Elastic impedance inversion from robust regression method

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Abstract: As in two terms AVO inversion, the linear fit of set of logarithm elastic impedance versus the sinus square of incidence angle have been used to extract elastic parameters .This way based on minimum least square sense, though very efficient ,gives optimal results when staying in Gaussian context. Unfortunately, the Gaussian distribution proves sometimes to be inexact on real data often affected by noises that create outliers and thus distort inversion results. In this paper, we introduce the one popular robust technique; the so-called M-estimators to deal with outliers. On synthetic elastic impedance (EI) data in which fours outliers have been added to far angles, the Andrews estimator gives the best results than Hubert estimator. From this observation, the Andrew estimator has been used to real seismic data and the inversion results are very stable. [Journal of American Science 2010; 6(9):713-718]. (ISSN: 1545-1003).

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1. Introduction

Since the introduction of EI by Connolly (1999), publications linked to this approach have not ceased to appear in literature. Two crucial points have been attracting researchers: the approximation of EI (Verwest et al., 2000, Whitcombe, 2002, Ma, 2004… Morozov, 2010) , and the estimation of elastic parameters or petrophysical parameters from EI. This paper is out of the scope of the first point; the readers are oriented to papers published by Yue&Liu, 2005, Zhang&Ni, 2006).As in two terms AVO inversion where the intercept and the gradient are computed using minimum linear square, Connolly and Cambois,2000, introduced the linear approximation of EI from which elastic parameters can be derived. Later, Mallick, 2000, demonstrated that in presence of 2% random noise,Connolly& Cambois method fails to extract the elastic model from synthetic data. The requirement to keep the exponent K constant at all interfaces and the assumption of convolutional model at nonnormal angles of incidence are the two factors limiting the inversion of elastic impedance (Mallick, 2001).

seismic real data can be damaged by certain types of noises; wavelet variation with offset (amplitude and phase), inaccurate NMO-correction, inaccurate estimation of incidence angles, multiples and converted waves. To overcome the first type of noise, each angle stack is inverted with its own wavelet and the remaining others kind of noises can create outliers in the data. In this situation, minimum least square method, though very efficient, can lead to inaccurate results.

In this paper, we introduce the one popular robust technique; the so-called M-estimators to deal with outliers. On synthetic elastic impedance (EI) data in which fours outliers have been added to far angles, the Andrews estimator gives the best results than Hubert estimator. From this observation, the Andrew estimator has been used to real seismic data and the inversion results are very stable.

2. 0 Material and Methods

2.1 Elastic Impedance approach

According to Connolly, 1999, the elastic impedance of plane P-wave can be approximated as follows:

$$EI = V_p^{(1+\tan^2\theta)} V_s^{(-8K\sin^2\theta)} \rho^{(1-4K\sin^2\theta)} \quad (1)$$

With $K=(V_s/V_p)^2$; EI, V_s,V_P and ρ are elastic impedance, shear velocity, compressional velocity and density respectively. When replacing $V_p^{(1+\tan^2\theta)}$ by $V_p^{(1+\sin^2\theta)}$, equation (1) is called the first order elastic impedance formulation. To reduce the dimensionality of EI, Whitecombe, 2002, introduced the normalization parameters to stabilize the variation of EI versus angle; thus equation (1) can be written as

$$EI = I_{p0} * \left(\frac{V_p}{V_{p0}}\right)^{(1+\tan^2\theta)} \left(\frac{V_s}{V_{s0}}\right)^{(-8K\sin^2\theta)} \left(\frac{\rho}{\rho_0}\right)^{(1-4K\sin^2\theta)} \quad (2)$$

The constant parameters with subscript zero, also called normalization parameters, are the average values along the entire log.

2.2 Elastic Impedance Inversion

The reflection coefficient $R_p(\theta)$ can be expressed in the same form as the normal incidence:

$$R_{ppi}(\theta) = \frac{EI_{i+1} - EI_i}{EI_{i+1} + EI_i} \approx \frac{1}{2}\ln\frac{EI_{i+1}}{EI_i} \quad (3)$$

Where EI_{i+1} is the elastic impedance of the upper layer and EI_i is the elastic impedance of the upper layer.

The success of any EI-inversion is largely depended on the quality of seismic traces, the wavelet estimation and the low frequency model which can be estimated in several manners (the most popular being the integration of well logs, interpreted seismic horizons and seismic velocities).

2.2.1 Quality of seismic trace

It well known that removing totally undesirable signal (noise) from seismic data is a thorny question that has been studying so long. To improve signal to noise ratio, offset gathers can be transform into limited angle gather stacks.

2.2.2 Wavelet estimation

From statistical way (i.e from seismic data) and/or from well logging, wavelet must be extracted separately at each limited angle stacks.

Low frequency model

From equation (2), the pseudo elastic impedance logs are computed to constrain the inversion of limited angle stacks. At well location, seismic limited angle stacks and the EI logs computed using the same incident angles are extrapolated via interpreted horizons to build low frequency models.

2.2.3 Inversion

Each angle-limited stack is inverted into elastic impedance using the linear sparse spike impedance inversion.

2.3 Estimation of elastic parameters from elastic impedance

The equation (2) can be transformed into the first order elastic impedance formulation:

$$\cos^2\ln(EI) = \ln(V_p\rho) - [4k\{\ln(\rho) + 2\ln(V_s)\} + \ln(\rho)]\sin^2(\theta) + 4k[\ln(\rho) + 2\ln(V_s)]\sin^4(\theta) \quad (4)$$

Restricting to angles less than 25°, such that $\tan^2(\theta) \approx \sin^2(\theta)$ and $\sin^4(\theta) \approx 0$, and assuming a background P-to S-wave velocity ratio of 2, i.e. K=0.25, equation (4) becomes

$$\ln(EI) \approx \ln(I_p) + [\ln(I_p) - 2\ln(I_s)]\sin^2(\theta) \quad (5)$$

Physically, the logarithm can only be applied to dimensionless variables; thus according to equation (3), equation (5) can be written as

$$\ln(EI/I_{p0}) = \ln(I_p/I_{p0}) + [\ln(I_p/I_{p0}) - 2\ln(I_s/I_{p0})]\sin^2(\theta) \quad (6)$$

Working with the equation (5) or equation (6), both P and S-impedance can be estimated accurately.

2.4 Regression robust

It well known that the ordinary least-squares method tries to minimize $\sum_i r_i^2$, which is unstable if there are outliers present in the data. The M-estimators (maximum likelihood type estimator), one of the most popular robust technique, try to reduce the effect of outliers by replacing the squared residuals r_i^2 (difference between observed data and measured data)by another function of the residuals:

$$\min \sum_i \rho(r_i) \quad (7)$$

Where ρ is a symmetric, positive-definite function with a unique minimum at zero, and is chosen to be less increasing than square. Equation (7) can be implemented as an iterated reweighted least-squares one. Then, taking the partials of $\rho(r_i)$ with respect to parameter vectors to be estimated, respectively, and setting them to zero, equation (7) yields the following equation:

$$\min \sum_i w(r_i^{(k-1)}) r_i^2 \quad (8)$$

Where the superscript (k) indicates the iteration number. The weight $w(r_i^{(k-1)})$ should be recomputed after each iteration in order to be used in the next iteration. W(x) $= \frac{\psi(x)}{x}$; $\psi(x)$ is called the influence function.

There exist a few commonly used influence functions, such as the influence function of Cauchy, Hubert, Andrews and so one (Press et al, 1986).

3.0 Results and Discussion

3.1 Application to synthetic data

Synthetic elastic impedance data have been generated using equation1 (first order elastic impedance formulation) for several incidence angles from the Vp log, the Vs log and the ρ log. At far incidence angles, four outliers have been added to Synthetic elastic impedance. The M-estimators (Cauchy estimator, Huber-estimator, Andrews estimator…) can detect the presence of outliers and don't take them into account when computing regression parameters.

It's clear from figure (1) and table (1) that Andrews estimators and Cauchy estimators can nearly recover the original values .the difference observed between Andrews estimators value, Cauchy estimators value and the original value in the s-impedance in table (1) is because the original elastic impedance values were computed using k=0.21.

If in figure (1) and table (1), one time sample have been used, in figure (2) the entire log have been considered .in order to create outliers , 7% of random noise have been added to far angles.

The inversion of P-impedance from EI (equation 6) doesn't suffer from inaccuracy, though the constant value of K used for inversion (k=0.25) is slightly different to the k value used to compute elastic impedance. All possibilities (Huber, Cauchy, Andrews, ordinary least square) give optimal results. Moreover, the situation is different when trying to invert S-impedance from EI. In the presence of noise at far angles, the inversion of S-impedance from ordinary least square may suffer from inaccuracy. Note the large misfit observed at near 2020m and at near 2600m (figure2-right). The slight misfit observed between Andrews- estimators , Cauchy estimators and the original data in the s-impedance in figure(2-right) is because the original elastic impedance values were computed using k=0.21(k=0.25 being used in inversion).

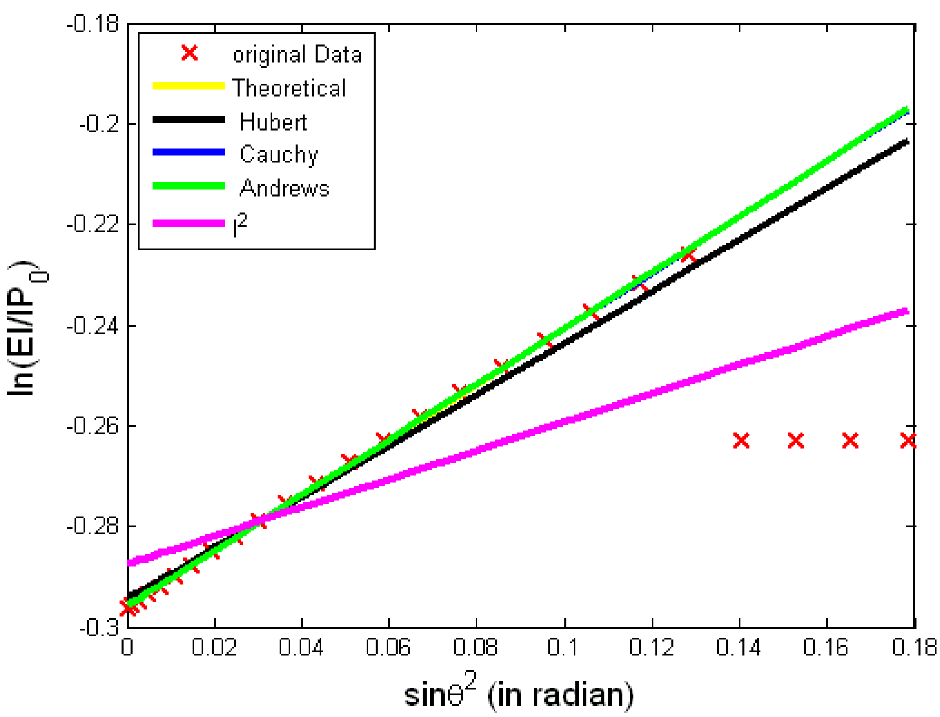

Figure 1. logarithm elastic impedance plotted against $\sin^2\theta$.Four red stars (outliers) has been added at far angles. Cauchy estimator, Huber-estimator, Andrews estimator can detect the presence of outliers and don't take them into account when computing regression parameters.

Table1. Comparison of regression parameters values

	$\ln(I_p/I_{pu})$	$\lvert\ln(I_p/I_{pu}) - 2\ln(I_s/I_{pu})\rvert$	I_p	I_S
Data			4955.84	1852.529
Theoretical	-0.2958	0.5527	4958.136	2315.66
L2	-0.2876	0.2838	4999.226	2325.23
Huber	-0.2946	0.511	4964.411	2068.3
Cauchy	-0.2959	0.5529	4958.109	2024.105
Andrews	-0.2959	0.55346	4958.036	2023.559

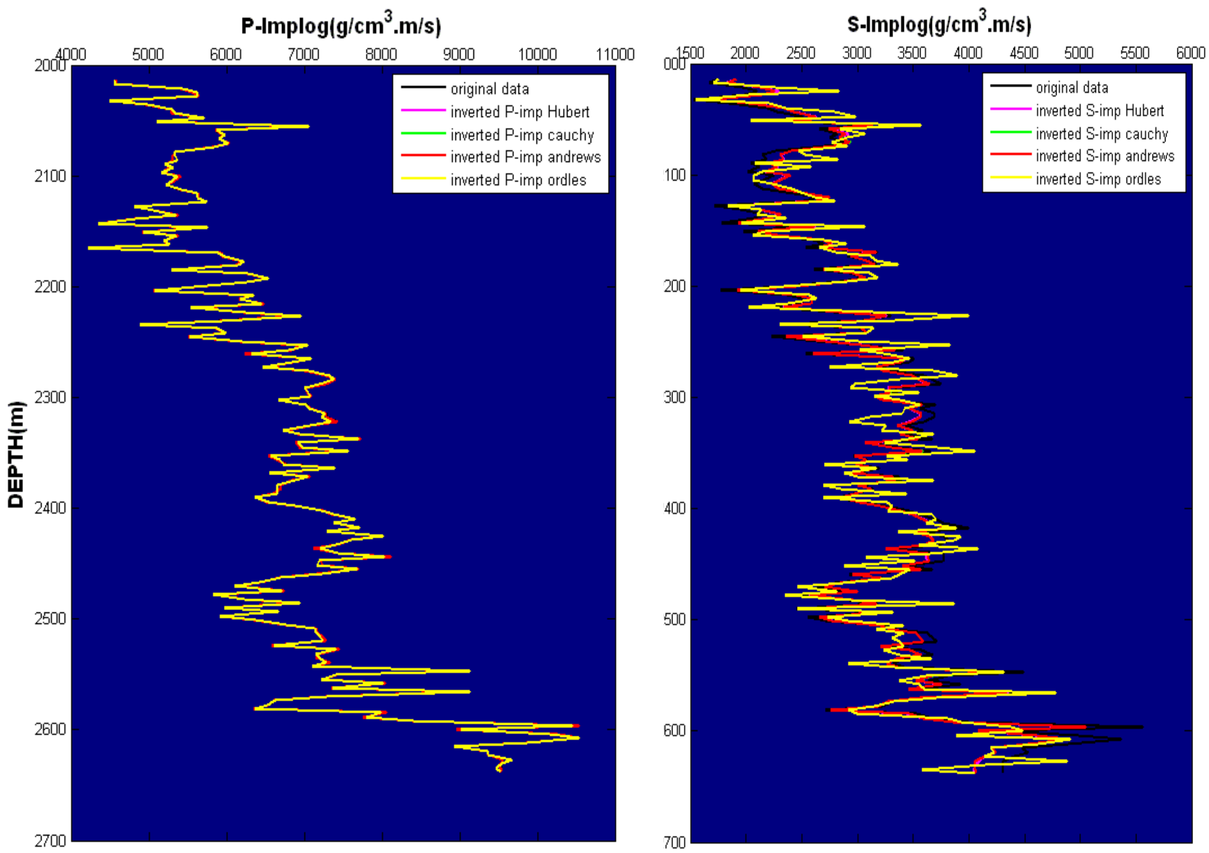

Figure2. Left: P-wave impedance from equation (6) using ordinary linear regression (yellow curve) and robust regression (Hubert-estimators in magenta curve, Cauchy estimators in green curve, Andrews-estimators in red curve) with the original data. Right: S-wave impedance from equation (6) using ordinary linear regression (yellow curve) and robust regression (Hubert-estimators in magenta curve, Cauchy estimators in green curve, Andrews-estimators in red curve) with the original data. Note the large misfit observed at near 2020m and at near 2600 m.

3.2 Application to real data

The real data is from a demo dataset distributed with the Hampson-Russell (H-R) inversion package. This 2 D prestack seismic data is inverted to give elastic impedance volumes. Five angle limited stacks have been generated, and so five elastic impedance volumes. Fitting equation (6) to the logarithm elastic impedance values provides P and S-impedances. From figures below, around 600ms and 650 ms (yellow ellipse), it is clear that P-impedance section (figure 3-left) highlights the upper and lower limits of sand gas , while this limits are blurred on S-impedance section ; supporting the idea that the low P-impedance values observed in this zone (yellow ellipse) correspond to the presence of gas.

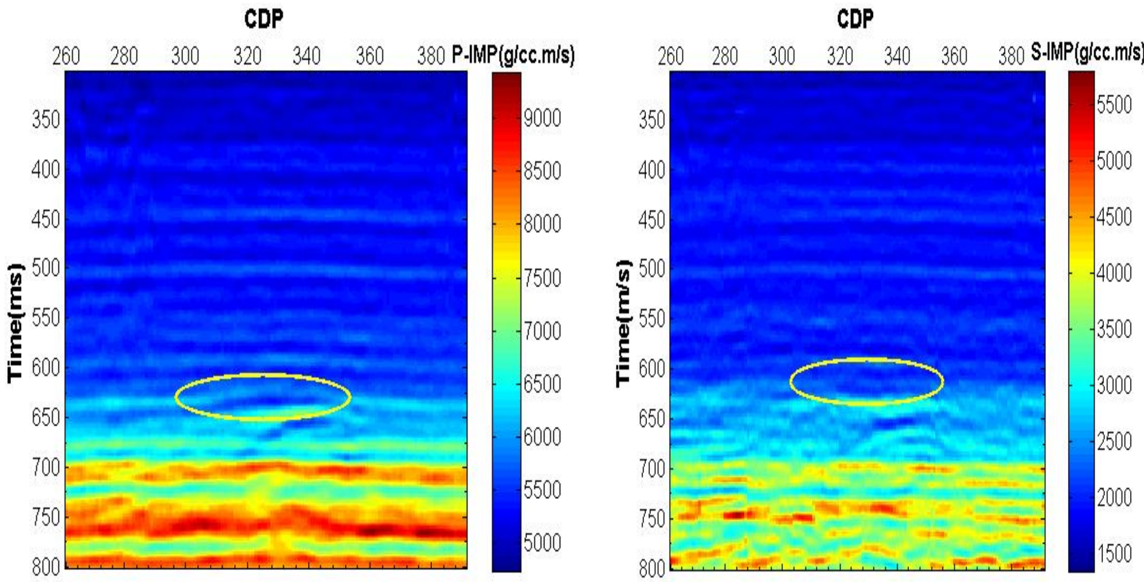

Figure3. Left: P-impedance section obtained from equation (6) using the robust regression. Right : S-impedance section obtained from equation (6) using the robust regression.

4. Conclusions

The thorny issue that often hampers the accuracy of elastic parameters derived from equations described above is the presence of noise. With several origins, it can create outliers in the data and thus distort inversion results. When data distribution (logarithm elastic impedance) obeys to Gaussian distribution, both linear regression and robust regression provide similar results, however when data distribution doesn't obey to Gaussian distribution, S-impedance derived from equation (6) using linear regression may suffer from inaccuracy. In this paper, Robust regression has been introduced to deal with such accuracy and the inversion results show that under outliers in the data, robust linear regression should be the right choice.

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