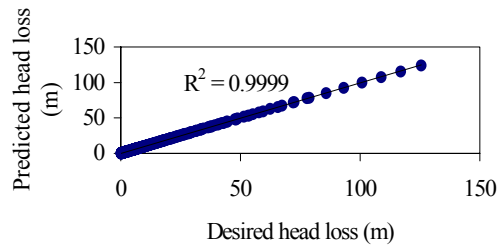

Figure 4. Predicted versus desired values of the head loss for galvanized iron pipes.
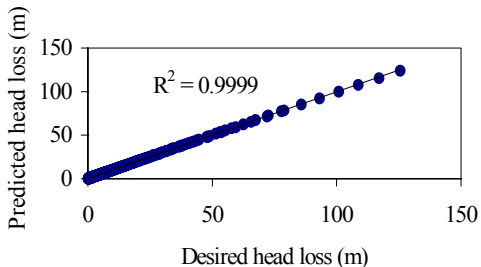

Figure 5. Predicted versus desired values of the head loss for cast iron pipes.
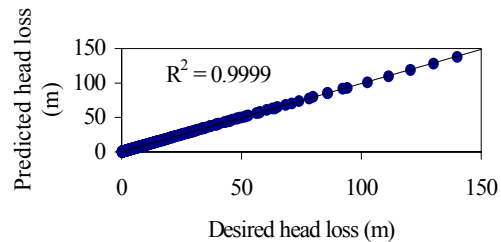

Figure 6. Predicted versus desired values of the head loss for concrete pipes