Structure and Some Geometric Properties of Generalized Cesáro Type Spaces Defined by Weighted Means

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Abstract: In this paper, we extend the Class of Cesáro sequence spaces $Ces[(p_n),(q_n)]$, introduced by Khan and Rahman to a generalized Cesáro type spaces $Ces[(a_n),(p_n),(q_n)]$ defined by weighted means $(a_n),(q_n)$ and of positive real number powers (p_n) with $\inf_n p_n > 0$. We define a modular functional on this generalized Cesáro sequence space and show that it is a complete paranomed space, and when equipped with the Luxemburg norm is a Banach space, possessing H-property, is not rotund and therefore not locally uniformly rotund. [Journal of American Science. 2010;6(10):7-12]. (ISSN: 1545-1003).

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Introduction

Let (X, ‖.‖) be a real Banach space and let B(X) (respe. S(X)) be the closed unit ball (resp. unit sphere) of X.

A point $x_0 \in S(X)$ is called an H-point of B(X) if for any sequence (x_n), $x_n \in B(X)$ such that $\| x_n \| \to 1$ as $n \to \infty$, the weak convergence of x_n to x_0 $(write\ x_n \xrightarrow{W} x_0)$ implies that $\| x_n - x_0 \| \to 0$ as $n \to \infty$.

If every point of S(X) is an H-point of B(X); then X is said to have H–property (Kadec-Klee). A point $x \in S(X)$ is called an extreme point of B(X), if for any $y, z \in S(X)$, the equality x= $\frac{y+z}{2}$ implies y=z.

A Banach space X is said to be Rotund (R) if for every point of S(X) is an extreme point of B(X). A point $x \in S(X)$ is called a locally uniformly rotund (LUR)-point, if for any sequence (x_n) in B(X) such that $\| x_n + x \| \to 2$ as n→∞, there holds that $\| x_n - x \| \to 0$ as n→∞. If every point of S(X) is a LUR-point of B(X), then the space X is called locally uniformly rotund (LUR). It is known that if X is LUR, then it is rotund (R) and possesses property (H). However the converse of this last statement is not true in general. By ω, we denote the space of all real or complex sequences and by ℕ= {0, 1,2, …}.

A linear topological space X over the real field ℝ is said to be a paranormed space if there is a sub additive function $g: X \to \mathbb{R}$ such that

$g(\theta) = 0$, $g(-x) = g(x)$ and for any sequence (x_n) in X such that

$g(x_n - x) \xrightarrow{n-\infty} 0$, and any sequence (α_n) in ℝ such that $|\alpha_n - \alpha| \xrightarrow{n-\infty} 0$, we get $g(\alpha_n x_n - \alpha x) \xrightarrow{n-\infty} 0$.

For these geometric notions and their role in mathematics we refer to the monographs [1], [2], [3], [4], and [5]. Some of these geometric properties were studied for orlicz spaces in [9], [10], [11], and [12].

In [5], Sanhan and Suantai investigated some geometrical properties

of $Ces((p_n))$ defined by

$Ces((p_n)) =$

$$\left\{ x \in \omega : \sum_{n=0}^{\infty} \left(\frac{1}{n+1} \sum_{k=2^n}^{2^{n+1}-1} |x_k| \right)^{p_n} < \infty \right\},$$ for any

bounded sequence (p_n) of positive real numbers, with $\inf_n p_n > 0$.

In [6] Khan and Rahman, generalized the space $Ces((p_n))$ by defining the space Ces $((p_n),(q_n))$, for positive sequences (p_n), (q_n) of real numbers, with $\inf_n p_n > 0$ by $Ces((p_n),(q_n)) =$

$$\left\{ x \in \omega : \sum_{n=0}^{\infty} \left(\frac{1}{Q_{2^n}} \sum_{k=2^n}^{2^{n+1}-1} q_k |x_k| \right)^{p_n} < \infty \right\},$$

where $Q_{2^n} = q_{2^n} + q_{2^n+1} + ... + q_{2^{n+1}-1}$.

Moreover they showed that $Ces((p_n),(q_n))$ is a paranomed space by the paranorm

$$g(x) = \left[\sum_{n=0}^{\infty} \left(\frac{1}{Q_{2^n}} \sum_{k=2^n}^{2^{n+1}-1} q_k |x_k| \right)^{p_n} < \infty \right]^{\frac{1}{M}},$$

where $M = \max\{1, H\}$, and $H = \sup_n P_n < \infty$.

For a real vector space X, a function $\sigma : X \to [0,\infty]$ is called modular, if it satisfies the following conditions:

(i) $\sigma(x) = 0 \Leftrightarrow x = 0$, $\forall x \in X$

(ii) $\sigma(\lambda x) = \sigma(x)$, for all $\lambda \in \mathbb{R}$ with $|\lambda| = 1$,

(iii) $\sigma(\lambda x + \beta y) \le \sigma(x) + \sigma(y)$, $\forall x, y \in X$, $\forall \lambda, \beta \ge 0;\ \lambda + \beta = 1$.

Further, the modular σ is called convex if

(iv) $\sigma(\lambda x + \beta y) \le \lambda\sigma(x) + \beta\sigma(y)$, $\forall x, y \in X$, $\forall \lambda, \beta \ge 0;\ \lambda + \beta = 1$.

We now introduce a generalized modular sequence space defined by weighted means.

Definition: let $(a_n),(q_n)$ and (p_n) be sequences of positive real numbers with $\inf_n p_n > 0$, we generalize the space Ces $((p_n),(q_n))$ by defining

$$Ces((a_n),(p_n),(q_n)) = \{x \in \omega : \sigma(\lambda x) < \infty, \text{for some } \lambda > 0\}$$

, where $\sigma(x) = \sum_{n=0}^{\infty} (a_n \sum_{k=2^n}^{2^{n+1}-1} q_k |x_k|)^{p_n}$. In the case when the sequence (p_n) is bounded we can simply write

$Ces((a_n),(p_n),(q_n)) =$

$$\left\{ x \in \omega : \sum_{n=0}^{\infty} (a_n \sum_{k=2^n}^{2^{n+1}-1} q_k |x_k|)^{p_n} < \infty \right\}.$$

The Luxemburg norm on the sequence space $Ces((a_n),(p_n),(q_n))$ is defined for any $x \in Ces((a_n),(p_n),(q_n))$ by:

$$\| x \| = \inf \left\{ \lambda > 0 : \sigma(\frac{x}{\lambda}) \le 1 \right\}.$$

Remarks:

(1) Taking

$$a_n = \frac{1}{n+1};\ q_n = 1, \forall n \in \mathbb{N}.$$

then $Ces((a_n),(p_n),(q_n)) = Ces(p_n)$.

(2)Taking $a_n = \frac{1}{Q_{2^n}}$,

where $Q_{2^n} = q_{2^n} + q_{2^n+1} + ... + q_{2^{n+1}-1}$, then

$Ces((a_n),(p_n),(q_n)) =$ $Ces((p_n),(q_n))$ studied by Khan and Rahman [13].

(3)Taking $a_n = \frac{1}{n+1}$, $q_n = 1$, $p_n = p$, $\forall n \in \mathbb{N}$, then $Ces((a_n),(p_n),(q_n)) =$ $Ces\ p$ studied by Lim [8].

Throughout this paper, the sequence (p_n) is considered to be bounded with $\inf_n p_n > 0$, and let $M = \max\{1, H\}, H = \sup_n p_n$.

For any bounded sequence of positive numbers (p_k), we have

$|a_k+b_k|^{p_k} \le 2^{\max(p_k,1)-1}(|a_k|^{p_k}+|b_k|^{p_k}) \le 2^{M-1}(|a_k|^{p_k}+|b_k|^{p_k})$, where $a_k, b_k \in \mathbb{R}$.

Lemma (1):

The functional σ is convex modular on $Ces[(a_n),(p_n),(q_n)]$.

Proof: Let $x, y \in Ces[(a_n),(p_n),(q_n)]$. It is obvious that;

(i) $\sigma(x)=0 \Leftrightarrow x=0$,

(ii) $\sigma(\lambda x)=\sum_{n=0}^{\infty}(a_n\sum_{k=2^n}^{2^{n+1}-1}q_k|\lambda x_k|)^{P_n}=$

$\sum_{n=0}^{\infty}|\lambda|^{P_n}(a_n\sum_{k=2^n}^{2^{n+1}-1}q_k|x_k|)^{P_n}=\sigma(x)$,

$\forall\lambda:|\lambda|=1$

(iii) Using the convexity of the function $t \longrightarrow |t|^{P_k}, \forall k \in \mathbb{N}$, we get

$$\sigma(\lambda x+\beta y)=\sum_{n=0}^{\infty}(a_n\sum_{k=2^n}^{2^{n+1}-1}q_k|\lambda x_k+\beta y_k|)^{P_n} \le$$

$$\le \sum_{n=0}^{\infty}\left[\lambda\left(a_n\sum_{k=2^n}^{2^{n+1}-1}q_k|x_k|\right)+\beta\left(a_n\sum_{k=2^n}^{2^{n+1}-1}q_k|y_k|\right)\right]^{P_n}$$

$=\lambda\sigma(x)+\beta\sigma(y)$,

for $\lambda,\beta \ge 0$ with $\lambda+\beta=1$.

Lemma (2): For any $x \in Ces[(a_n),(p_n),(q_n)]$, the functional σ on $Ces[(a_n),(p_n),(q_n)]$ satisfies the following properties:

(i) If 0<r<1, then $r^H\sigma\left(\frac{x}{r}\right) \le \sigma(x)$

and $\sigma(rx) \le r\sigma(x)$,

(ii) if r>1, then $\sigma(x) \le r^H\sigma\left(\frac{x}{r}\right)$,

(iii) if r≥1, then $\sigma(x) \le r\sigma(x) \le \sigma(rx)$.

Proof **:** (i) For 0<r<1, we get

$$\sigma(x)=\sum_{n=0}^{\infty}\left(a_n\sum_{k=2^n}^{2^{n+1}-1}q_k|x_k|\right)^{P_n}$$

$$=\sum_{n=0}^{\infty}r^{P_n}\left(a_n\sum_{k=2^n}^{2^{n+1}-1}q_k\left|\frac{x_k}{r}\right|\right)^{P_n} \ge r^H\sigma\left(\frac{x}{r}\right).$$

(ii) For r>1, we get

$$\sigma(x)=\sum_{n=0}^{\infty}\left(a_n\sum_{k=2^n}^{2^{n+1}-1}q_k|x_k|\right)^{P_n}=$$

$$\sum_{n=0}^{\infty}\left(a_n r\sum_{k=2^n}^{2^{n+1}-1}q_k\left|\frac{x_k}{r}\right|\right)^{P_n}$$

$$\le r^H\sum_{n=0}^{\infty}\left(a_n\sum_{k=2^n}^{2^{n+1}-1}q_k\left|\frac{x_k}{r}\right|\right)^{P_n} \le r^H\sigma\left(\frac{x}{r}\right).$$

(iii) It is clear that (iii) is satisfied by the convexity of σ.

Lemma (3): For any $x \in Ces[(a_n),(p_n),(q_n)]$, the following assertions are satisfied:

(i) If $\|x\|<1$, then $\sigma(x) \le \|x\|$,

(ii) if $\|x\|>1$, then $\sigma(x) \ge \|x\|$,

(iii) $\|x\|=1$ if and only if $\sigma(x)=1$,

(iv) if $0<r<1$ and $\|x\|>r$, then $\sigma(x)>r^H$,

(v) if $r \ge 1$ and $\|x\|<r$, then $\sigma(x)<r^H$.

Proof : It can be proved with standard techniques in a similar way as in [5,13].

Lemma(4): Let (x_n) be a sequence in $Ces[(a_n),(p_n),(q_n)]$,

(i) if $\lim_{n\to\infty}\|x_n\|=1$, then $\lim_{n\to\infty}\sigma(x_n)=1$,

(ii) if $\lim_{n\to\infty}\sigma(x_n)=0$, then $\lim_{n\to\infty}\|x_n\|=0$.

Proof: (i) Suppose that $\lim_{n\to\infty}\|x_n\|=1$. Then for any $\varepsilon \in (0,1)$ there exists n_o such that

$1-\varepsilon<\|x_n\|<1+\varepsilon \ \forall \ n \ge n_0$. By lemma (3), $(1-\varepsilon)^H<\sigma(x_n)<(1+\varepsilon)^H$ implies that

$\underset{n\to\infty}{Lim}\,\sigma(x_n)=1.$

(ii) If $\underset{n\to\infty}{Lim}\| x_n \|\neq 0$, then there is an $\varepsilon\in(0,1)$ and a subsequence (x_{n_k}) such that $\| x_{n_k} \|>\varepsilon^H \ \forall\ k\in\mathbb{N}$. This implies that $\underset{n\to\infty}{Lim}\,\sigma(x_{n_k})\neq 0$ and

Hence $\underset{n\to\infty}{Lim}\,\sigma(x_n)\neq 0$.

Lemma(5): Let $x, x_n \in Ces[(a_n),(p_n),(q_n)]$, $\forall n\in\mathbb{N}$ .If $\sigma(x_n)\to\sigma(x)$ as $n\to\infty$ and $x_n(i)\to x(i)$ as $n\to\infty\ \forall\ i\in\mathbb{N}$, then $x_n\to x$ as $n\to\infty$.

Proof:

Since, $\sigma(x)=\sum_{r=0}^{\infty}(a_r\sum_{k=2^r}^{2^{r+1}-1}q_k\,|\,x(k)\,|)^{P_r}<\infty$, then for $\varepsilon>0$, there exists $r_0\in\mathbb{N}$ such tha

$$\sum_{r=r_0+1}^{\infty}(a_r\sum_{k=2^r}^{2^{r+1}-1}q_k\,|\,x(k)\,|)^{P_r}<\frac{\varepsilon}{3(2^{M+1})},\quad (1)$$

Since

$$\sigma(x_n)-\sum_{r=0}^{r_0}(a_r\sum_{k=2^r}^{2^{r+1}-1}q_k\,|\,x_n(k)\,|)^{P_r}\to\sigma(x)-\sum_{r=0}^{r_0}(a_r\sum_{k=2^r}^{2^{r+1}-1}q_k\,|\,x(k)\,|)^{P_r}$$

as $n\to\infty$ and $x_n(k)\to x(k)$ as $n\to\infty$,

$\forall\ k\in\mathbb{N}$, there exists $r_0\in\mathbb{N}$ such that $\forall\, r\geq r_0$

$$\left|\sum_{r=r_0+1}^{\infty}(a_r\sum_{k=2^r}^{2^{r+1}-1}q_k\,|\,x_n(k)\,|)^{P_r}-\sum_{r=r_0+1}^{\infty}(a_r\sum_{k=2^r}^{2^{r+1}-1}q_k\,|\,x(k)\,|)^{P_r}\right|<\frac{\varepsilon}{3(2^M)}$$

. (2)

Since $x_n(k)\to x(k)$ as $n\to\infty$ then for every $n\geq n_0$ we get $|\,x_n(k)-x(k)\,|<\varepsilon$ for some n_0. As a result we get

$$\sum_{r=0}^{r_0}(a_r\sum_{k=2^r}^{2^{r+1}-1}q_k\,|\,x_n(k)-x(k)\,|)^{P_r}<\frac{\varepsilon}{3}$$

$\forall\, n\geq n_0$. (3)

From (1), (2) and (3) it follows that for $n\geq n_0$, we have

$$\sigma(x_n-x)=\sum_{r=0}^{\infty}\left(a_r\sum_{k=2^r}^{2^{r+1}-1}q_k\,|\,x_n(k)-x(k)\,|\right)^{P_r}=$$

$$\sum_{r=0}^{r_0}(a_r\sum_{k=2^r}^{2^{r+1}-1}q_k\,|\,x_n(k)-x(k)\,|)^{P_r}+\sum_{r=r_0+1}^{\infty}(a_r\sum_{k=2^r}^{2^{r+1}-1}q_k\,|\,x_n(k)-x(k)\,|)^{P_r}$$

$$<\frac{\varepsilon}{3}+2^M\left[\sum_{r=r_0+1}^{\infty}\left(a_r\sum_{k=2^r}^{2^{r+1}-1}q_k\,|\,x_n(k)\,|\right)^{P_r}+\sum_{r=r_0+1}^{\infty}\left(a_r\sum_{k=2^r}^{2^{r+1}-1}q_k\,|\,x(k)\,|\right)^{P_r}\right]$$

$$<\frac{\varepsilon}{3}+2^M\left[2\sum_{r=r_0+1}^{\infty}\left(a_r\sum_{k=2^r}^{2^{r+1}-1}q_k\,|\,x(k)\,|\right)^{P_r}+\frac{\varepsilon}{3(2^M)}\right]$$

$$<\frac{\varepsilon}{3}+\frac{\varepsilon}{3}+\frac{\varepsilon}{3}=\varepsilon$$

This shows that $\underset{n\to\infty}{Lim}\,\sigma(x_n-x)=0$ and by lemma 4 (ii), we get $\underset{n\to\infty}{Lim}\| x_n-x \|=0$.

Main results

Theorem (1): $Ces[(a_n),(p_n),(q_n)]$ is a Banach space with respect to the Luxemburg norm defined by$\| x \|=\inf\left\{\rho>0:\sigma\left(\frac{x}{\rho}\right)\leq 1\right\}$.

Proof: Let $x_n=(x_n(k))_{k=1}^{\infty}, n=0,1,2,...$ be a Cauchy sequence in $Ces[(a_n),(p_n),(q_n)]$ according to the Luxemburg norm. Thus $\forall\,\varepsilon\in(0,1)$ $\exists$ n_o such that $\| x_n-x_m \|<\varepsilon^M\ \forall\ m,n\geq n_0$. By Lemma 3(i) we obtain

$$\sigma(x_n-x_m)<\| x_n-x_m \|<\varepsilon^M\ \forall\ m,n\geq n_0. \quad (4)$$

That is $\sum_{r=0}^{\infty}\left(a_r\sum_{k=2^r}^{2^{r+1}-1}q_k\,|\,x_n(k)-x_m(k)\,|\right)^{P_r}<\varepsilon^M$

$\forall\, m,n\geq n_0$. For any k we get

$|\,x_n(k)-x_m(k)\,|<\varepsilon\,\forall\, m,n\geq n_0$, and the sequence $(x_n(k))$ is a Cauchy sequence of real numbers. Let $x(k)=\lim_{n\to\infty} x_n(k)$, then from inequality (4), we can write

$$\sum_{r=0}^{\infty}\left(a_n\sum_{k=2^r}^{2^{r+1}-1} q_k\,|x_n(k)-x(k)|\right)^{p_r} < \varepsilon^M ,$$

$\forall n\geq n_0$.That is, $\sigma(x_n-x)<\varepsilon^M \Rightarrow x_n\to x$ as $n\to\infty$.

By the following calculations,

$$\sum_{r=0}^{\infty}\left(a_r\sum_{k=2^r}^{2^{r+1}-1} q_k\,|x(k)|\right)^{p_r} = \sum_{r=0}^{\infty}\left(a_r\left(\sum_{k=2^r}^{2^{r+1}-1} q_k\,|x(k)-x_n(k)|+\sum_{k=2^r}^{2^{r+1}-1} q_k\,|x_n(k)|\right)\right)^{p_r}$$

$$\leq 2^{M-1}\left[\left(\sum_{r=0}^{\infty}\left(a_r\sum_{k=2^r}^{2^{r+1}-1} q_k\,|x(k)-x_n(k)|\right)^{p_r}\right)+\sum_{r=0}^{\infty}\left(a_r\sum_{k=2^r}^{2^{r+1}-1} q_k\,|x_n(k)|\right)^{p_r}\right]$$

$< \varepsilon$,

we see that the sequence x_n converges to

$x=(x(k))\in Ces[(a_n),(p_n),(q_n)]$.This completes the proof.

Theorem(2)**:**The space $Ces[(a_n),(p_n),(q_n)]$ has the property Kadec-Klee (H-property).

Proof. Let $x\in S(Ces[(a_n),(p_n),(q_n)])$ and $x\in B(Ces[(a_n),(p_n),(q_n)])$ $\forall n\in\mathbb{N}$ such that $\|x_n\|\to 1$ and $x_n \xrightarrow{W} x$ as $n\to\infty$. From Lemma 3(iii), and Lemma 4(i) we get $\sigma(x)=1$ and that $\sigma(x_n)\to\sigma(x)$ as $n\to\infty$. Since

$x_n \xrightarrow{W} x$ and the i^{th}-coordinate mapping Π_i: $Ces[(a_n),(p_n),(q_n)]\to\mathbb{R}$ defined by $\Pi_i(x)=x_i$ is a continuous linear functional, it follows that $x_n(i)\to x(i)$ as $n\to\infty$ for all $i\in\mathbb{N}$. Thus we obtain by Lemma 5 that $x_n\to x$ as $n\to\infty$.

Theorem (3) The space $Ces[(a_n),(p_n),(q_n)]$ is not rotund, and so is not LUR.

Proof: It is sufficient to give a counter example. Choosing

$x=\left(\frac{1}{a_0q_1\sqrt[p_0]{2}},0,\frac{1}{a_1q_3\sqrt[p_1]{2}},0,0,0,.....\right)$ and

$y=\left(\frac{1}{a_0q_1\sqrt[p_0]{2}},\frac{1}{a_1q_2\sqrt[p_1]{2}},0,0,0,.....\right)$, we see that

$x,y\in S(Ces[(a_n),(p_n),(q_n)])$, and their

midpoint $(x+y)/2\in S(Ces[(a_n),(p_n),(q_n)])$. This shows that (x+y)/2 while belonging to $S(Ces[(a_n),(p_n),(q_n)])$, is not an extreme point for $B(Ces[(a_n),(p_n),(q_n)])$.

Corollary

(1) $Ces(p)$ is not rotund, see [5].

(2) $Ces[(p_n),(q_n)]$ is not rotund, see [13].

Finally, we get the following:

Theorem (4): The space $Ces[(a_n),(p_n),(q_n)]$ is a complete linear metric space with respect to the paranorm defined by

$$g(x)=\left[\sum_{n=0}^{\infty}\left(a_n\sum_{k=2^n}^{2^{n+1}-1} q_k|x_k|\right)^{p_n}\right]^{\frac{1}{M}}.$$

Proof: The proof of linearity of

$Ces[(a_n),(p_n),(q_n)]$ with respect to the coordinate wise addition and multiplication follows from the following inequalities which are satisfied for all $x,y\in Ces[(a_n),(p_n),(q_n)]$

$$\left[\sum_{n=0}^{\infty}\left(a_n\sum_{k=2^n}^{2^{n+1}-1} q_k|x_k+y_k|\right)^{p_n}\right]^{\frac{1}{M}} \leq \left[\sum_{n=0}^{\infty}\left(a_n\sum_{k=2^n}^{2^{n+1}-1} q_k|x_k|\right)^{p_n}\right]^{\frac{1}{M}}+\left[\sum_{n=0}^{\infty}\left(a_n\sum_{k=2^n}^{2^{n+1}-1} q_k|y_k|\right)^{p_n}\right]^{\frac{1}{M}}$$

(5), and $|\alpha|^{p_n}\leq\max\{1,|\alpha|^M\}$ for any $\alpha\in\mathbb{R}$.

We now verify that g(x) is a paranorm over the space $Ces[(a_n),(p_n),(q_n)]$. In fact,

(i) $g(\theta)=0$ (obvious)

(ii) $g(-x)=g(x)$, $\forall x\in Ces[(a_n),(p_n),(q_n)]$

(iii) $g(x+y)\leq g(x)+g(y)$, $\forall x,y\in Ces[(a_n),(p_n),(q_n)]$, follows from the inequality (5).

(iv) Let (x_m) be any sequence in $Ces[(a_n),(p_n),(q_n)]$ such that $g(x_m - x)\xrightarrow{m-\infty}0$; let (α_m) be any sequence in $\mathbb{R}$ such that $|\alpha_m-\alpha|\xrightarrow{m-\infty}0$, since $x_m = x+(x_m - x)$ then we get

$g(x_m)\le g(x)+g(x_m-x)$. Hence $\{g(x_m)\}$ is bounded and we have

$$g(\alpha_m x_m-\alpha x)=\left[\sum_{n=0}^{\infty}\left(a_n\sum_{k=2^n}^{2^{n+1}-1}q_k\left|\alpha_m x_m(k)-\alpha x(k)\right|\right)^{p_n}\right]^{\frac{1}{M}}$$

$$=\left[\sum_{n=0}^{\infty}\left(a_n\sum_{k=2^n}^{2^{n+1}-1}q_k\left|(\alpha_m-\alpha)(x_m(k))+\alpha(x_m(k)-x(k))\right|\right)^{p_n}\right]^{\frac{1}{M}},$$

this tends to zero as $m\to\infty$.

The completeness of the space $Ces[(a_n),(p_n),(q_n)]$ is a routine verification by using standard techniques as theorem (1).

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