

# Performance of an endoreversible Atkinson cycle with variable specific heat ratio of working fluid

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**Abstract:** The performance of an air standard Atkinson cycle is analyzed using finite-time thermodynamics. In the endoreversible cycle model, the linear relation between the specific heat ratio of the working fluid and its temperature, and the heat transfer loss are considered. The relations between the net work output, the thermal efficiency, and the compression ratio are indicated by numerical examples. Moreover, the effects of variable specific heats of the working fluid on the endoreversible cycle performance are analyzed. The results show that the effect of the temperature dependent specific heat of the working fluid on the endoreversible cycle performance is significant. The conclusions of this investigation are of importance when considering the designs of actual Atkinson engines. [Journal of American Science 2010;6(2):12-17]. (ISSN: 1545-1003).

**Key words:** Atkinson heat-engine; Finite-time processes; Heat loss; Performance; Thermodynamics

## 1. Introduction

Recently, the analysis and optimization of thermodynamic cycles for different optimization objectives has made tremendous progress by using finite-time thermodynamics (Aizenbud and Band, 1982; Bejan, 1996; Chen et al., 1999; Wu et al., 1999; Chen and Sun 2004; Aragon-Gonzalez et al., 2006). Leff (1987) determined the thermal efficiency of a reversible Atkinson-engine cycle at maximum work output. A power density maximization of a reversible Atkinson cycle has been performed by Chen et al. (1998a). Their results showed that the efficiency at maximum power density is always greater than that at maximum power, and the design parameters at maximum power density lead to smaller and more efficient Atkinson engines with larger pressure ratios. Al-Sarkhi et al. (2002) compared the performance characteristic curves of the Atkinson cycle with those of the Miller and Joule–Brayton cycles by using numerical examples, and outlined the effect of maximizing power density on the performance of the cycle efficiency. Qin et al. (2003) derived the performance characteristics of a universal generalized cycle model, which included the Atkinson cycle, with heat-transfer loss. Wang and Hou (2005) studied the performance analysis and comparison of an Atkinson cycle coupled to variable temperature heat reservoirs under maximum work and maximum power density conditions, assuming a constant specific heat, too. Their results showed an engine design based on maximum power density is better than that based on maximum work conditions, from the view points of engine size

and thermal efficiency. Ge et al. (2005a) derived the performance characteristics of a universal generalized cycle model, which included the Atkinson cycle with heat transfer and friction-like term losses. Zhao and Chen (2006) performed analysis and parametric optimum criteria of an irreversible Atkinson heat engine using finite time thermodynamics. Performance analysis of an Atkinson cycle with heat transfer, friction and variable specific heats of the working fluid was studied by Ge et al. (2006). Their results showed that the effects of variable specific heats of working fluid and friction-like term losses on the irreversible cycle performance are significant. Ge et al. (2007) analyzed the effects of the heat transfer and variable specific heats of working fluid on the performance of an endoreversible Atkinson cycle. Hou (2007) compared the performances of air standard Atkinson and Otto cycles with heat transfer loss considerations. Lin and Hou (2007) investigated the effects of heat loss, as characterized by a percentage of fuel's energy, friction and variable specific heats of the working fluid, on the performance of an air standard Atkinson cycle under the restriction of the maximum cycle-temperature. Chen et al. (2007) built a class of generalized irreversible universal steady flow heat engine cycle model consisting of two heating branches, two cooling branches, and two adiabatic branches with consideration of the losses of heat resistance, heat leakage, and internal irreversibility. The performance characteristics of Diesel, Otto, Brayton, Atkinson, Dual and Miller cycles were derived. Chen et al. (2008) analyzed and

compared the performance characteristics of endoreversible and irreversible reciprocating Diesel, Otto, Atkinson, Brayton, Braysson, Carnot, dual, and Miller cycles with constant and variable specific heats of the working fluid. Thermodynamic analysis of an ideal air-standard Atkinson cycle with temperature dependant specific heat is presented by Al-Sarkhi et al. (2008). This paper outlines the effect of maximizing power density on the performance of the cycle efficiency. Ge et al. (2008a, 2008b) analyzed the performance of an air standard Otto and Diesel cycles. In the irreversible cycle model, the non-linear relation between the specific heat of the working fluid and its temperature, the friction loss computed according to the mean velocity of the piston, the internal irreversibility described by using the compression and expansion efficiencies, and the heat transfer loss are considered. Ust (2009) made a comparative performance analysis and optimization of irreversible Atkinson cycle under maximum power density and maximum power conditions.

All of the above mentioned research, the specific heats at constant pressure and volume of working fluid are assumed to be constants or functions of temperature alone and have the linear and or the non-linear forms. But when calculating the chemical heat released in combustion at each instant of time for internal combustion engine, the specific heat ratio is generally modeled as a linear function of mean charge temperature (Gatowski et al., 1984; Ebrahimi, 2006). The model has been widely used and the phenomena that it takes into account are well known (Klein, 2004). However, since the specific heat ratio has a great influence on the heat release peak and on the shape of the heat release curve (Brunt, 1998), many researchers have elaborated different mathematical equations to describe the dependence of specific heat ratio from temperature (Gatowski et al., 1984; Brunt, 1998; Egnell, 1998; Klein, 2004; Klein and Erikson, 2004; Ceviz and Kaymaz, 2005). It should be mentioned here that the most important thermodynamic property used in the heat release calculations for engines is the specific heat ratio (Ceviz and Kaymaz, 2005). Therefore, the objective of this study is to examine the effect of variable specific heat ratio on the net work output and the thermal efficiency of air standard Atkinson cycle.

## 2. Cycle model

The Atkinson cycle engine is a type of internal

combustion engine, which was designed and built by James Atkinson in 1882 (Ge et al., 2005a). The Atkinson cycle, one of the most heat-efficient, high-expansion ratio cycles, is designed to provide efficiency at the expense of power. The Atkinson cycle allows the intake, compression, power, and exhaust strokes of the four-stroke cycle to occur in a single turn of the crankshaft. By the use of clever mechanical linkages, the expansion ratio is greater than the compression ratio, resulting in greater efficiency than with engines using the alternative Otto cycle. The cycle for this engine is depicted in figure 1. The cycle is also called the Sargent cycle by several physics oriented thermodynamic books (Ge et al. 2007).

Figure 1 presents pressure-volume ( $P-V$ ) and temperature-entropy ( $T-S$ ) diagrams for the thermodynamic processes performed by an air standard Atkinson cycle. Process ( $1 \rightarrow 2$ ) is an adiabatic (isentropic) compression; process ( $2 \rightarrow 3$ ) is a heat addition at a constant volume; process ( $3 \rightarrow 4$ ) adiabatic (isentropic) expansion; process ( $4 \rightarrow 1$ ) is heat rejection at a constant pressure.

As already mentioned in the previous section, it can be supposed that the specific heat ratio of the working fluid is function of temperature alone and has the linear forms:

$$\gamma = \gamma_0 - k_1 T \quad (1)$$

where  $\gamma$  is the specific heat ratio and  $T$  is the absolute temperature.  $\gamma_0$  and  $k_1$  are constants.

The heat added to the working fluid, during processes ( $2 \rightarrow 3$ ) is

$$Q_{in} = M \left( \int_{T_2}^{T_3} c_v dT \right) = M \int_{T_2}^{T_3} \left( \frac{R}{\gamma_0 - k_1 T - 1} \right) dT = \frac{MR}{k_1} \ln \left( \frac{\gamma_0 - k_1 T_2 - 1}{\gamma_0 - k_1 T_3 - 1} \right) \quad (2)$$

where  $M$  is the molar number of the working fluid.  $R$  and  $c_p$  are molar gas constant and molar specific heat at constant volume for the working fluid, respectively.

The heat rejected in the isobaric heat rejection process ( $4 \rightarrow 1$ ) may be written as

$$Q_{out} = M \int_{T_1}^{T_4} c_p dT = M \int_{T_1}^{T_4} \left( \frac{(\gamma_0 - k_1 T) R}{\gamma_0 - k_1 T - 1} \right) dT = MR \left[ T_4 - T_1 + \frac{1}{k_1} \ln \left( \frac{\gamma_0 - k_1 T_1 - 1}{\gamma_0 - k_1 T_4 - 1} \right) \right] \quad (3)$$

where  $c_p$  is the molar specific heat at constant

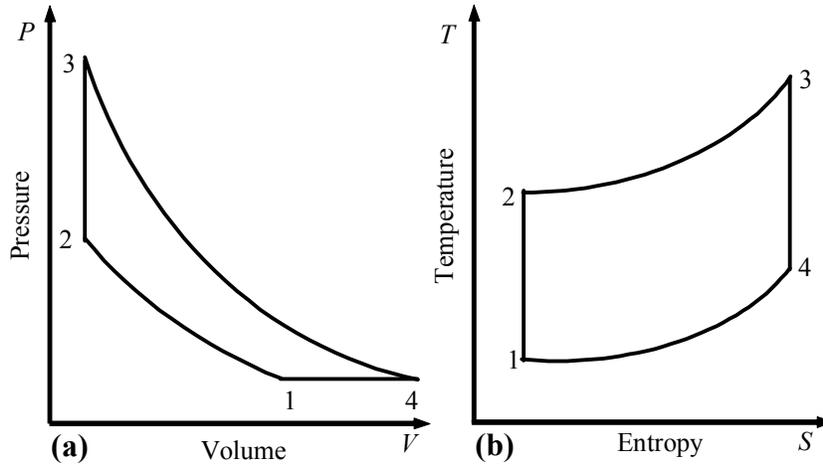


Figure 1. (a)  $P-V$  diagram; (b)  $T-S$  diagram for the air standard Atkinson cycle

pressure for the working fluid.

According to references (Ge et al., 2006; Ebrahimi 2009), the equation for a reversible adiabatic process with variable specific heat ratio can be writing as follows:

$$TV^{\gamma-1} = (T + dT)(V + dV)^{\gamma-1} \quad (4)$$

Re-arranging equations (1) and (4), we get the following equation

$$T_i(\gamma - k_1 T_j - 1) = T_j(\gamma - k_1 T_i - 1) \left( \frac{V_j}{V_i} \right)^{\gamma-1} \quad (5)$$

The specific compression,  $r_c$ , and compression,  $r_c^*$ , ratios are defined as

$$r_c = V_1/V_2 \quad (6)$$

and

$$r_c^* = \frac{V_4}{V_2} = \frac{T_4}{T_1} r_c \quad (7)$$

Therefore, the equations for processes (1 → 2) and (3 → 4) are shown, respectively, by the following:

$$T_1(\gamma - k_1 T_2 - 1)(r_c)^{\gamma-1} = T_2(\gamma - k_1 T_1 - 1) \quad (8)$$

$$T_3(\gamma - k_1 T_4 - 1) = T_4(\gamma - k_1 T_3 - 1) \left( \frac{T_4}{T_1} r_c \right)^{\gamma-1} \quad (9)$$

The energy transferred to the working fluid during combustion is given by the following linear relation (Chen et al. 1998b; Ge et al., 2008a).

$$Q_{in} = M [A - B(T_2 + T_3)] \quad (10)$$

where  $A$  and  $B$  are two constants related to combustion and heat transfer which are function of engine speed. From equation (10), it can be seen that  $Q_{in}$  contained two parts: the first part is  $MA$ , the

released heat by combustion per second, and the second part is the heat leak loss per second,  $Q_{leak} = MB(T_2 + T_3)$ .

From equations (2) and (3), the net work output of the Atkinson cycle engine is given by:

$$W_{out} = Q_{in} - Q_{out} = \frac{MR}{k_1} \ln \left( \frac{(\gamma - k_1 T_2 - 1)(\gamma - k_1 T_5 - 1)}{(\gamma - k_1 T_4 - 1)(\gamma - k_1 T_1 - 1)} \right) + MR(T_4 - T_3) \quad (11)$$

The thermal efficiency of the Atkinson cycle engine is expressed by:

$$\eta_{th} = \frac{\frac{1}{k_1} \ln \left( \frac{(\gamma - k_1 T_2 - 1)(\gamma - k_1 T_5 - 1)}{(\gamma - k_1 T_4 - 1)(\gamma - k_1 T_1 - 1)} \right) + T_4 - T_3}{\frac{1}{k_1} \ln \left( \frac{\gamma - k_1 T_2 - 1}{\gamma - k_1 T_4 - 1} \right) + T_4 - T_3} \quad (12)$$

When the values of  $r_c$  and  $T_1$  are given,  $T_2$  can be obtained from equation (8), then, substituting equation (2) into equation (10) yields  $T_3$ . The last unknown is  $T_4$ , which can be deduced from equation (9). Finally, by substituting  $T_1$ ,  $T_2$ ,  $T_3$  and  $T_4$  into equations (11) and (12), respectively, the net work output and thermal efficiency of the Atkinson cycle engine can be obtained. Therefore, the relations between the net work output, the thermal efficiency and the compression ratio can be derived.

### 3. Numerical examples and discussion

According to references (Ebrahimi, 2009, Ghatak and Chakraborty, 2007; Ge et al., 2007, Chen et al., 2006; Ge et al., 2005b) the following constants and ranges of parameters are used in the calculations:  $T_1 = 360 K$ ,  $\gamma = 1.31-1.41$ ,  $A = 60000 J.mol^{-1}$ ,

$M = 1.57 \times 10^{-5} \text{ kmol}$ ,  $k_1 = 0.00003 - 0.00009 \text{ K}^{-1}$  and  $B = 28 \text{ J.mol}^{-1} \text{ K}^{-1}$ . Numerical examples are shown as follows.

Figures 2-5 show the effect of the parameters  $\gamma_0$  and  $k_1$  related to the variable specific heat ratio of the working fluid on the Atkinson cycle performance with considerations of heat transfer. From these figures, it can be found that  $\gamma_0$  and  $k_1$  play a key role on the work output and the thermal efficiency. It is clearly seen that the effects of  $\gamma_0$  and  $k_1$  on the work output and thermal efficiency are related to compression ratio.

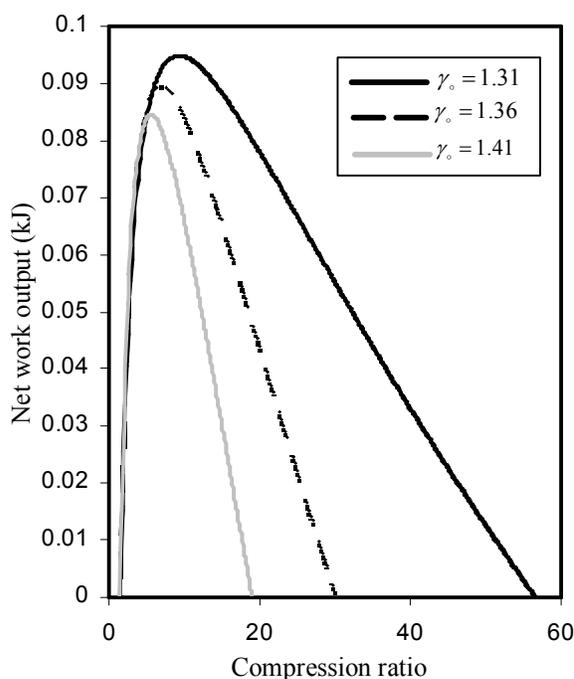


Figure 2. Effect of  $\gamma_0$  on the variation of the net work output with compression ratio ( $k_1 = 0.00006 \text{ K}^{-1}$ )

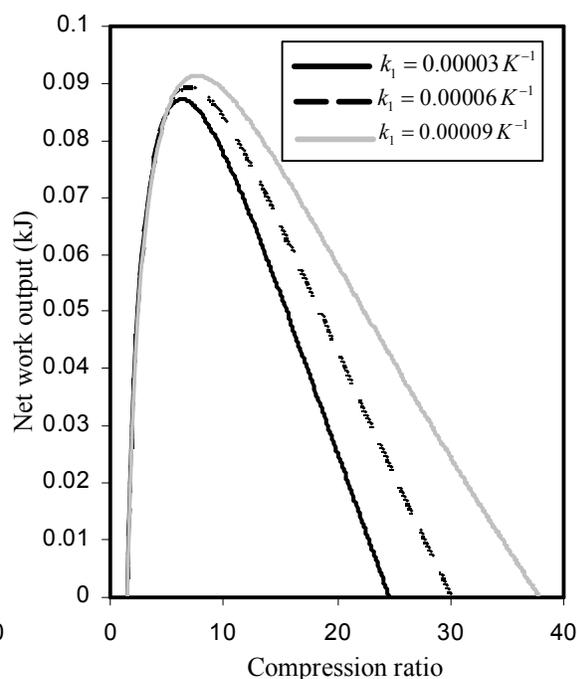


Figure 3. Effect of  $k_1$  on the variation of the net work output with compression ratio ( $\gamma_0 = 1.36$ )

The effects of  $\gamma_0$  and  $k_1$  on the net work output are shown in Figures 2 and 3. It can be found from these figures that the net work output versus compression ratio characteristic is approximately parabolic like curves. In other words, the net work output increases with increasing compression ratio, reach their maximum values and then decreases with further increase in compression ratio. It can also be found from the figures 2 and 3 that if compression ratio is less than certain value, the increase (decrease) of  $\gamma_0$  ( $k_1$ ) will make the net work output bigger, due to the increase in the ratio of the heat added to the heat rejected. In contrast, if compression ratio exceeds certain value, the increase (decrease) of  $\gamma_0$  ( $k_1$ ) will make the net work output less, because of decrease in the ratio of the heat added

They reflect the performance characteristics of an endoreversible Atkinson cycle engine. It should be noted that the heat added and the heat rejected by the working fluid decrease with increases of  $\gamma_0$ , while increase with increasing  $k_1$ . (see Eqs. (2) and (3)). It can be seen that the effect of  $\gamma_0$  is more than that of  $k_1$  on the net work output and thermal efficiency. It should be mentioned here that for a fixed  $k_1$ , a larger  $\gamma_0$  corresponds to a greater value of the specific heat ratio and for a given  $\gamma_0$ , a larger  $k_1$  corresponds to a lower value of the specific heat ratio.

to the heat rejected. One can see that the maximum net work output, the working range of the cycle and the optimal compression ratio corresponding to maximum net work output decrease (increase) about 11% (4.8%) and 66.5% (54.7%), 43.6% (25%) when  $\gamma_0$  ( $k_1$ ) increases 7.6% (200%). This is due to the fact that the ratio of heat added to heat rejected increases (decreases) with increasing  $\gamma_0$  ( $k_1$ ) in this case. It should be noted here that both the heat added and the heat rejected by the working fluid decrease with increasing  $\gamma_0$  (see Eq. (4)), and increase with increase of  $k_1$  (see Eq. (5)). The effects of  $\gamma_0$  and  $k_1$  on the thermal efficiency are shown in Figures 4 and 5. It can be found that the thermal efficiency increases with the increase of  $\gamma_0$  and the decrease of  $k_1$  throughout the compression

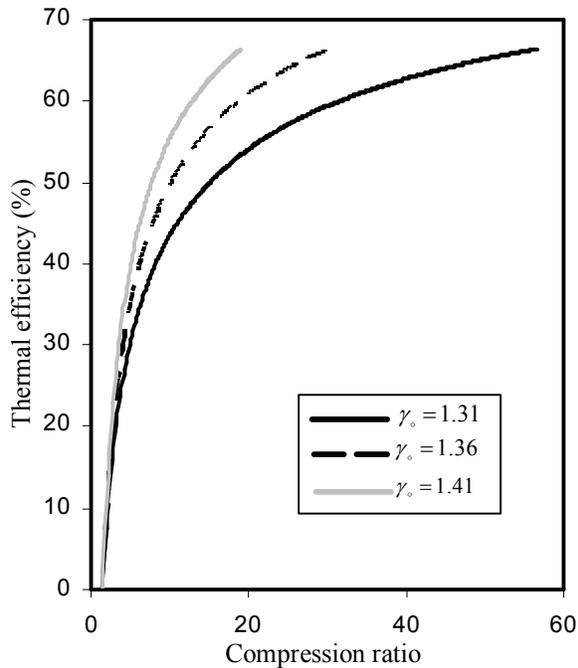


Figure 2. Effect of  $\gamma_0$  on the variation of the thermal efficiency with compression ratio ( $k_1 = 0.00006 K^{-1}$ )

ratio range. On average, the thermal efficiency increases (decreases) by about 29.7% (9.3%) when  $\gamma_0$  ( $k_1$ ) increases (increases) 7.6% (200%) over a range of compression ratios from 1.4 (1.4) to 18.9 (24.6).

#### 4. Conclusion

In this paper, an endoreversible air standard Atkinson cycle model taking considerations of heat transfer loss and the variable specific heat ratio of working fluid is presented. The relations between the net work output and the compression ratio and between the thermal efficiency and the compression ratio of the cycle are derived. The effects of the cycle parameters, such as  $\gamma_0$  and  $k_1$ , on the net work output and the efficiency were analyzed by detailed numerical examples. The results obtained may provide a theoretical basis for both the optimal design and operation of real Atkinson heat engines.

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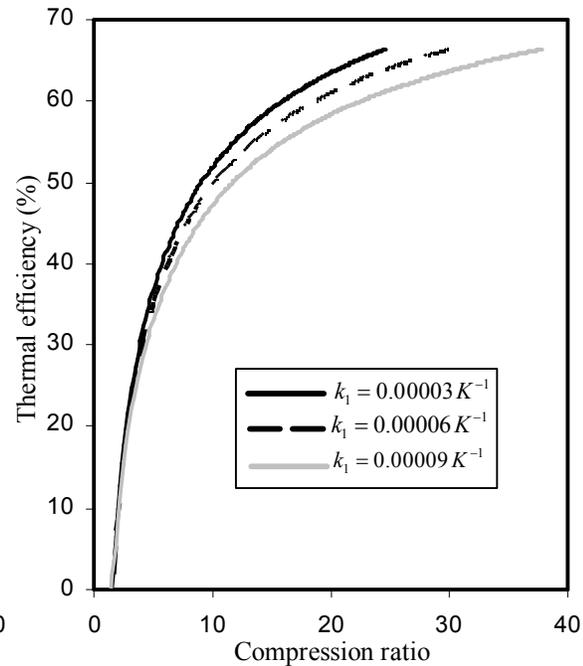


Figure 3. Effect of  $k_1$  on the variation of the thermal efficiency with compression ratio ( $\gamma_0 = 1.36$ )

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