Effects of Chemical Reaction and Heat Radiation on the MHD Flow of Viscoelastic Fluid through a Porous Medium Over a Horizontal Stretching Flat Plate

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Abstract: This paper presents a study of the Magnetohydrodynamic flow of non-Newtonian incompressible fluid obeying (Walters' liquid) model with mass and heat transfer over an infinite porous horizontal stretching sheet under radiation, heat generation (absorption) and chemical reaction. The governing differential equations which describe the motion of the problem are converted into dimensionless formulas by using a similarity transformation method and solved analytically by using The Kummer's function. The parameters of viscoelastic dissipation, internal heat generation /absorption, constant magnetic field, radiation, chemical reaction and permeability of the porous medium are included and discussed numerically in the governing equations of momentum, energy and concentration. The effects of the elasticity, porosity, heat, radiation, reaction effect and magnetic interaction parameters with Eckert, Prandtl and Schmidt numbers on the velocity, temperature (in the two cases PST and PHF) and concentration distributions have been discussed and illustrated graphically. [Journal of American Science 2010;6(9):126-136]. (ISSN: 1545-1003).

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1. Introduction

Boundary layer behavior over a moving continuous solid surface is an important type of flow occurring in a number of engineering processes. To be more specific, heat treated materials traveling between a feed roll and a wind-up roll, aerodynamic extrusion of plastic sheets, glass fiber and paper production, cooling of an infinite metallic plate in a cooling path, manufacturing of polymeric sheets are examples for practical applications of continuous moving flat surfaces. Since the pioneering work of Sakiadis [1], various aspects of the problem have been investigated by many authors. Mass transfer's analyses at the stretched sheet were enclosed in their studies by Erickson et al. [2] and relevant experimental results were reported by Tsou et al. [3] regarding several aspects for the flow and heat transfer boundary layer problems in a continuously moving sheet. Crane [4] and Gupta [5] have analyzed the stretching problem with constant surface temperature while Soundalgekar [6] investigated the Stokes problem for a viscoelastic fluid. This flow was examined by Siddappa and Khapate [7] for a special class of non-Newtonian fluids Known as second-order fluids which are viscoelastic in nature. Rajagopal et al. [8] independently examined the same flow as in Ref. [7] and obtained similarity solutions of the boundary equations numerically for the case of small viscoelastic parameter. K_1. It is shown that skin-friction decreases with increase in K_1. Dandapat and Gupta [9] examined the same problem with heat transfer. In Ref. [9], an exact analytical solution of the non-linear equation governing this self-similar flow which is consistent with the numerical results in Ref. [8] is given and the solutions for the temperature for various values of K_1 are presented. Later, Cortell [10] extended the work of Dandapat and Gupta [9] to study the heat transfer in an incompressible second-order fluid caused by a stretching sheet with a view to examining the influence of the viscoelastic parameter on that flow. It is found that temperature distribution depends on K_1, in accordance with the results in Ref. [9].

In the case of fluids of differential type (see Ref. [11]), the equations of motion are in general one order higher than the Navier–Stokes equations and, in general, need additional boundary conditions to determine the solution completely. These important issues were studied in detail by Rajagopal [11,12] and Rajagopal and Gupta [13]. The effects of heat generation/absorption become important in view of various physical problems (see Vajravelu and Hadjinicolaou [14]) and those effects have been assumed to be constant, space-dependent or temperaturedependent (Vajravelu and Hadjinicolaou [15]). Even, very recently, the mixed convection boundary layer flow of a Newtonian, electrically conducting fluid over an inclined continuously stretching sheet with power–law temperature variation in the presence of magnetic field, internal heat generation/absorption and wall suction/injection is analyzed by Abo-Eldahab and El Aziz [16]. In the present research, we extend the problem investigated in Ref. [16] to viscoelastic fluid flows. Furthermore, Char [17] studied MHD flow of a viscoelastic fluid over a stretching sheet, however, only the thermal diffusion is considered in the energy equation; later, Sarma and Rao [18], Vajravelu and Roper [19] and Cortell [20,21] analyzed the effects of work due to deformation in such an equation. Another effect which bears great importance on heat transfer is the viscous dissipation. The determination of the temperature distribution when the internal friction is not negligible is of utmost significance in different industrial fields, such as chemical and food processing, oil exploitation and bio-engineering. Consequently, the effects of viscous dissipation are also included in the energy equation.

On the other hand, the effect of radiation on viscoelastic boundary-layer flow and heat transfer problems can be quite significant at high operating temperature. In view of this, viscoelastic flow and heat transfer over a flat plate with constant suction, thermal radiation and without viscous dissipation were studied by Raptis and Perdikis [22]. Viscous dissipation and radiation were considered by Raptis [23] and the effect of radiation was also included in Ref. [24] and in Ref. [25]. Very recently, researches in these fields have been conducted by many investigators [26–30]; however, the effects of work due to deformation on viscoelastic flows and heat transfer in the presence of radiation, viscous dissipation and non-uniform heat source/sink have not been studied in recent years. In the present paper a proper sign for the normal stress modulus (i.e., $\alpha_1 \geq 0$) is used and, as we will see in Section 3, the effects of viscous dissipation, uniform transverse magnetic field, internal heat generation/absorption and thermal radiation are included in the energy equation. This last effect has been enclosed in this study by employing the Rosseland approximation [31]. Furthermore, we augment the boundary conditions to the flow problem and then, momentum and heat transfer in an incompressible and thermodynamically compatible second order fluid, which is termed as second grade fluid (see Ref. [20]), past a stretching sheet, are analyzed.

This paper runs as follows. In Section 2, we shall consider the mathematical analysis of the flow and some exact solutions of the boundary layer second grade fluid flow over a linearly stretching continuous surface; in Section 3 we shall examine the thermal problem when all the effects cited above are included in the energy equation for two cases of boundary heating: (a) prescribed surface temperature (PST case) and (b) prescribed heat flux (PHF case); In Section 4, we shall solve the concentration equation with a chemical reaction of order one apply on the fluid flow; furthermore, similar solutions are obtained for both stream function and temperature and the influence on the numerical results of those additional effects above-mentioned will also be discussed.

2. Flow analysis

An incompressible homogeneous fluid of second order
has a constitutive equation given by [32],[33]:

$$\boldsymbol{T} = -p\boldsymbol{I} + \mu\boldsymbol{A}_1 + \alpha_1\boldsymbol{A}_2 + \alpha_2\boldsymbol{A}_1^2 . \quad (1)$$

Here $\boldsymbol{T}$ is the stress tensor, p the pressure, $\boldsymbol{I}$ the coefficient of viscosity, α_1, α_2 are the normal stress module and $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ are defined as

$$\boldsymbol{A_1} = (grad\,\mathbf{v}) + (grad\,\mathbf{v})^T \quad (2)$$

$$\boldsymbol{A_2} = \left(\frac{d}{dt}\boldsymbol{A_1}\right) + \boldsymbol{A_1}\cdot(grad\,\mathbf{v}) + (grad\,\mathbf{v})^T\cdot\boldsymbol{A_1} \quad (3)$$

Here $\boldsymbol{v}$ denotes the velocity field and d/dt is the material time derivative. Some assumptions concerning the sign of a1 in the model (1) will be necessary. For thermodynamic reasons (see Ref. [34]), the material parameter a1 must be positive. If the fluid of second order modeled by Eq. (1) is to be compatible with thermodynamics and is to satisfy the Clausius–Duhem inequality for all motions and the assumption that the specific Helmholtz free energy of the fluid is a minimum when it is locally at rest, then

$$\mu \geq 0,\ \alpha_1 \geq 0,\ \alpha_1 + \alpha_2 = 0. \quad (4)$$

The constitutive equation given by Eq. (1) is capable of modeling a non-Newtonian fluid which possesses viscoelastic ($\mu > 0$; $\alpha_1 > 0$) properties. In our analysis we assume that the fluid is thermodynamically compatible ($\alpha_1 \geq 0$); we consider the flow of an incompressible second grade fluid past a flat and impermeable sheet coinciding with the plane y = 0, the flow being confined to y > 0. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. The steady two-dimensional boundary layer equations for this fluid, in the usual notation, are:

The continuity Equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (5)$$

The momentum Equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\mu}{k_p}u-\frac{\sigma B_y^2}{\rho}u+ \frac{\alpha_1}{\rho}\left(\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}+v\frac{\partial^3 u}{\partial y^3}\right) \quad (6)$$

The Energy Equation

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=K_o\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+ Q\left(T-T_\infty\right)-\frac{\partial q_r}{\partial y}+\sigma B_y^2 u^2 \quad (7)$$

The concentration Equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}+k_1\left(C-C_\infty\right) \quad (8)$$

It should be also mentioned that the continuous and momentum Equations (5) and (6) are very well described in the papers by Harris [35] and Sadeghy and Sharifi [36], whereas Equation (7) can be found in the book by Bejan [37], see also Ref.[38]. The oundary conditions of Eqs. (5) and (6) are

$$y=0, u=u_w=\lambda x, v=v_w=-\sqrt{\lambda\upsilon}\left(\frac{m-1}{m}\right) \quad (9)$$

$$y\to\infty, u\to 0, \frac{\partial u}{\partial y}\to 0 \quad (10)$$

where x *and* y are the Cartesian coordinates along and normal to the plate, respectively, u *and* v are the velocity components along $x-$*and* $y-$ axes, respectively, υ is the Kinematic viscosity, T is the fluid temperature, ρ is the mass density of the fluid, D is the molecular diffusivity, C is the mass concentration of the species of the flow, σ is the electric conductivity, Q is the volumetric rate of heat generation/absorption, C_p is the specific heat at constant pressure, B_y is the uniform magnetic field which distributed by the flow, k_p is the permeability of the fluid, k_1 is the reaction rate coefficient, q_r is the radiation heat flux, K_o is the thermal conductivity, λ and m are constants, T_∞ is the temperature of ambient fluid and C_∞ is the concentration of ambient fluid.

In the boundary condition Equation (9), it should be noted that $m>1$ corresponds to suction ($v_w<0$), where $m<1$ corresponds to blowing ($v_w>0$). In the case when the parameter $m=1$ corresponds to suction ($v_w=0$), the stretching sheet is impermeable. In this study, set all of parameter $m=1$ simplified the problem. The second boundary condition Eq. (10) is the augmented condition since the flow is in an unbounded domain, which has been discussed by Garg and Rajagobal [39]. A similarity solution for the velocity will be obtained if one introduces a set of transformations, such that

$$u=\lambda x f'(\eta) \quad \& \quad v=-\sqrt{\upsilon\lambda}f(\eta) \quad \text{and}$$

$$\eta=\sqrt{\frac{\lambda}{\upsilon}}y \quad (11)$$

further more, the velocity components in terms of velocity function $\psi(x,y)$ are defined as

$$u=\frac{\partial\psi}{\partial y}\ , \quad v=-\frac{\partial\psi}{\partial x}\ , \quad \psi(x,y)=\sqrt{\lambda\upsilon}xf(\eta) \quad (12)$$

equations (11) and (12) satisfies the continuity equation (5), substituting (12) into (6), we have

$$f'^2-ff''=f'''-K(2f'f'''-ff''''-f''^2)-Mf' \quad (13)$$

with

$$M=M_n+1/k \quad (14)$$

where $K=\frac{\lambda\alpha_1}{\rho\upsilon}$ is the viscoelastic parameter, $M_n=\frac{\sigma B_y^2}{\rho\lambda}$ is the magnetic parameter and $k=\frac{k_p\lambda}{\mu}$ is the porosity parameter. The corresponding boundary conditions become

$$f=0\ ,\quad f'=1 \qquad at \quad \eta=0 \quad (15)$$

$$f'\to 0\ ,\quad f''\to 0 \qquad as \quad \eta\to\infty \quad (16)$$

Then the solution of the momentum equation (13) is

$$f(\eta)=\frac{1}{\alpha}\left(1-e^{-\alpha\eta}\right) \quad (17)$$

which satisfied the boundary conditions (15) and (16), substituting (17) into (11) we get the velocity components takes the form

$$u=\lambda x e^{-\alpha\eta} \quad \& \quad v=\frac{-\sqrt{\lambda\upsilon}}{\alpha}\left(1-e^{-\alpha\eta}\right) \quad (18)$$

where,

$$\alpha = \sqrt{(1+M)/(1-K)} \tag{19}$$

3. Solution of heat transfer equation

The governing boundary layer equation with temperature dependent heat generation (absorption), constant magnetic field and Radiation is given by Eq. (7), by using Rosseland approximation the radiation heat flux has given by

$$q_r = \frac{-4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{20}$$

where σ^* and k^* respectively, the Stephan-Boltzmann constant and mean absorption coefficient. Further, we assume that the temperature difference within the flow is such that T^4 may be expand in a Taylor series. Hence, expanding T^4 about T_∞ and neglecting higher order terms we get

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{21}$$

Now using Equations (20) and (21), Equation (7) becomes

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=K_o\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+ \\ Q(T-T_\infty)+\frac{16\sigma T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2}+\sigma B_y^2 u^2 \tag{22}$$

The thermal boundary conditions depend on the type of heating process under consideration.

Here we consider two different heating processes, namely: **PST** and **PHF**.

3.1. Case A: prescribed surface temperature (PST)

In the **PST** case se define non-dimensional temperature variable as

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{23}$$

In order to deal with non-isothermal stretching boundary in **PST** case we consider the boundary conditions on temperature as

$$T=T_w=T_\infty+A_o\left(\frac{x}{\ell}\right)^2 \qquad at \quad y=0 \tag{24}$$

$$T\to T_\infty \qquad as \qquad y\to\infty \tag{25}$$

where T_w is the temperature of the wall, A_o is a constant whose value depends on the properties of the fluid and ℓ is a characteristic length. Now, by using the transformations given by equations (11) and (23) in the equation (22). This leads to the non-dimensional form of temperature equation as follows.

$$\left(\frac{3N_r+4}{3P_r N_r}\right)\theta''(\eta)+f\,\theta'(\eta)-(2f'-\beta)\theta(\eta)= \\ -E_C(f'')^2-M_nE_C(f')^2 \tag{26}$$

where $N_r=\frac{K_oK^*}{4\sigma T_\infty^3}$ is the radiation parameter, $P_r=\frac{\mu C_p}{K_o}$ is the Prandtl number, $E_c=\frac{\lambda^2\ell^2}{A_oC_p}$ is the Eckert number and $\beta=\frac{Q}{\rho C_p\lambda}$ is the heat source/sink parameter. By using the dimensionless variable of Eq. (23) in Eqs. (24) and (25) we get the corresponding dimensionless boundary conditions as

$$\theta(0)=1 \quad and \quad \theta(\infty)=0 \tag{27}$$

Further, using the equation (17) in equation (26) we get

$$a\,\theta''(\eta)+\frac{1}{\alpha}\left(1-e^{-\alpha\eta}\right)\theta'(\eta)-\left(2e^{-\alpha\eta}-\beta\right)\theta(\eta)= \\ -E_C(\alpha^2+M_n)e^{-2\alpha\eta} \tag{28}$$

where, $$a=\left(\frac{3N_r+4}{3P_rN_r}\right) \tag{29}$$

Defining new variables

$$\xi=-\frac{1}{a\alpha^2}e^{-\alpha\eta} \tag{30}$$

Using the transformation given by equation (30) in equation (28), we derive the governing equation for the temperature in the form

$$\xi\theta''(\xi)+\left(1-\frac{1}{a\alpha^2}-\xi\right)\theta'(\xi)+\left(2+\frac{\beta}{a\alpha^2\xi}\right)\theta(\xi)= \\ E_C\,\alpha^2 a\left(\alpha^2+M_n\right)\xi \tag{31}$$

with the corresponding boundary conditions as

$$\theta\left(\frac{-1}{a\alpha^2}\right)=1 \qquad and \qquad \theta(0)=0 \tag{32}$$

The equations (31) and (32) constitute a non-homogenous boundary value problem. Denoting the solution of the homogenous part of equation (31) by $\theta_C(\xi)$

i.e

$$\xi\,\theta_C\,''(\xi)+\left(1-\frac{1}{a\alpha^2}-\xi\right)\theta_C\,'(\xi)+\left(2+\frac{\beta}{a\alpha^2\xi}\right)\theta_C\,(\xi)=0 \quad (33)$$

and further introduce the transformation $\theta_C\left(\xi\right)=\xi^{\delta}\,\omega\left(\xi\right)$, we obtain the confluent hyper-geometric equation of the form

$$\xi\Omega''(\xi)+\left(1+b_o-\xi\right)\Omega'(\xi)-\frac{1}{2}\left(a_o+b_o-4\right)\Omega(\xi)=0 \quad (34)$$

where

$$\delta=\frac{a_o+b_o}{2},\ a_o=\frac{1}{a\alpha^2}\ \text{ and }\ b_o^2=a_o^2-4a_o\beta \quad (35)$$

the solution of equation (34) is

$$\Omega\left(\xi\right)=M\left(\frac{a_o+b_o-4}{2},1+b_o,\xi\right) \quad (36)$$

where M is the Kummer's function (Abramowitz and Stegun [40]) and it is defined by

$$M\left(a_o,b_o,z\right)=1+\sum_{n=1}^{\infty}\frac{\left(a_o\right)_n z^n}{\left(b_o\right)_n n!}$$

$$\left(a_o\right)_n=a_o\left(a_o+1\right)\left(a_o+2\right),...,\left(a_o+n-1\right)$$

$$\left(b_o\right)_n=b_o\left(b_o+1\right)\left(b_o+2\right),...,\left(b_o+n-1\right) \quad (37)$$

then the solution of Eq. (33) as follows

$$\theta_C\left(\xi\right)=\xi^{\delta}M\left(\delta-2,1+b_o,\xi\right) \quad (38)$$

the particular integral solution of the equation (31) is

$$\theta_p\left(\xi\right)=-\frac{E_C\,\alpha^2\left(\alpha^2+M_n\right)a}{P_r\left(4-2a_o+\beta a_o\right)}\xi^2 \quad (39)$$

Hence, the solution of equation (31) is

$$\theta\left(\eta\right)=\theta_p\left(\eta\right)+c_1\theta_C\left(\eta\right) \quad (40)$$

now using the boundary conditions of equation (32) and changing the variable ξ to η we obtain the solution of Eq. (28) in the following form of confluent hyper-geometric function

$$\theta(\eta)=-Ae^{-2\alpha\eta}+\left(1+A\right)e^{-\alpha\delta\eta}\frac{M\left(\delta-2,1+b_o,-a_oe^{-\alpha\eta}\right)}{M\left(\delta-2,1+b_o,-a_o\right)} \quad (41)$$

where $$A=\frac{E_C\left(\alpha^2+M_n\right)a_o}{P_r\left(4+a_o\left(\beta-2\right)\right)} \quad (42)$$

The dimensionless wall temperature gradient $\theta'\left(0\right)$ is obtained as

$$\theta'\left(0\right)=2\alpha A-\alpha\delta\left(1+A\right)+a_o\,\alpha\left(2-\delta\right)\left(1+A\right)*\frac{M\left(\delta-1,2+b_o,-a_o\right)}{M\left(\delta-2,1+b_o,-a_o\right)} \quad (43)$$

The Dimensional local heat flux q_w is defined as

$$q_w=-\kappa\left(\frac{\partial T}{\partial y}\right)_w=\kappa\sqrt{\frac{\lambda}{\upsilon}}\left(T_w-T_\infty\right)\left[-\theta'\left(0\right)\right] \quad (44)$$

3.2. Case B: Prescribed power law heat flux (PHF)

In the **PHF** case se define non-dimensional temperature variable as

$$g\left(\eta\right)=\left(T-T_\infty\right)/\left(E_o\left(\tfrac{x}{\ell}\right)^2\tfrac{1}{K_o}\sqrt{\tfrac{\upsilon}{\lambda}}\right) \quad (45)$$

in order to deal with non-isothermal stretching boundary in **PHF** case we consider the boundary conditions on temperature as

$$q_w=-\kappa\left(\frac{\partial T}{\partial y}\right)_w=E_o\left(\frac{x}{\ell}\right)^2 \quad at \quad y=0 \quad (46)$$

$$T\to T_\infty \quad as \quad y\to\infty \quad (47)$$

where E_o is a constant whose value depends on the properties of the fluid and ℓ is a characteristic length. Now, by using the transformations given by equations (11) and (45) in the equation (22). This leads to the non-dimensional form of temperature equation as follows.

$$a\,g''(\eta)+f\,g'(\eta)-\left(2f'-\beta\right)g(\eta)=-E_C^{'}\left[f''^2-M_nf'^2\right] \quad (48)$$

By using the dimensionless variable of Eq. (45) in Equations (46) and (47) we get the corresponding dimensionless boundary conditions as

$$\frac{dg}{d\eta}=g'=-1 \quad at \quad \eta=0 \quad (49)$$

$$g\to 0 \quad as \quad \eta\to\infty \quad (50)$$

further, using the equation (17) in equation (48) we get

$$a\,g''(\eta)+\frac{1}{\alpha}\left(1-e^{-\alpha\eta}\right)g'(\eta)-\left(2e^{-\alpha\eta}-\beta\right)g(\eta)=-E_C^{'}(\alpha^2+M_n)e^{-2\alpha\eta} \quad (51)$$

where, $E_C^{'}$ is the scaled Eckert number for PHF case.

Defining new variables

$$\xi = -a_o e^{-\alpha\eta} \qquad (52)$$

Using the transformation given by equation (52) in equation (51), we derive the governing equation for the temperature in the form

$$\xi g''(\xi)+(1-a_o-\xi)g'(\xi)+\left(2+\frac{a_o\beta}{\xi}\right)g(\xi)=\frac{E_C'}{a_o}\left(\alpha^2+M_n\right)\xi \qquad (53)$$

with the corresponding boundary conditions as

$$g'(-a_o)=-\alpha a \quad \& \quad g(0)=0 \qquad (54)$$

The analytical solution of equation (53), subject to the corresponding boundary conditions of equation (54), is obtained in the following form of confluent hypergeometric function of the similarity variable η.

$$g(\eta)=-A'e^{-2\alpha\eta}-\frac{(1+2\alpha A')e^{-\alpha\delta\eta}M\left(\delta-2,1+b_o,-a_o e^{-\alpha\eta}\right)}{\alpha\left[a_o(2-\delta)M(\delta-1,2+b_o,-a_o)-\delta M(\delta-2,1+b_o,-a_o)\right]} \qquad (55)$$

where $A'=\dfrac{E_C'\left(\alpha^2+M_n\right)a_o}{P_r\left(4+a_o(\beta-2)\right)}$ (56)

The expression for dimensionless wall temperature is obtained as

$$g(0)=-A'-\frac{(1+2\alpha A')M\left(\delta-2,1+b_o,-a_o\right)}{\alpha\left[a_o(2-\delta)M(\delta-1,2+b_o,-a_o)-\delta M(\delta-2,1+b_o,-a_o)\right]} \qquad (57)$$

4. Solution of mass transfer equation

We define a dimensionless concentration $\varphi(\eta)$ as

$$C=C_\infty+(C_w-C_\infty)\varphi(\eta) \qquad (58)$$

with the Boundary conditions as follows

$$C=C_w=C_\infty+A_o x^{\lambda} \quad at \quad y=0 \qquad (59)$$

$$C\to C_\infty \quad as \quad y\to\infty \qquad (60)$$

where C_w and C_∞ are the concentration at the wall and infinity respectively and A_o is constant, at $\lambda=1$ which is the case in Ref. [41] .To solve the mass transfer equation (8), substituting equations (11), (12) and (58) into the equation (8) and the boundary conditions (59) and (60), we have

$$\varphi''(\eta)+S_C f\,\varphi'(\eta)+S_C(\gamma-\lambda f')\varphi(\eta)=0 \qquad (61)$$

where $S_C=\frac{\upsilon}{D}$ is the Schmidt Number and $\gamma=\frac{k_1}{\lambda}$ is the chemistry reaction parameter.

the corresponding boundary conditions are

$$\varphi=1 \quad at \quad \eta=0 \quad \text{and}$$

$$\varphi\to 0 \quad as \quad \eta\to\infty \qquad (62)$$

Substituting from equation (17) in equation (60), we get

$$\varphi''(\eta)+\frac{S_C}{\alpha}\left(1-e^{-\alpha\eta}\right)\varphi'(\eta)+S_C(\gamma-\lambda e^{-\alpha\eta})\varphi(\eta)=0 \qquad (63)$$

we transform equations (60) and (61), using the relationship $\xi=-a_1 e^{-\alpha\eta}$ with $a_1=\dfrac{S_C}{\alpha^2}$, to yield

$$\xi\varphi''(\xi)+(1-a_1-\xi)\varphi'(\xi)+\left(\frac{a_1\gamma}{\xi}+\lambda\right)\varphi(\xi)=0 \qquad (64)$$

and

$$\varphi(-a)=1 \ and \ \varphi(\xi\to 0)=0 \qquad (65)$$

the solution, after further transforming the above equation into standard Kummer's equation, is given by

$$\varphi(\xi)=\left(\frac{\xi}{a_1}\right)^{\frac{1}{2}(a_1+b_1)}\frac{M\left(\frac{1}{2}(a_1+b_1-2\lambda),1+b_1,\xi\right)}{M\left(\frac{1}{2}(a_1+b_1-2\lambda),1+b_1,-a_1\right)} \qquad (66)$$

or, in terms of η, as

$$\varphi(\eta)=e^{-\frac{1}{2}(a_1+b_1)\alpha\eta}\frac{M\left(\left(\frac{a_1+b_1}{2}-\lambda\right),1+b_1,-a_1 e^{-\alpha\eta}\right)}{M\left(\left(\frac{a_1+b_1}{2}-\lambda\right),1+b_1,-a_1\right)} \qquad (67)$$

where $b_1=\sqrt{a_1}\sqrt{a_1-4\gamma}$ (68)

it is important to find the mass transfer rate J_w, after obtaining the concentration field, as

$$J_w=-\rho D\frac{\partial C}{\partial y}(0)=-\rho DAx\sqrt{\left(\lambda/\upsilon\right)}\,\varphi'(0) \qquad (69)$$

where the concentration gradient at the sheet is

$$\varphi'(0)=-\frac{1}{2}\alpha(a_1+b_1)+\frac{a_1\alpha\varepsilon M\left[1-\varepsilon,2+b_1,-a_1\right]}{M\left[-\varepsilon,1+b_1,-a_1\right]} \qquad (70)$$

Where

$$\varepsilon = \left(\lambda - \frac{(a_1 + b_1)}{2} \right) \qquad (71)$$

5. Results and Discussion

The Magnetohydrodynamic flow of non-Newtonian viscoelastic fluid with heat and mass transfer over infinite porous horizontal stretching sheet under radiation, heat generation / absorption and chemical reaction is governed by nine parameters, namely, K the viscoelastic parameter, M_n the magnetic parameter, β the heat parameter, k the porosity parameter, N_r the radiation parameter, γ the chemical reaction parameter , E_C the Eckert number, P_r the Prandtl number and S_c the Schmidt number. An insight into the effects of these parameters of the flow field can be obtained by the study of the velocity components, temperature and mass concentration distributions. The components of the velocity $f(\eta)$ *and* $f'(\eta)$ have been plotted against the dimension η for several sets of the values of the parameters K, M_n *and* k . Fig. (1) Show that the velocity components decrease with an increases in the magnetic parameter M_n . In Fig. (2), the variation of the velocity components $f(\eta)$ *and* $f'(\eta)$ with k (the porosity parameter) seen and show that the velocity components increase with an increases of k . Fig. (3) Show that the velocity components decrease with an increases in the viscoelastic parameter K . In the Figures (4), (5), (8) and (10) presents the variation of the dimensionless Temperature $\theta(\eta)$ in PST case with the viscoelastic parameter K , the porosity parameter k , the Eckert number E_C and the heat parameter β respectively for constant the other parameters can be seen, Figures (4), (5), (8) and (10) show that $\theta(\eta)$ increase with an increase of these parameters. But the Figures (6), (7) and (9) illustrate the effects of the magnetic parameter M_n , the Prandtl number P_r and the radiation parameter N_r on the dimensionless temperature $\theta(\eta)$. From these Figures, it can be observed that the effects of M_n, P_r and N_r decrease the temperature distribution $\theta(\eta)$. Figures (11), (12) and (15) present the dimensionless mass concentration profiles $\varphi(\eta)$ for selected values of the parameters M_n the magnetic interaction parameter, K the viscoelastic parameter and γ the chemical reaction parameter respectively with fixed the other parameters. It is shown that the dimensionless mass concentration at a given point in the fluid is increase with increase the parameters M_n, K and γ. From figures (13) and (14) one sees that the effect of the porosity parameter k and the Schmidt number S_C are to increase the dimensionless mass concentration $\varphi(\eta)$ of the fluid flow.

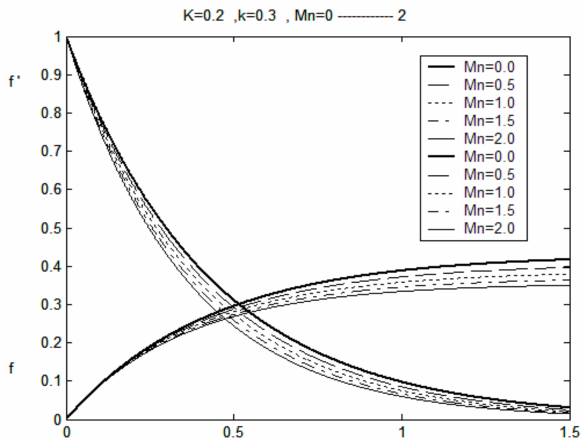

Fig. (1): The components of the velocity distribution for varies values of the Magnetic parameter M_n

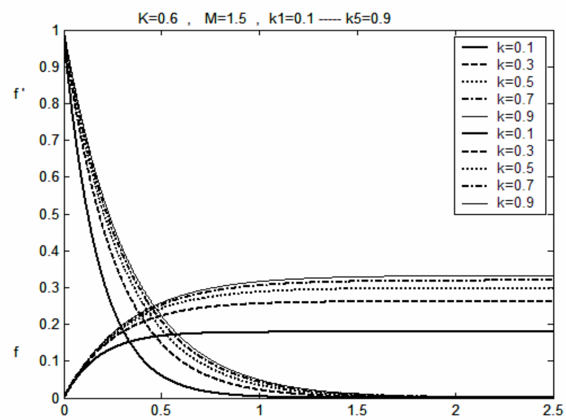

Fig. (2): The components of the velocity distribution for varies values of the porosity parameter k

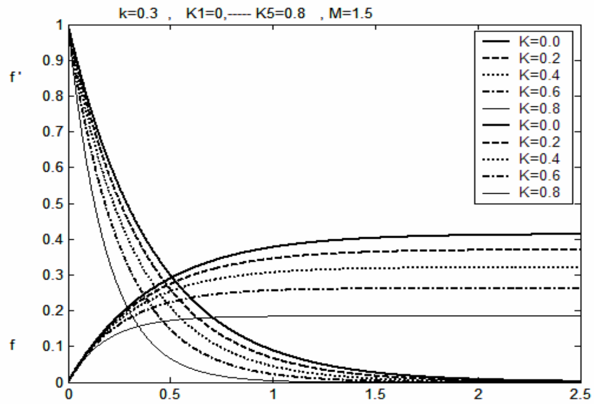

Fig. (3): The components of the velocity distribution for varies values of the viscoelastic parameter *K*

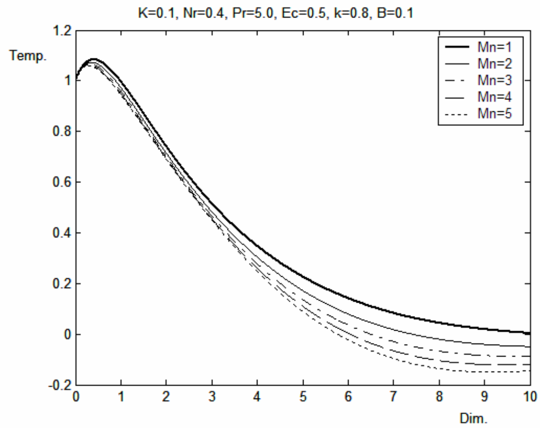

Fig. (6): Temperature distribution for varies values of the magnetic parameter M_n

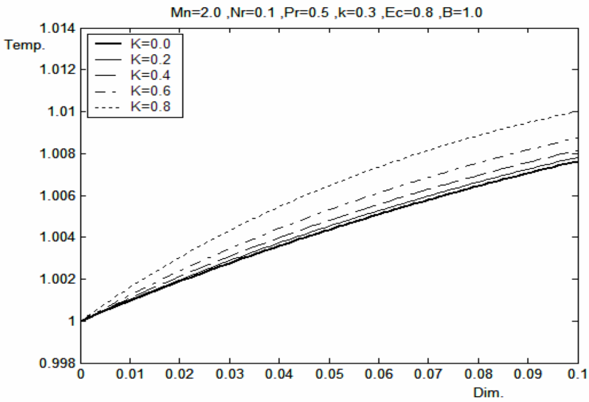

Fig. (4): Temperature distribution for varies values of the viscoelastic parameter *K*

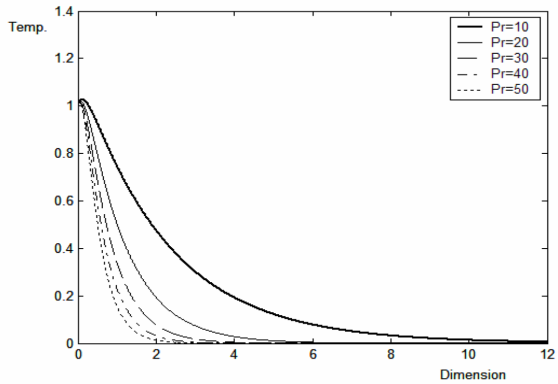

Fig. (7): Temperature distribution for varies values of the Prandtl Number P_r

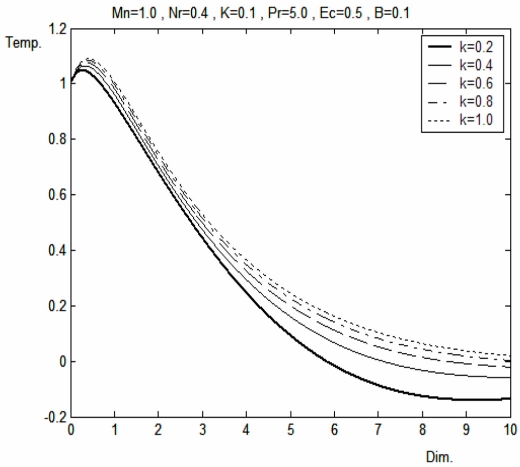

Fig. (5): Temperature distribution for varies values of the porosity parameter *k*

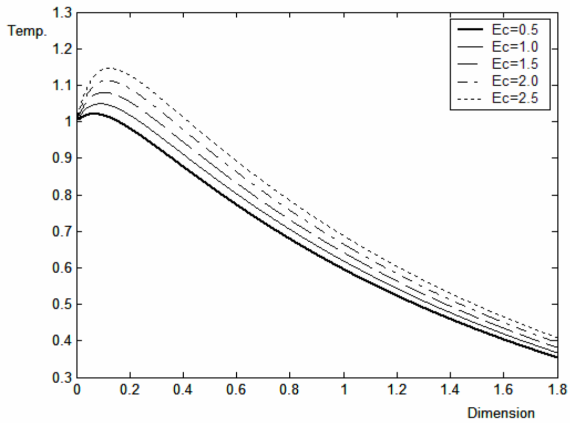

Fig. (8): Temperature distribution for varies values of the Eckert Number E_C

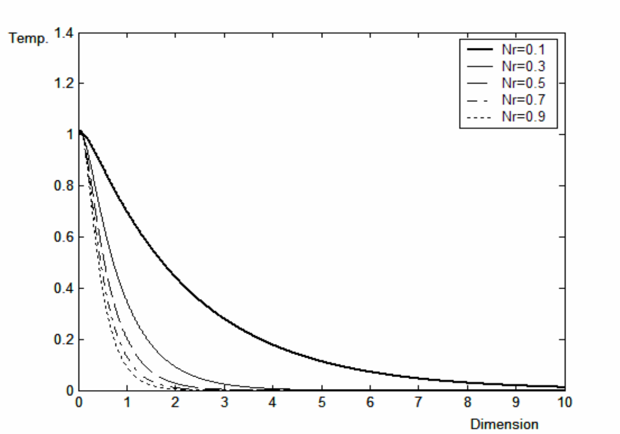

Fig. (9): Temperature distribution for varies values of the radiation parameter N_r

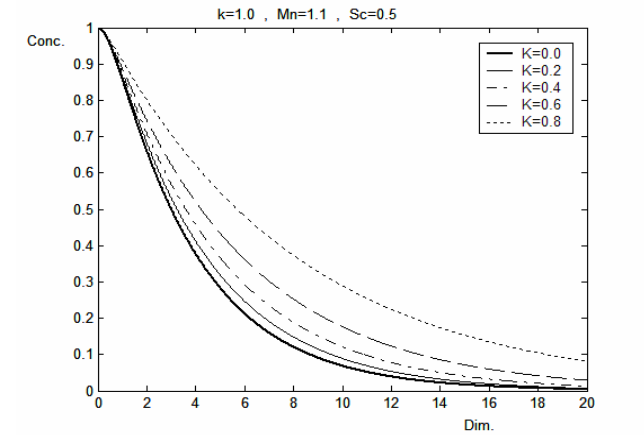

Fig. (12): Concentration distribution for varies Values of the viscoelastic parameter K

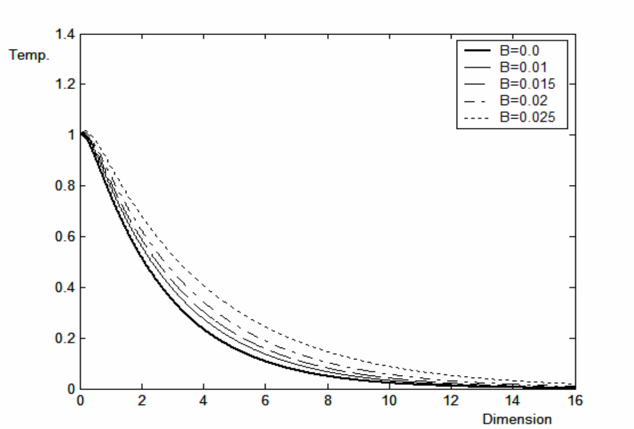

Fig. (10): Temperature distribution for varies values of the heat parameter β

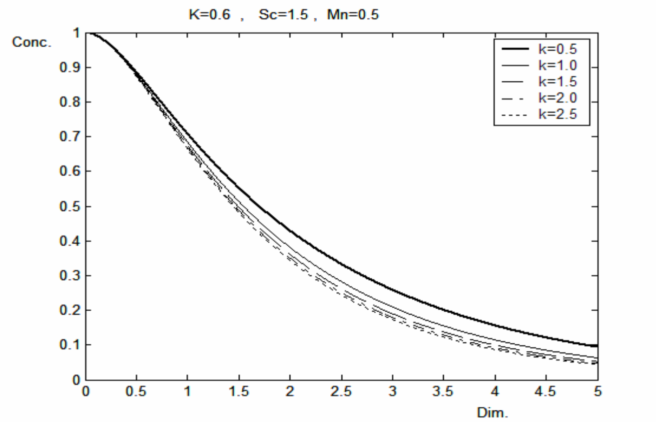

Fig. (13): Concentration distribution for varies Values of the porosity parameter k

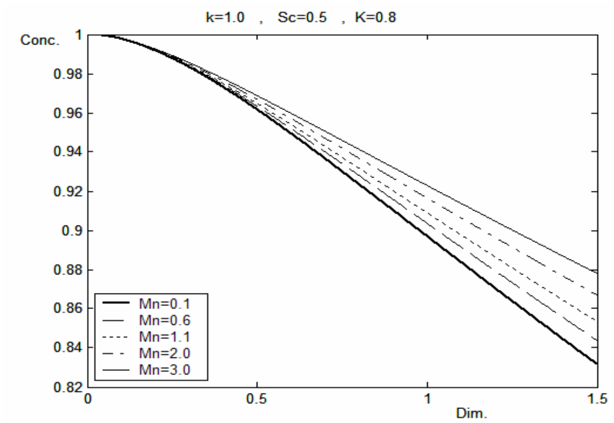

Fig. (11): Concentration distribution for varies Values of the magnetic parameter M_n

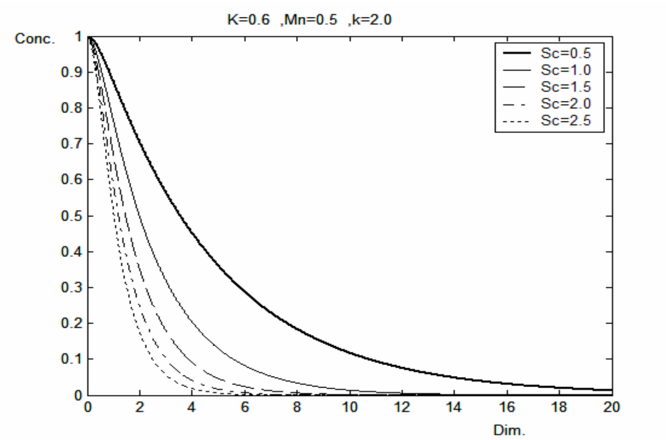

Fig. (14): Concentration distribution for varies Values of the Schmidt Number S_C

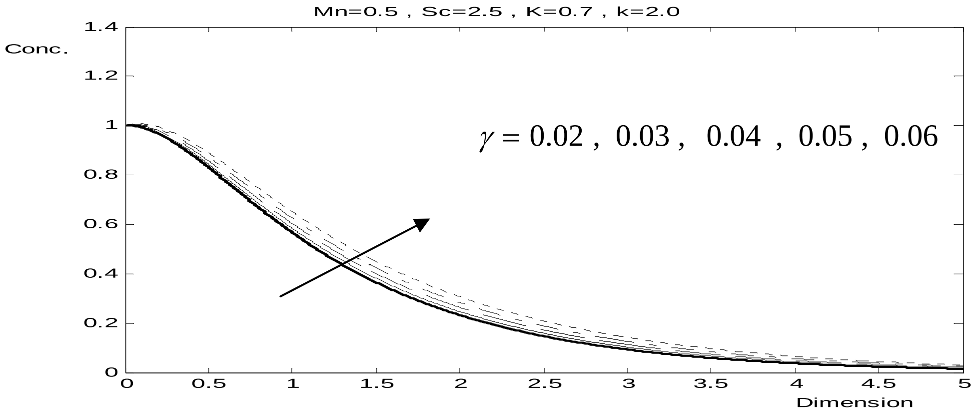

Fig. (15): Concentration distribution for varies Values of the reaction parameter γ

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