Model for Computational Analysis of the Quantity of Water Lost by Evaporation during Oven-Drying of Clay

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Abstract: Model for computational analysis of the quantity of water lost by evaporation during oven drying of clay has been derived. The model;

\[ \beta = \exp[(\ln t)^{0.998} - 2.9206] \]

indicates that the quantity of evaporated water during the drying process is dependent on the drying time, the evaporating surface being constant. It was found that the validity of the model is rooted on the expression \((\log \alpha + \ln \beta)^{N} = \ln t\) where both sides of the expression are correspondingly almost equal. The maximum deviation of the model-predicted quantity of evaporated water from the corresponding experimental value is less than 20% which is quite within the acceptable deviation range of experimental results, hence depicting the usefulness of the model. Water evaporation rate evaluated from experimental and model-predicted results are 0.0488 and 0.0530g/min respectively, indicating proximate agreement. [Journal of American Science 2010;6(6):38-42]. (ISSN: 1545-1003).

Keywords: Model, Water, Evaporation, Oven Drying, Clay

1. Introduction

Reed (1988) described firing as having three stages through which it proceeds; preliminary reactions which include binder burnout, elimination of gaseous product of decomposition and oxidation, sintering as well as cooling which may include thermal and chemical annealing.

Several works (Barsoum, 1999; Viewey and Larry, 1978; Keey, 1978) have been carried out on shrinkage of clay during drying. In all these works, porosity has been shown to influence the swelling and shrinkage behaviour of clay products of different geometry. It has been reported (Reed, 1988) that drying occurs in three stages; increasing rate, constant and decreasing rate. He pointed out that during the increasing rate; evaporation rate is higher than evaporating surface hence more water is lost. At constant rate, the evaporation rate and evaporation surface are constant. He posited that shrinkage occurs at this stage. Keey (1978) also in a similar study suggested that at this stage, free water is removed between the particles and the inter-particle separation decreases, resulting in shrinkage. During the decreasing rate, particles make contacts as water is removed, which causes shrinkage to cease.

Model for calculating the volume shrinkage resulting from the initial air-drying of wet clay has been derived (Nwoye, 2008). The model;

\[ \theta = \gamma^3 - 3\gamma^2 + 3\gamma \]  

(1)

calculates the volume shrinkage when the value of dried shrinkage \(\gamma\), experienced during air-drying of wet clays is known. The model was found to be third-order polynomial in nature. Olokoro clay was found to have the highest shrinkage during the air drying condition, followed by Ukpor clay while Otamiri clay has the lowest shrinkage. Volume shrinkage was discovered to increase with increase in dried shrinkage until maximum volume shrinkage was reached, hence a direct relationship.

A mathematical model for evaluating internal volume shrinkage of fired clays has been derived by Nwoye (2008). The model;

\[ \beta = \alpha^3 - 3\alpha^2 + 3\alpha \]  

(2)

evaluates the volume shrinkage \(\beta\) when the value of fired shrinkage \(\alpha\), resulting from intense firing (to a temperature of 1200°C). The model was found to be third-order polynomial in nature.

Nwoye et al. (2008) derived a model for the evaluation of overall volume shrinkage in molded clay products (from initial air-drying stage to completion of firing at a temperature of 1200°C). It was observed that the overall volume shrinkage values predicted by the model were in agreement with those calculated using conventional equations. The model;

\[ S_T = \alpha^2 + \gamma^2 - 3(\alpha^2 + \gamma^2) + 3(\alpha + \gamma) \]  

(3)

depends on direct values of the dried \(\gamma\) and fired shrinkage \(\alpha\) for its precision. Overall volume shrinkage was found to increase with increase in dried

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Log $\alpha + \ln \beta = (\ln t)^N$ (8)

Introducing the value of $N$ into equation (8)

Log $\alpha + \ln \beta = (\ln t)^{1.002}$ (9)

Log $\alpha + \ln \beta = (\ln t)^{0.998}$ (10)

$ln \beta = (\ln t)^{0.998} - \ln \alpha$ (11)

$\beta = \exp((\ln t)^{0.998} - \ln \alpha)$ (12)

Introducing the value of $\alpha$ into equation (12) reduces it to;

$\beta = \exp((\ln t)^{0.998} - 2.9206)$ (13)

Where

$\beta =$ Weight of water lost by evaporation during the drying process (g)

$\alpha =$ Area of evaporating surface (mm$^2$)

$N =$ 1.002; (Collapsibility coefficient of binder-clay particle boundary at the drying temperature of 80$^\circ$C) determined in the experiment (Nwoye, 2007).

$t =$ Drying time (mins.),

<table>
<thead>
<tr>
<th>(t)</th>
<th>(a)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>833</td>
<td>2.00</td>
</tr>
<tr>
<td>50</td>
<td>833</td>
<td>2.80</td>
</tr>
<tr>
<td>70</td>
<td>833</td>
<td>3.70</td>
</tr>
<tr>
<td>90</td>
<td>833</td>
<td>4.70</td>
</tr>
<tr>
<td>110</td>
<td>833</td>
<td>5.90</td>
</tr>
<tr>
<td>130</td>
<td>833</td>
<td>5.90</td>
</tr>
</tbody>
</table>

Table 1: Variation of quantity of evaporated water with drying time. (Nwoye, 2007)

3. Boundary and Initial Conditions

Consider a rectangular shaped clay product of length 49mm, width 17mm, and breadth 9mm exposed to drying in the furnace while it was in wet condition. Initially, atmospheric levels of oxygen are assumed. Atmospheric pressure was assumed to be acting on the clay samples during the drying process (since the furnace is not air-tight). The grain size of clay particles used is 425µm, weight of clay and binder (bentonite) used (for each rectangular product); 100g and 10g respectively, quantity of water used for mixing: 2% (of total weight), drying temperature used; 80$^\circ$C, area of evaporating surface; 833mm$^2$ and range of drying time used; (30-130 mins.). The boundary conditions are: atmospheric levels of oxygen at the top and bottom of the clay samples since they are dried under the atmospheric condition. No external force due to compression or tension was applied to the drying clays. The sides of the particles and the rectangular shaped clay products are taken to be symmetries.
4. Model Validation
The formulated model was validated by direct analysis and comparison of the model-predicted $\beta$ values and those from the experiment for equality or near equality. Analysis and comparison between these $\beta$ values reveal deviations of model-predicted $\beta$ from those of the experimental values. This is believed to be due to the fact that the surface properties of the clay and the physiochemical interactions between the clay and binder, which were found to have played vital role during the evaporation process (Nwoye, 2007) were not considered during the model formulation. This necessitated the introduction of correction factor, to bring the model-predicted $\beta$ value to that of the corresponding experimental value (Table 3).

Deviation (Dv) (%) of model-predicted $\beta$ values from the experimental $\beta$ values is given by

$$Dv = \left( \frac{Pw - Ew}{Ew} \right) \times 100$$  \hspace{1cm} (14)

Where $Pw$ = Quantity of water evaporated as predicted by model
$Ew$ = Quantity of water evaporated as obtained from experiment (g) (Nwoye, 2007)

Correction factor (Cf) is the negative of the deviation i.e

$$Cf = -Dv$$  \hspace{1cm} (15)

Therefore

$$Cf = -100 \left( \frac{Pw - Ew}{Ew} \right)$$  \hspace{1cm} (16)

Introduction of the value of Cf from equation (16) into the model gives exactly the corresponding experimental value of $\beta$ (Nwoye, 2007).

5. Results and Discussion
The derived model is equation (13). Computational analysis of the experimental data (Nwoye, 2007) shown in Table 1, gave rise to Table 2.

Table 2: Variation of $\ln t$ with ($\ln \alpha + \ln \beta$)$^N$

<table>
<thead>
<tr>
<th>$\ln t$</th>
<th>$\ln \alpha$</th>
<th>$\ln \beta$</th>
<th>($\ln \alpha + \ln \beta$)$^N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4012</td>
<td>2.9206</td>
<td>0.6931</td>
<td>3.6230</td>
</tr>
<tr>
<td>3.9120</td>
<td>2.9206</td>
<td>1.0296</td>
<td>3.9611</td>
</tr>
<tr>
<td>4.2485</td>
<td>2.9206</td>
<td>1.3083</td>
<td>4.2411</td>
</tr>
<tr>
<td>4.4998</td>
<td>2.9206</td>
<td>1.5476</td>
<td>4.4816</td>
</tr>
<tr>
<td>4.7005</td>
<td>2.9206</td>
<td>1.7750</td>
<td>4.7101</td>
</tr>
<tr>
<td>4.8675</td>
<td>2.9206</td>
<td>1.7750</td>
<td>4.7101</td>
</tr>
</tbody>
</table>

An ideal comparison of the quantities of water evaporated per unit rise in the drying time as obtained from experiment and as predicted by the model for the purpose of testing the validity of the model is achieved by considering the $R^2$ values. The values of the correlation coefficient, R calculated from the equation;

$$R = \sqrt{R^2}$$  \hspace{1cm} (17)

using the r-squared values (coefficient of determination) from Figures 1 and 2 which show a better correlation (1.0000) with model-predicted quantity of water evaporated per unit rise in the drying time compared to that obtained from experiment (0.9851).

This suggests that the model predicts more accurate, reliable and ideal quantity of evaporated water than the actual experiment despite its deviations from the experimental values.

Figure 1: Variation of quantity of water evaporated with drying time as obtained from experiment (Nwoye, 2007)

Figure 2: Variation of quantity of water evaporated with drying time as obtained from derived model

Figure 3: Comparison of the quantities of water evaporated as obtained from experiment (Nwoye, 2007) and derived model

Figure 3 shows that values of $E$ obtained from the experiment and those from the model are generally quite close hence depicting the reliability and validity of the model. However, Figure 3 shows that the
quantities of water evaporated per unit rise in the drying time as obtained from the experiment (Nwoye, 2007), designated by the line ExD and as predicted by the model (line MoD) are in very good agreement within a drying time range 30-130mins.

5.1 Evaporation per unit rise in drying time
Water evaporated per unit rise in drying time resulting from drying of the clay at a temperature 80°C for a time range 30-130mins. was determined following comparison of the evaporation per unit rise in drying time obtained by calculations involving experimental results, and model-predicted results obtained directly from the model.
Evaporation per unit rise in the drying time, E_t (g/min) was calculated from the equation;

\[ E_t = \frac{E}{t} \]  

(18)

Therefore, a plot of mass of water evaporated E against drying time t, as in Figure 1 using experimental results in Table 1, gives a slope, S at points (2.0, 30) and (5.9, 110) and following their substitution into the mathematical expression;

\[ S = \frac{\Delta E}{\Delta T} \]  

(19)

Eqn. (19) is detailed as

\[ S = E_2 - E_1 / T_2 - T_1 \]  

(20)

Where

\[ \Delta E = \text{Change in the quantities of water evaporated} \]  

between two drying time values \( t_2, t_1 \). Considering the points (2.0, 30) and (5.9, 110) for \( (E_1, T_1) \) and \( (E_2, T_2) \) respectively, and substituting them into eqn. (20), gives the slope as 0.0488g/min which is the quantity of water evaporated per unit rise in the drying time during the actual experimental drying process. Also similar plot (as in Figure 2) using model-predicted results gives a slope. Considering points (1.6036, 30) and (5.8436, 110) for \( (E_1, T_1) \) and \( (E_2, T_2) \) respectively and substituting them into eqn. (20) gives the value of slope, S as 0.0530g/min. This is the model-predicted quantity of water evaporated per unit rise in the drying time during the drying of the clay. A comparison of these two quantities of water evaporated per unit rise in the drying time shows proximate agreement. This indicates a very high degree of validity for the model as a reliable tool for predicting the quantity of water evaporated as well as the evaporation per unit rise in the drying time during drying of Ukpor clay at a temperature of 80°C within a time range 30-110 mins.

A comparison of the values of \( \beta \) obtained from the experiment and those from the derived model (Table 3) shows maximum deviation less than 20% which is quite within the acceptable deviation range of experimental results, hence depicting the usefulness of the model. It was found that the validity of the model is rooted in the expression \((\log \alpha + \ln \beta)^N\) where both sides of the equation are correspondingly almost equal. Table 2 also agrees with equation (7) following the values of \((\log \alpha + \ln \beta)^N\) and int evaluated from Table 1 as a result of corresponding computational analysis.

### Table 3: Variation of model-predicted quantities of evaporated water with the associated deviation and correction factor

<table>
<thead>
<tr>
<th>( \beta_M )</th>
<th>( \Delta ) E</th>
<th>( \Delta ) Cf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6036</td>
<td>-19.82</td>
<td>+19.82</td>
</tr>
<tr>
<td>2.6665</td>
<td>-4.77</td>
<td>+4.77</td>
</tr>
<tr>
<td>3.7271</td>
<td>+0.73</td>
<td>-0.73</td>
</tr>
<tr>
<td>4.7860</td>
<td>+1.83</td>
<td>-1.83</td>
</tr>
<tr>
<td>5.8436</td>
<td>-0.96</td>
<td>+0.96</td>
</tr>
<tr>
<td>6.9002</td>
<td>+16.95</td>
<td>-16.95</td>
</tr>
</tbody>
</table>

\( \beta_M = \text{Weight of water evaporated as predicted by the derived model} \)

### 6. Conclusion

The model computes the quantity of water lost by evaporation during oven-drying of Ukpor (Nigeria) clay at 80°C within the time range: 30-130 mins. It was found that the validity of the model is rooted in the expression \((\log \alpha + \ln \beta)^N\) where both sides of the expression are correspondingly almost equal. The maximum deviation of the model-predicted quantity of evaporated water from the corresponding experimental value is less than 20% which is quite within the acceptable deviation range of experimental results.

Further works should incorporate more process parameters into the model with the aim of reducing the deviations of the model-predicted \( \beta \) values from those of the experimental.

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### References


