Fault Determination Using One Dimensional Wavelet Analysis

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Abstract: Faults play an important role in mineral exploration and volcanic activities. Their identification, a major problem in the world of geosciences, is significant to both geologists and geophysicists. Multiscale wavelet analysis, a powerful tool for filtering and denoising, has been applied to solve many problems in geophysics. Wavelet transforms have advantages to traditional Fourier methods in analyzing physical situations where the signal contains discontinued and sharp spikes. In this paper we advance the use of one dimensional multiscale wavelet for the identification of faults from potential field data. The method is based on the power of the discrete wavelet utilizing the concept of breakline and discontinuity (edge detection) and uses the Daubachies wavelet. The method is applied to synthetic data and real potential field data from Dagang, southern China yielding very good results. [Journal of American Science 2010;6(7):177-182]. (ISSN: 1545-1003).

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1. Introduction

Faults and fractures are essential in the interpretation of potential field data. The identification and mapping of shallow and deep faults are necessary in the petroleum industry, earthquake detection and ground water displacement.

Exploration for a wide range of mineral deposits is critically dependent on knowledge of the location and age of fractures and faults. Oil and gas fields in many sedimentary basins are distributed along fault-controlled linear trends (Lyatsky, H. 2004), and fault identification is often used effectively for target-area selection in hydrocarbon exploration. Similarly, mineral deposits in various geologic settings are commonly associated with fluid-conducting faults.

Generally, the main features used to discriminate faults according to gravity or magnetic anomalies are linear gradient trends, boundaries of anomalies, an anomalous linear transitional belt, a linear anomaly belt, and so forth. Therefore, linear trends and their parametric information play an important role in gravity and magnetic-data interpretation. However, owing to geologic- or geophysical conditions, these linear features are not always apparent in images. Thus, we sometimes need to apply methods of enhancing lineaments to aid geologic interpretation. The commonly used methods are anomaly separation (continuation or filtering), correlative analysis, and directional derivatives.

Traditional methods, upward continuation and derivatives, of fault inference rely on the experience of the interpreter, the different nature of the anomaly, dramatic changes with the positive and negative anomaly; abnormal intensity of the adjacent line mutation, abnormalities with the level of dislocation axis and so forth.

These methods are prone to uncertainties and are often unreliable. In downward continuation abnormality is often flatten, information greatly reduced, and there is blurriness in characteristics resulting to inference difficulty.

To mitigate these uncertainties and making it simple for both the experience and inexperience interpreters, we are proposing the use of one dimensional discrete wavelet analysis for the inference of faults based on potential field data.

2.0 Method

2.1 Multiscale Wavelet

Wavelets are mathematical functions which split data into different frequency components and then each component is studied with a resolution to match its scale. Wavelet transforms have advantages to traditional Fourier methods in analyzing physical situations where the signal contains discontinued and sharp spikes. Wavelets were developed independently in the fields of mathematics, physics, electrica1 engineering and geophysics. Interchanges between these fields during the last ten years have led to many new wavelet application such as image compression, turbulence, human vision, radar and earthquake prediction,. The application of wavelet to potential field (gravity and magnetic) over the years have gained favorable results where exploration is concern.

We consider the space $L^2(R)$ of measurable functions $f(x)$, defined on the real line R that satisfies

$$f(x) \in L^2(R) \Rightarrow \int_{-\infty}^{\infty} |f(x)|^2 dx < \infty \qquad (1)$$

The continuous wavelet transform *(cwt)* of $f(x)$ is given as,

$$cwt_{(a,b)} = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(x) \psi\left(\frac{a-b}{a}\right) dx \qquad (2)$$

Where ψ is called the mother wavelet. The variables a and b are integers that scale and dilate the mother wavelet (ψ). The scale index a indicates the wavelet's width, and the location index b gives its position. Setting $a = 2^{-j}$ and $b = a.k$ in Eq.(2), the discrete wavelet transform *(DWT)* is written as,

$$DWT_{j,k}(a,b) = \sum\sum f(x) 2^{-j/2} \psi(2^{-j} x - k) \qquad (3)$$

The dilations and translations are chosen based on power two, so-called dyadic scales and positions, which make the analysis efficient and accurate.

Mallat (1989) introduced an efficient algorithm to the DWT known as the Multi-Resolution Analysis (MRA). Multi-scale representation of signal $f(x)$ may be achieved in different scales of the frequency domain by means of an orthogonal family of functions $\phi(x)$. In MRA, $L^2(R)$ is nested subspaces:

$$A_j \ldots \subset A_{-2} \subset A_{-1} \subset A_0 \subset A_1 \subset A_2 \subset \ldots \qquad (4)$$

such that the closure of their union is $L^2(R)$,

$$\bigcup_{j=-\infty}^{\infty} A_j = L^2(R) \qquad (5)$$

and their intersection contains only the zero function

$$\bigcap_{j=-\infty}^{\infty} A_j = \{0\} \qquad (6)$$

Considering the dyadic case, when each subspace A_j is twice as large as A_{j+l}, a function $f(x)$ that belongs to one of the subspaces A_j has the following properties:

$f(x)$ member of A_j $\Leftrightarrow$ dilation $f(2x)$ member of A_{j+1} (7)

$f(x)$ member of A_0 $\Leftrightarrow$ translation $f(x + 1)$ member of A_0 (8)

If we can find a function $\phi(x) \in Ao$ such that the set of functions consisting of $\phi(x)$ and its integer translates then,

$$\{\phi(x-k)\}_{k \text{ member of } z} \qquad (9)$$

form a basis for the space Ao, we call it a scaling function. For the other subspaces A_j (with $j\neq 0$) we define

$$\phi(x) = 2^{-j/2} \phi(2^{-j} x - k);\ j, k \text{ member of } z \qquad (10)$$

We can express $\phi(x)$ and $\psi(x)$ in terms of basis functions of Aj

$$\phi(x) = 2 \sum_{k=0}^{n=1} h_k \phi(2x - k) \qquad (11)$$

$$\psi(x) = 2 \sum_{k}^{n=1} g_k \phi(2x - k) \qquad (12)$$

Due to the multi-resolution analysis, these relations are also valid between $Aj+1$, Aj and Dj for arbitrary j. h_k and g_k are called filter coefficients that uniquely define the scaling function $\phi(x)$ and the wavelet $\psi(x)$.

2.2 Multiscale Edge Detection via Wavelet Transform

Potential field theory is suitable for multiscale wavelet analysis. In both gravity and magnetic data most of the information is contained in irregularities in the analytical signal which in turn, map the boundaries of contrasting properties of subsurface rocks (contacts, faults, etc).. Multiscale edge mapping is an automatic process of picking edges in potential field data at a variety of different scales (Hornby. P., Boschetti, F. and Horowitz, G. F., 1999., Archibald, N. J., Gow, P. and Boschetti, F., 1999).

Many traditional potential field operations have an elegant and compact expression in wavelet domain, the most obvious being the equivalence between the concept of change of scale and upward continuation. This results in a mathematically common and rigorous framework for different potential field applications. The following authors have dealt extensively with multiscale and its application to edge diction; Blakely and Simpson (1986) for modeling. Hornby *et al* (1999) for a formal presentation of the underlying theory, to Archibald *et al.* (1999) for the application of multiscale edges to the enhanced visual interpretation of potential field data, to Boschetti, F., Moresi, L. and Covil, K., (1999) for applications to feature-based signal processing, Boschetti, F., Hornby, P. and Horowitz, F., (2000) for application to mathematical inversion, and Martelet. G., Sailhac. P., Moreau., F. and Diament. M., (2001) for application of 1D DWT to boundaries characterization.

2.3 Spectral Analysis

Spectral analysis of potential field data has been used extensively over the years to derive depth to certain geological features, such as magnetic basement (Spector &

Grant 1970, Hahn & Mishra 1976, Connard *et al*, 1983, Garcia and Ness 1994) or the curie-temperature isotherm (Shuey *et al.* 1997, Blakely 1998, Okubo and Matsunaga 1994). Spector and Grant (1970) provided a foundation for these techniques. A matched filter seeks to deconvolve the signal from one such source using parameters determined from the observations. Matched filtering is a Fourier transform filtering method which uses model parameters determined from the natural log of the power spectrum in designing the appropriate filters. This method uses the power spectrum graph of a magnetic grid in the spectral domain. Magnetic and gravity sources at similar depths show straight-line segments.

This method assumes that you can summarize the power spectrum in terms of two straight line segments, characterizing the regional and shallow sources. The slope (fig. 1) of a line segment indicates the depth of the sources that it characterizes. The intercept with the vertical axis is an indication of the intensity of the source at that depth.

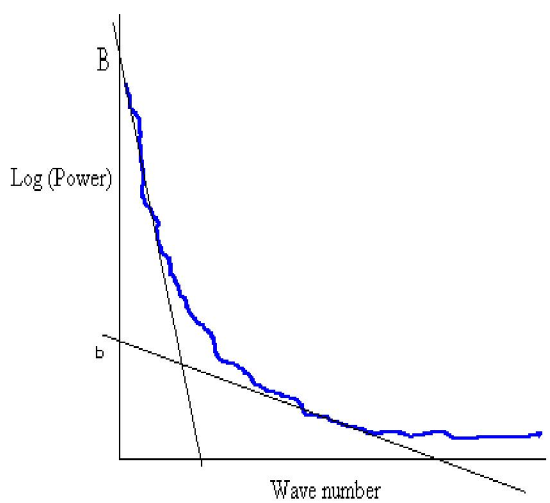


Figure 1: Spectral analysis separation filter

Where B is the Y intercept of the line segment representing the regional sources; b is the Y intercept of the line segment representing the shallow sources; H is the slope of the line segment representing the regional sources; h is the slope of the line segment representing the shallow sources.

3.0 Results

This research is directed at locating deep faults from potential field data employing the discrete wavelet to model faults and actual fiend data obtaining great result as seen below.

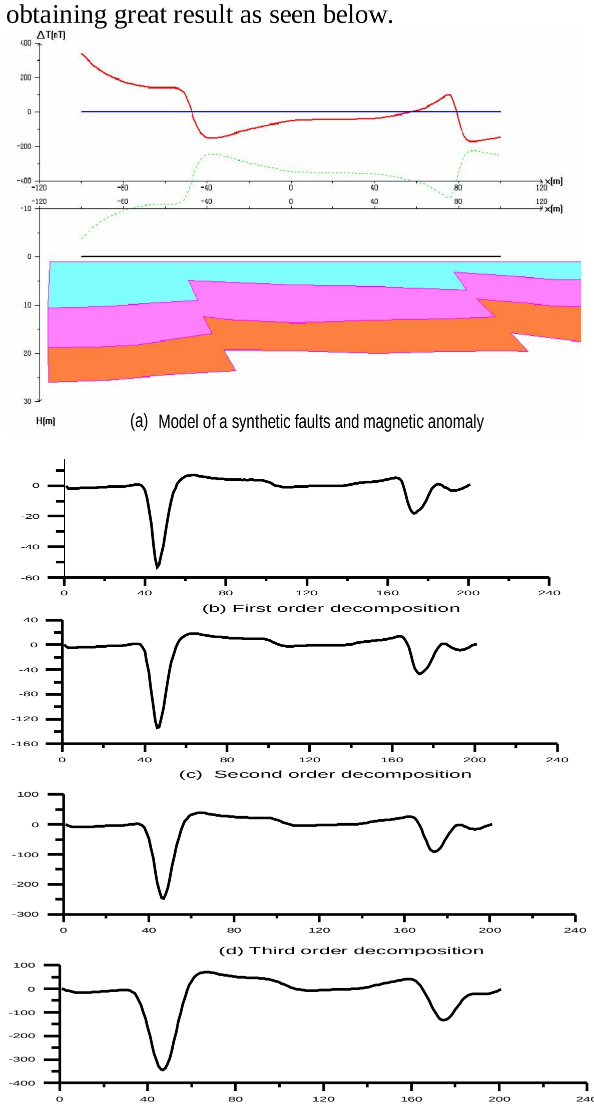
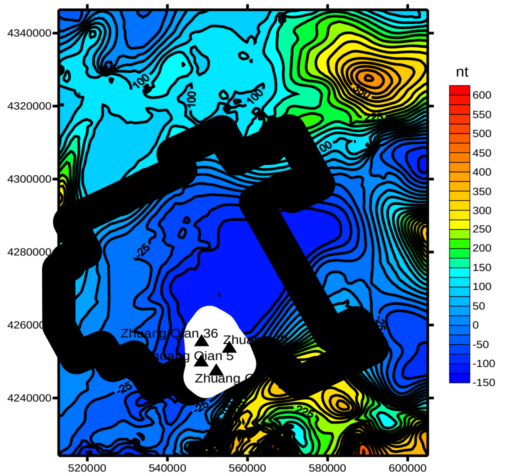


Figure 2: Model fault and decomposition result (a) model showing three strata with magnetic anomaly (b) first order detail (C) second order detail (d) third order detail (e) fourth order detail

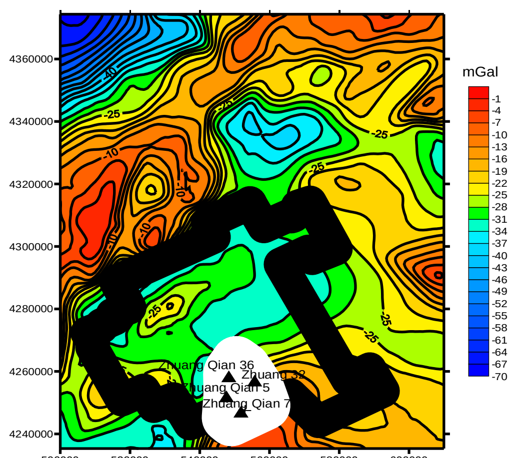
From the decomposition of the model one clearly sees where faults are located as shown in the four details (fig. 2). However, it is very sharp in the first and second levels pointing out exact positions but the third and fourth do not. Points of relative inflection or change in the trend are also noticed in the first detail. Power spectral analysis shows depths estimate of the details be to be 0.1km, 0.17km, 0.25km, 0.4km and 0.6km for the fist to the fourth details respectively.

Multiscale edge analysis indicates that the location and amplitude of edges contain the same information as the original profile. Accordingly, information about causative sources can be obtained by analysing the multiscale edge.

The total magnetic and bourger gravity maps show Dagang oilfield indicated with black lines and the research area is denoted by white lines as shown in figure 3. The oil field is replete with complex fault system that varies from north to south. Our research area, around the Qikou depression is characterized with diabase related reservoir overflow volcanic rocks.



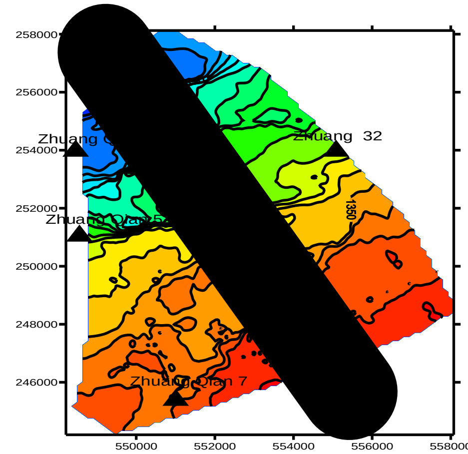
(a) magnetic anomaly map



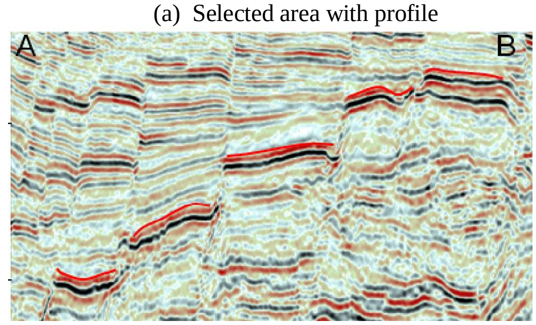
(b) Bouger gravity map

Figure 3: Magnetic anomaly (top) and bourger gravity (bottom) maps of the Dagang area with area of research marked white and few well positions

A profile (fig. 4) across our area of interest is taken and decomposed via daubachies wavelet (db2) at level 4 and compared to a seismic line of the same area.



(a) Selected area with profile



(b) Seismic cross section of the profile

Figure 4: (a) Selected area of interest and (b) seismic line matching the location indication location of faults along the profile AB. A whole body of igneous rock is separated by four faults.

The results of the decomposition (fig.5 and 6) on both gravity and magnetic data respectively show the location of breakpoints indicating faults as depicted in the seismic section. The troughs indicate a brought change in the density of the strata while the peaks signify the midpoint of a given finite mass. The higher the level of decomposition , the less apparent the break point or fault position since faults are associated with high frequency, thus a good result may be obtain at lower levels. The apparent depths of the faults using spectral analysis range from 200m to 1km.

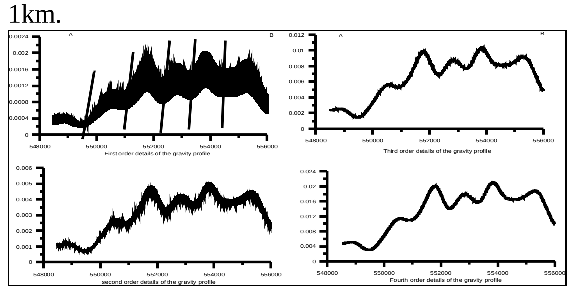


Figure 5: Decomposition of gravity profile of our area of interest (a) first order details with position of faults (b)second order details (c)third order details and (d) fourth order details. Note: the vertical lines are drawn to indicate positions of the faults.

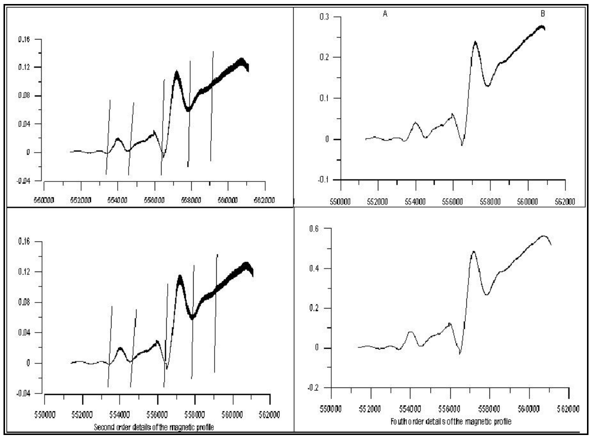


Figure 6: Magnetic profile decomposition (a) first order details with faults locations (b) second order details with faults locations(c) third order details (d) fourth order details. Note: The vertical lines indicate the positions of the faults.

The second, third and fourth levels of the magnetic decomposition clearly indicate points of interest showing changes in susceptibility and break points or edges. The positions of the faults are indicated by troughs/minima as with those of the gravity decomposition. Source discontinuity or dis-homogeneity are marked by minima or decay in the coefficient of the details (Moreau, F. Gilbert, M. Holschneider and Saracco, G. 1999) However, the first order detail is not too distinct due in part to the very small difference in magnetic susceptibility and noise. The amplitudes and change in the gravity decomposition is an indication of sharp density contrast between each section and their separation in terms of depth difference

4.0 Discussion

The multiresolution analysis is obtained by applying the inverse wavelet transform to the coefficients of each level. Note that the multiresolution is not translation invariant; a shifted version of the same signal can give different multiresolution representations. The choice of the type of wavelet is very important in the processing of data. We use the Haar or the Daubachies wavelets in this research. This particular family of wavelet is suitable for the processing and detection of break points and edges, as the processing scales are linked to the speed of the change.

The deterministic part of the signal may undergo abrupt changes such as a jump, or a sharp change in the first or second derivative. In image processing, one of the major problems is edge detection, which also involves detecting abrupt changes. Also in this category, we find signals with very rapid evolutions such as transient signals in dynamic systems. The main characteristic of these phenomena is that the change is localized in time or in space. This is shown in the decomposition of the potential field data. The first and second order details localized points of discontinuity as expected when dealing with discrete wavelet. The results indicate that faults and edges can easily be identified and localized. Where potential field is concerned the density contrast is evident in the amplitudes generated as a consequence of the decomposition.

Noise reduction (Leblanc and Morris, 2001, J.C.S. de Oliveira Lyrio, L. Tenorio and Y. Li, 2004) which is an important attribute of the DWT is also shown in the decomposed signals. The first two details in both the gravity and magnetic results (fig 5 & 6) clearly manifest same. There are marked differences and noise is greatly reduced from first to fourth orders..

It should be noted that short wavelets are often more effective than long ones in detecting a signal rupture. The shapes of discontinuities that can be identified by the smallest wavelets are simpler than those that can be identified by the longest wavelets. The presence of noise, which is after all a fairly common situation in signal processing, makes identification of discontinuities more complicated. If the first levels of the decomposition can be used to eliminate a large part of the noise, the rupture is sometimes visible at deeper levels in the decomposition.

5.0 Conclusion

We have shown that one dimensional discrete wavelet analysis can be effectively applied to infer faults from potential field data. This was demonstrated using synthetic fault models and actual field data with very good results obtained. The method is easy to use, pointing out hidden faults and reducing the propensity for errors in potential field interpretation. Their depth estimates using power spectral range from 200m to 1km from the shallowest to the deepest. We are of the strong conviction that this method will be useful at all levels of interpretations. However, one should be careful when selecting the levels of decomposition and the type of wavelet family and note that the multiresolution analysis is obtained by applying the inverse wavelet transform to the coefficients of each level.

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