Separation Distance Determination of Torsional Adjacent Buildings Using Random Vibration Theory

Mojtaba Labibzadeh [1], Navid Siahplolo [1], Amin Khajehdezfuly [1]

[1.] Department of Civil Engineering, Faculty of Engineering, Shahid Chamran University, Ahvaz, Iran
Labibzadeh_m@scu.ac.ir

Abstract: One of the phenomena observed during strong earthquakes is the pounding of adjacent buildings which has been known as the pounding in dynamic of structures science. This pounding sometimes can lead to significant pounding forces which ultimately resulted into the destruction of adjacent buildings. Various methods for prevention against to the pounding were having been proposed. Most simple and practical method to mitigate this damaging force is to provide adequate separation distance between adjacent buildings which has become a standard criterion in all structural codes. Various dynamic analyses such as time history, spectral response and random vibration method have been applied for determining of that separation distance. In the mentioned methods often the torsion behavior of buildings due to their inherent asymmetrical forms has been neglected. In this work an attempt has been performed to calculate the required separation distance in asymmetric buildings via some analytical relations developments using the torsional-lateral behavior. The most important factor which previous studies did not consider is the effect of similarity of torsional modes of two adjacent building in reducing the necessary separation distance assignment. The random vibration method was implemented as the analytic solution method and the effects of various parameters such as eccentricity, damping and natural frequency were taken into account in separation distance determination and the obtained results were compared to UBC97, IBC 2006 and standard No.2800 of Iran structural codes. The results were presented in the form of some graphs. It is important to note that throughout the entire of study the linear elastic response analysis was applied. [Journal of American Science. 2010;6(10):184-194]. (ISSN: 1545-1003).

Keywords: Pounding force, Separation distance, Random Vibration Method, Torsional behavior

1. Introduction

Insufficient distance between adjacent buildings would probably result into striking of the buildings during strong earthquakes and subsequently create an excessive dynamic force called pounding. Pounding is the collision of adjacent buildings during an earthquake due to insufficient lateral clearance. Pounding has been the cause of a number of mid-rise building collapses, most notably in the 1985 Mexico City Earthquake (Chen and Lui, 2006, Kasai and Maison, 1989). In buildings with a torsional-lateral behavior it is necessary to calculate and implement proper separation distance between adjacent buildings in contrast to the case in which the buildings with only have lateral deflections where usually a simply predefined linear varied separation distance can be used (UBC97, 1997,IBC2006, 2006,2008Iran 2007). In the former, a coupled torsional-lateral response is being created which may result into the increase or decrease of the relative displacement between adjacent buildings.

After the Loma preita earthquake in 1989, the pounding force and the seeking of practical solutions to mitigate such a force were being seriously taken into account. In order to mitigate the pounding force scientists have proposed various methods, the most important being:

1. Creating of a suitable separation distance (the placing of two buildings at a calculated distance from one another)
2. Placing of the story of each floor for both buildings at same story level
3. Unifying the response of both structures by joining the building via a link beam
4. Utilizing of Bomber walls
5. Foreseeing and implementation of sufficient lateral resistant elements in order to restrict the extent of the relative displacement of the structure

It may be deduced that between mentioned methods, the first one is in most cases the most economic solution method and in this paper we are going to focus on it. The separation distance between adjacent buildings is calculated on the basis of a series of variables such as the building's mass , story stiffness, damping , height of each story, largeness, earthquake time duration , and the

behavior characteristics of the building itself whether shear or lateral-torsional. Standard building code specifications provide various methods for the estimating/ or calculating of adequate separation distance between buildings for example in the UBC97, it is specified that the separation distance between two buildings should be calculated on the basis of the algebraic sum of the non-linear relative displacement of the two adjacent buildings . In the IBC2006, it has been stated that the sum of squared lateral displacements between two adjacent buildings should be used as the basis for calculating the required separation distance; however when adjacent buildings show tandem vibration characteristics the calculated separation distance would provide to be excessive in extent. According to the Iranian Seismic Resistance Building Code, the separation distance between adjacent buildings is defined as the five thousandth of the total height of each story from base level. This distance must be provided from the property line of the building in each side. It is also mentioned in the code that this distance must be always greater than the calculated lateral displacements based on the proper nonlinear analysis (2800Iran, 2007).
Most of previous researches used only lateral mode of kinematics of two adjacent building in determination of separation distance. None of them considered torsional-lateral behavior of two adjacent building in their separation distance calculations (Ohta, et. al., 2006, Mahmoud and Jankowski, 2009, Shehata, 2006, Jankowski, 2005, Lopez and Soong, 2009). In this paper this effect will be examined.

2. Material and Methods

2.1. The Relative Displacement Function

The relative displacement function of building **a** in comparison to building **b** is as follows (Chopra, 2004):

$$\Delta(t) = u_b(t) + (\frac{D_b}{2})\theta_b(t) - u_a(t) - (\frac{D_a}{2})\theta_a(t) \qquad (1)$$

In this equation $u_b(t)$ and $u_a(t)$ are the horizontal displacement of the center of mass for building **a** and **b** respectively, whereas $\theta_b(t)$ and $\theta_a(t)$ are the amount of the rotational displacement of the center of mass for building **a** and **b**. also Da and D_b are the length of building **a** and **b** vertical to the earthquake direction.
The displacement Function in time $t+\tau$ as per equation (1) can be replaced by $t+\tau$ instead of t thus developing the following equation:

$$\Delta(t+\tau) = u_b(t+\tau) + (\frac{D_b}{2})\theta_b(t+\tau) - u_a(t+\tau) - (\frac{D_a}{2})\theta_a(t+\tau) \qquad (2)$$

u_a, u_b in equation (1) and (2) can be found using modal Analysis method, thus the equation (1) becomes :

$$\Delta(t) = \sum_{j=1}^{2N_b} \varphi_b(N_a, j)\cdot y_{bj}(t) + \sum_{j=1}^{2N_b}(\frac{D_b}{2})\cdot\varphi_b(N_a+N_b, j)\cdot y_{bj}(t) - \sum_{k=1}^{2N_a}\varphi_a(N_a,k)\cdot y_{ak}(t) - \sum_{k=1}^{2N_a}(\frac{D_a}{2})\cdot\varphi_a(2N_a,k)\cdot y_{ak}(t) \qquad (3)$$

$\Delta(t+\tau)$ is obtained from (3) by replacing t by $t+\tau$ .In relation (3) $\varphi_b(N_a, j)$ and $\varphi_b(N_a+N_b, j)$ are corresponding component of jth lateral and torsion degree of freedom in the j mode for building **b** at the level of the roof of building **a** while $\varphi_a(N_a,k)$ and $\varphi_a(2N_a,k)$ are corresponding component of jth lateral and torsion degree of freedom in the kth mode for building **a.** Furthermore $y_{bj}(t)$ and $y_{aj}(t)$ are the normalized coordinate of the Jth mode for buildings **a** and **b**, respectively.
N_b and N_a both refer to the number of stories for both buildings **a** and **b**, respectively. In order to determine the relative velocity function, we must differentiate the relative displacement function over time.

2.2. The Auto- Correlation Function

The auto- correlation function of the relative displacement function of adjacent structures is:

$$R_{\Delta\Delta} = E[\Delta(t)\cdot\Delta(t+\tau)] \qquad (4)$$

By substituting (1) and (2) in (4) and the rewriting of the equation using the modal method, the auto-Correlation function can be stated as:

$$R_{\Delta\Delta} = E[\{\sum_{j=1}^{2N_b}\varphi_b(N_a,j)\cdot y_{bj}(t) + \sum_{j=1}^{2N_b}(\frac{D_b}{2})\cdot\varphi_b(N_a+N_b,j)\cdot y_{bj}(t) - \sum_{k=1}^{2N_a}\varphi_a(N_a,k)\cdot y_{ak}(t) - \sum_{k=1}^{2N_a}(\frac{D_a}{2})\cdot\varphi_a(2N_a,k)\cdot y_{ak}(t)\}$$
$$\times\{\sum_{j=1}^{2N_b}\varphi_b(N_a,j)\times y_{bj}(t+\tau) + \sum_{j=1}^{2N_b}(\frac{D_b}{2})\cdot\varphi_b(N_a+N_b,j)\cdot y_{bj}(t+\tau) - \sum_{k=1}^{2N_a}\varphi_a(N_a,k)\cdot y_{ak}(t+\tau) - \sum_{k=1}^{2N_a}(\frac{D_a}{2})\cdot\varphi_a(2N_a,k)\cdot y_{ak}(t+\tau)\}] \qquad (5)$$

Equation (5) is rewritten as follow for simplicity:

$$R_{\Delta\Delta} = R_{u_{b,Na}u_{b,Na}} + R_{u_{b,Na}\theta_{b,Na+Nb}} - R_{u_{b,Na}u_{a,Na}} - R_{u_{b,Na}\theta_{a,2Na}} + R_{\theta_{b,Na+Nb}u_{b,Na}} + R_{\theta_{b,Na+Nb}\theta_{b,Na+Nb}} - R_{\theta_{b,Na+Nb}u_{a,Na}} - R_{\theta_{b,Na+Nb}\theta_{a,2Na}} - R_{u_{a,Na}u_{b,Na}} - R_{u_{a,Na}\theta_{b,Na+Nb}} + R_{u_{a,Na}u_{a,Na}} + R_{u_{a,Na}\theta_{a,2Na}} - R_{\theta_{a,2Na}u_{b,Na}} - R_{\theta_{a,2Na}\theta_{b,Na+Nb}} + R_{\theta_{a,2Na}u_{a,Na}} + R_{\theta_{a,2Na}\theta_{a,2Na}} \qquad (6)$$

In above equation for example $R_{u_{a,Na}u_{a,Na}}$ is the cross correlation function of displacement building **a** at floor Na and displacement building **a** at floor Na .The auto- correlation function has 16 terms. The first term is calculated as follow and other term's calculations are similar.

$$R_{u_{b,Na}u_{b,Na}} = E[\sum_{j=1}^{2Nb}\sum_{k=1}^{2Nb}\varphi_b(N_a,j)\varphi_b(N_a+N_b,k)\times$$
$$y_{bj}(t)y_{bk}(t+\tau)] \quad (7)$$

The relationship between the elements of the equation can be restated:

$$R_{u_{b,Na}u_{b,Na}} = \sum_{j=1}^{2Nb}\sum_{k=1}^{2Nb}\varphi_b(N_a,j)\varphi_b(N_a,k)\times$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}E[p_{bk}(t-\theta_2)p_{bj}(t-\theta_1)]h_{bj}(\theta_1)h_{bk}(\theta_2)d\theta_1 d\theta_2 \quad (8)$$

For building b;

$$E[p_{bj}(t-\theta_1)p_{bk}(t+\tau-\theta_2)]=$$
$$E[\sum_{r=1}^{Nb}\sum_{s=1}^{Nb}\varphi_b(r,j)\varphi_b(s,k)f_{br}(t-\theta_1)f_{bs}(t+\tau-\theta_2)] \quad (9)$$

This becomes significance due to the following:

$$R_{f_{br}f_{bs}} = E[f_{br}(t)f_{bs}(t+\tau)] \quad (10)$$

In the above equation $f_{br}(t)$ and $f_{bs}(t)$ are respectively rth and sth of the vector for the external force of building b, hence the final form for equation (8) would be as below:

$$R_{u_{b,Na}u_{b,Na}} == \sum_{j=1}^{2Nb}\sum_{k=1}^{2Nb}\varphi_b(N_a,j)\varphi_b(N_a,k)\times$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\sum_{r=1}^{Nb}\sum_{s=1}^{Nb}\varphi_b(r,j)\varphi_b(s,k)R_{f_{br}f_{bs}})h_{bj}(\theta_1)h_{bk}(\theta_2)d\theta_1 d\theta_2 \quad (11)$$

In this equation the functions $h_{aj}(\theta_1)$ and $h_{bj}(\theta_1)$ are respectively the unit impulse response functions of the jth mode of buildings ***a*** and ***b***. Other terms in relation 8 can be defined through a similar way. In order to determine the auto correlation function of the relative velocity of systems similar calculations can be applied.

3.2. The Power Spectral Density Function (PSD)

The Earthquake Spectral Density Function of the relative displacement is the Fourier transition of the relative displacement autocorrelation function of the adjacent buildings, thus:

$$S_{\Delta\Delta} = \frac{1}{2\pi}\int_{-\infty}^{\infty}R_{\Delta\Delta}(\tau)e^{-i\omega\tau}d\tau \quad (12)$$

Therefore by inserting (6) into (12), the equation can be rewritten as follows:

$$S_{\Delta\Delta} = S_{u_{b,Na}u_{b,Na}}(\omega)+S_{u_{b,Na}\theta_{b,Na+Nb}}(\omega)-S_{u_{b,Na}u_{a,Na}}(\omega)-S_{u_{b,Na}\theta_{a,2Na}}(\omega)$$
$$+S_{\theta_{b,Na+Nb}u_{b,Na}}(\omega)+S_{\theta_{b,Na+Nb}\theta_{b,Na+Nb}}(\omega)-S_{\theta_{b,Na+Nb}u_{a,Na}}(\omega)-S_{\theta_{b,Na+Nb}\theta_{a,2Na}}(\omega)$$
$$-S_{u_{a,Na}u_{b,Na}}(\omega)-S_{u_{a,Na}\theta_{b,Na+Nb}}(\omega)+S_{u_{a,Na}u_{a,Na}}(\omega)+S_{u_{a,Na}\theta_{a,2Na}}(\omega)$$
$$-S_{\theta_{a,2Na}u_{b,Na}}(\omega)-S_{\theta_{a,2Na}\theta_{b,Na+Nb}}(\omega)+S_{\theta_{a,2Na}u_{a,Na}}(\omega)+S_{\theta_{a,2Na}\theta_{a,2Na}}(\omega) \quad (13)$$

In the above equation, for instance we have:

$$S_{u_{b,Na}\theta_{b,Na+Nb}} = \frac{1}{2\pi}\int_{-\infty}^{\infty}R_{u_{b,Na}\theta_{b,Na+Nb}}e^{-i\omega\tau}d\tau \quad (14)$$

In which by replacing amounts such as $R_{u_{b,Na}\theta_{b,Na+Nb}}$ we can obtain:

$$S_{u_{b,Na}\theta_{b,Na+Nb}} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\sum_{j=1}^{2Nb}\sum_{k=1}^{2Nb}(\frac{D_b}{2})\varphi_b(N_a,j)\varphi_b(N_a+N_b,k)$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{\frac{1}{2\pi}R_{f_{br}f_{bs}}(\tau-\theta_2+\theta_1)e^{-i\omega(\tau-\theta_2+\theta_1)}d(\tau-\theta_2+\theta_1)\}$$
$$h_{bj}(\theta_1)h_{bk}(\theta_2)e^{i\omega\theta_1}e^{-i\omega\theta_2}d\theta_1 d\theta_2 \quad (15)$$

Due to the fact that the vibration mode shape remains constant over time, it is possible to replace the Integral and summation in (15), thus obtaining:

$$S_{u_{b,Na}\theta_{b,Na+Nb}} = \sum_{j=1}^{2Nb}\sum_{k=1}^{2Nb}(\frac{D_b}{2})\varphi_b(N_a,j)\varphi_b(N_a+N_b,k)$$
$$\sum_{r=1}^{Nb}\sum_{s=1}^{Nb}\varphi_b(r,j)\varphi_b(s,k)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{\frac{1}{2\pi}R_{f_{br}f_{bs}}(\tau-\theta_2+\theta_1)e^{-i\omega(\tau-\theta_2+\theta_1)}$$
$$d(\tau-\theta_2+\theta_1)\}h_{bj}(\theta_1)h_{bk}(\theta_2)e^{i\omega\theta_1}e^{-i\omega\theta_2}d\theta_1 d\theta_2 \quad (16)$$

Using (14) the latest relation becomes:

$$S_{u_{b,Na}\theta_{b,Na+Nb}}(\omega) = \sum_{j=1}^{2Nb}\sum_{k=1}^{2Nb}(\frac{D_b}{2})\varphi_b(N_a,j)\varphi_b(N_a+N_b,k)$$
$$\{\sum_{r=1}^{Nb}\sum_{s=1}^{Nb}\varphi_b(r,j)\varphi_b(s,k)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}S_{f_{br}f_{bs}}(\omega)h_{bj}(\theta_1)e^{i\omega\theta_1}h_{bk}(\theta_2)e^{-i\omega\theta_2}d\theta_1 d\theta_2\} \quad (17)$$

Since the variable θ_1 and θ_2 are two independent variables, the double integral in eq.17 can be replaced by two single integral:

$$S_{u_{b,Na}\theta_{b,Na+Nb}}(\omega) = \sum_{j=1}^{2Nb}\sum_{k=1}^{2Nb}(\frac{D_b}{2})\varphi_b(N_a,j)\varphi_b(N_a+N_b,k)$$
$$\{\sum_{r=1}^{Nb}\sum_{s=1}^{Nb}\varphi_b(r,j)\varphi_b(s,k)S_{f_{br}f_{bs}}(\omega)\}\int_{-\infty}^{\infty}h_{bj}(\theta_1)e^{i\omega\theta_1}d\theta_1\int_{-\infty}^{\infty}h_{bk}(\theta_2)e^{-i\omega\theta_2}d\theta_2 \quad (18)$$

Now by using the definition of complex frequency response functions, the equation (18) can be rewritten as follows:

$$S_{u_{b,Na}\theta_{b,Na+Nb}}(\omega) = \sum_{j=1}^{2Nb}\sum_{k=1}^{2Nb}(\frac{D_b}{2})\varphi_b(N_a,j)\varphi_b(N_a+N_b,k)\times \quad (19)$$
$$\{\sum_{r=1}^{Nb}\sum_{s=1}^{Nb}\varphi_b(r,j)\varphi_b(s,k)S_{f_{br}f_{bs}}(\omega)\}\overset{*}{H}_{bj}(\omega)H_{bk}(\omega)$$

The other part of earthquake SPD function and cross SPD of the relative displacement of buildings ***a*** and ***b*** are capable of being estimated using the abovementioned calculations. In eq.19, $H_{bk}(\omega)$ and $\overset{*}{H}_{bk}(\omega)$ are the complex frequency response functions (Transference Function) of building ***b*** in the kth mode and its conjugate, respectively is as follows:

$$H_{bk} = \int_{-\infty}^{\infty}h_{bk}e^{-i\omega t}dt = \frac{1}{K_{bk}+i\omega C_{bk}-\omega^2 M_{bk}} \quad (20)$$

$$S_{f_{ar}f_{as}}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{E}[f_{ar}(t)f_{as}(t+\tau)]e^{-i\omega\tau}d\tau \quad (21)$$

$$S_{f_{br}f_{bs}}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{E}[f_{br}(t)f_{bs}(t+\tau)]e^{-i\omega\tau}d\tau \quad (22)$$

$S_{f_{ar}f_{bs}}(\omega)$, in fact, is the intersecting spectral density of *r*th (the vector for the external force of building **a**) and *s*th (the vector for the external force of building **b**) and $S_{f_{br}f_{as}}(\omega)$ is the function of the combined rate of the Frequency response of building b in the *r*th mode (the vector for the external force of building **b**) with corresponding quantity of building **a** in the *s*th t(he vector for the external force of building **a**). Thus from equations (23) and (24) the following results are obtained:

$$S_{f_{ar}f_{bs}}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{E}[f_{ar}(t)f_{bs}(t+\tau)]e^{-i\omega\tau}d\tau \tag{23}$$

$$S_{f_{br}f_{as}}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{E}[f_{br}(t)f_{as}(t+\tau)]e^{-i\omega\tau}d\tau \tag{24}$$

The earthquakes spectral density function for the relative velocity of the two corners of the adjacent buildings can be derived from the direct Fourier transform of coupling between the angles of the adjacent buildings and calculated as follows:

$$S_{\ddot{\Delta}\ddot{\Delta}}=\frac{1}{2\pi}\int_{-\infty}^{\infty}R_{\ddot{\Delta}\ddot{\Delta}}e^{-i\omega\tau}d\tau \tag{25}$$

4.2. The Mean Square of Relative Displacement Function

The Mean square of Relative Displacement Function which can be influenced by the relative displacement of adjacent buildings is defined as follows:

$$\sigma^2{}_z=\int_{-\infty}^{\infty}S_{\Delta\Delta}(\omega)d\omega \tag{26}$$

By inserting (13) INTO (26) the Mean squared of total displacement squares influenced by relative displacement is obtained as:

$$\sigma^2{}_z=\int_{-\infty}^{\infty}\{S_{u_{b,Na}u_{b,Na}}(\omega)+S_{u_{b,Na}\theta_{b,Na+Nb}}(\omega)-S_{u_{b,Na}u_{a,Na}}(\omega)$$
$$-S_{u_{b,Na}\theta_{a,2Na}}(\omega)+S_{\theta_{b,Na+Nb}u_{b,Na}}(\omega)+S_{\theta_{b,Na+Nb}\theta_{b,Na+Nb}}(\omega)-$$
$$S_{\theta_{b,Na+Nb}u_{a,Na}}(\omega)-S_{\theta_{b,Na+Nb}\theta_{a,2Na}}(\omega)-S_{u_{a,Na}u_{b,Na}}(\omega)-S_{u_{a,Na}\theta_{b,Na+Nb}}(\omega)$$
$$+S_{u_{a,Na}u_{a,Na}}(\omega)+S_{u_{a,Na}\theta_{a,2Na}}(\omega)-S_{\theta_{a,2Na}u_{b,Na}}(\omega)-S_{\theta_{a,2Na}\theta_{b,Na+Nb}}(\omega)$$
$$+S_{\theta_{a,2Na}u_{a,Na}}(\omega)+S_{\theta_{a,2Na}\theta_{a,2Na}}(\omega)\}d\omega \tag{27}$$

5.2. Explicit Form of Equation Formulation

The external force imposed upon the structure which is itself influenced by the induced earthquake excitations is as follows:

$$f(t)=\ddot{u}_g(t)m \tag{28}$$

In the above equation $[m]_{2N\times 2N}$ and $\ddot{u}_g(t)$ are the mass matrix of the structure and the earthquake's acceleration vector respectively. By using above formulation, equation (21) becomes:

$$S_{f_{ar}f_{as}}=\frac{1}{2\pi}\int_{-\infty}^{\infty}E[m_{ar}\ddot{u}_g(t)m_{as}\ddot{u}_g(t+\tau)]e^{-i\omega\tau}d\tau \tag{29}$$

Or

$$S_{f_{ar}f_{as}}=m_{ar}m_{as}\frac{1}{2\pi}\int_{-\infty}^{\infty}E[\ddot{u}_g(t)\ddot{u}_g(t+\tau)]e^{-i\omega\tau}d\tau \tag{30}$$

Regards to the definition of the earthquake spectral density function in (31) we have:

$$S_g(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}E[\ddot{u}_g(t)\ddot{u}_g(t+\tau)]e^{-i\omega\tau}d\tau \tag{31}$$

Thus (29) can be stated as:

$$S_{f_{ar}f_{as}}=m_{ar}m_{as}S_g(\omega) \tag{32}$$

And by the same way we shall obtain:

$$S_{f_{br}f_{bs}}=m_{br}m_{bs}S_g(\omega) \tag{33}$$

$$S_{f_{ar}f_{bs}}=m_{ar}m_{bs}S_g(\omega) \tag{34}$$

$$S_{f_{br}f_{as}}=m_{br}m_{as}S_g(\omega) \tag{35}$$

In the above equations, m_{ar} and m_{bs} are the mass of floor *r*th of building **a** and the mass of floor *s*th of building **b** respectively. The equation $S_g(\omega)$ is the earthquake spectral density function. It should be noted that in our study the earthquake consider as white noise process, within constant PSDF.

If the results from (33) and (35) are superimposed for each of the parameters in the equation of (27) and if the occurrence of the earthquake is in the form of white noise with a spectral density of S_0, thus we will have the following result for the first parameter:

$$\int_{-\infty}^{\infty}S_{u_{b,Na}u_{b,Na}}(\omega)d\omega=\sum_{j=1}^{2Nb}\sum_{k=1}^{2Nb}\varphi_b(N_a,j)\varphi_b(N_a,k)\times$$
$$\{\sum_{r=1}^{Nb}\sum_{s=1}^{Nb}\varphi_b(r,j)\varphi_b(s,k)\times\int_{-\infty}^{\infty}S_{f_{br}f_{bs}}(\omega)\overset{*}{H}_{bj}(\omega)H_{bk}(\omega)d\omega\}=$$
$$\sum_{j=1}^{2Nb}\sum_{k=1}^{2Nb}\varphi_b(N_a,j)\varphi_b(N_a,k)\times\{\sum_{r=1}^{Nb}\sum_{s=1}^{Nb}\varphi_b(r,j)\varphi_b(s,k)m_{br}m_{bs}\}$$
$$\times\int_{-\infty}^{\infty}S_0(\omega)\overset{*}{H}_{bj}(\omega)H_{bk}(\omega)d\omega \tag{36}$$

The above equation can replace by a simple summation and double summation as follow;

$$\int_{-\infty}^{\infty}S_{u_{b,Na}u_{b,Na}}(\omega)d\omega=\sum_{j=1}^{2Nb}\varphi_b{}^2(N_a,k)\{\sum_{r=1}^{Nb}\sum_{s=1}^{Nb}\varphi_b(r,j)\varphi_b(s,j)m_{br}m_{bs}\}$$
$$\times\int_{-\infty}^{\infty}S_0|H_{bj}(\omega)|^2d\omega+\sum_{j=1}^{2Nb-1}\sum_{k=j+1}^{2Nb}\varphi_b(N_a,j)\varphi_b(N_a,k)\times$$
$$\{\sum_{r=1}^{Nb}\sum_{s=1}^{Nb}\varphi_b(r,j)\varphi_b(s,k)m_{br}m_{bs}\times\int_{-\infty}^{\infty}S_0[H_{bj}(\omega)\times\overset{*}{H}_{bk}(\omega)$$
$$+\overset{*}{H}_{bj}(\omega)\times H_{bk}(\omega)]d\omega\} \tag{37}$$

As can be seen in equation (37) the most difficult aspect of calculating the relationship is determining its mathematical Integral, hence it is necessary to simplify the calculations as follows:

$$I = \int_{-\infty}^{\infty} S_0(\omega)[H_{bj}(\omega)\times \overset{*}{H}_{bk}(\omega)+\overset{*}{H}_{bj}(\omega)\times H_{bk}(\omega)]d\omega\} \quad (38)$$

The equation in parentheses in (38) can be rewritten as follows:

$$A=[H_{bj}(\omega)\times\overset{*}{H}_{bk}(\omega)+\overset{*}{H}_{bj}(\omega)\times H_{bk}(\omega)]=$$

$$[H_{bj}(\omega)\times\overset{*}{H}_{bk}(\omega)\frac{\overset{*}{H}_{bj}(\omega)\times H_{bk}(\omega)}{\overset{*}{H}_{bj}(\omega)\times H_{bk}(\omega)}+\overset{*}{H}_{bj}(\omega)\times H_{bk}(\omega)\frac{H_{bj}(\omega)\times\overset{*}{H}_{bk}(\omega)}{[H_{bj}(\omega)\times\overset{*}{H}_{bk}(\omega)}]$$

$$=\left|H_{bj}(\omega)\right|^2\left|H_{bk}(\omega)\right|^2[\frac{1}{\overset{*}{H}_{bj}(\omega)\times H_{bk}(\omega)}+\frac{1}{[H_{bj}(\omega)\times\overset{*}{H}_{bk}(\omega)}]$$

Thus the above relationship in **A** can be modified to state:

$$A=\left|H_{bj}(\omega)\right|^2\left|H_{bk}(\omega)\right|^2$$
$$[\frac{1}{\overset{*}{H}_{bj}(\omega)\times H_{bk}(\omega)}+\frac{1}{[H_{bj}(\omega)\times\overset{*}{H}_{bk}(\omega)}]$$

(39)

In the equation above $\left|H_{bj}(\omega)\right|^2$ and $\left|H_{bk}(\omega)\right|^2$ are the quantity of the complex frequency response function of *k*th and *j*th vibration modes of building **b**. Equations $H_{bk}(\omega)$ and $H_{bj}(\omega)$ are the complex frequency response function of *k*th and *j*th vibration modes of building **b** and the equations $\overset{*}{H}_{bj}(\omega)$ and $\overset{*}{H}_{bk}(\omega)$ are the conjugate complex frequency response function. On the other hand as per the definitions provided we have:

$$M_j=\omega_j^2\cdot K_j$$

$$C_j=2\xi_j\omega_j M_j$$

It is worth to note that parameters C_j, M_j and K_j are respectively the modal dampening, the mass and the stiffness in the *j*th mode of the structure under study. By inserting the aforementioned in the complex frequency response function of *k*th and *j*th vibration modes of building **b**, the following equations are obtained:

$$H_{bj}=\frac{1}{M_{bj}((\omega^2-\omega_{bj}{}^2)+2i\,\xi_{bj}\,\omega\omega_{bj})} \quad (40)$$

$$H_{bk}=\frac{1}{M_{bk}((\omega^2-\omega_{bk}{}^2)+2i\,\xi_{bk}\,\omega\omega_{bk})} \quad (41)$$

Using the aforementioned relationships and by inserting these equations in (39), the following equation is redefined as:

$$[\frac{1}{\overset{*}{H}_{bj}(\omega)\times H_{bk}(\omega)}+\frac{1}{[H_{bj}(\omega)\times\overset{*}{H}_{bk}(\omega)}]=$$
$$2M_{bj}M_{bk}\times[(\omega^2-\omega_{bj}{}^2)\times(\omega^2-\omega_{bk}{}^2)+4\omega^2\xi_{bk}\omega_{bk}\xi_{bj}\omega_{bj}]$$

(42)

The statement in parentheses in (42) can be defined as:

$$N(\omega)=[(\omega^2-\omega_{bj}{}^2)\times(\omega^2-\omega_{bk}{}^2)+4\omega^2\xi_{bk}\omega_{bk}\xi_{bj}\omega_{bj}]$$

(43)

ω_{bj} And ξ_{bj} are respectively the angular frequency and the coefficient of damping for the j^{th} mode of building **b**; hence it can be stated that the final simplified equation would be:

$$A=2M_{bj}M_{bk}N(\omega)\left|H_{bj}(\omega)\right|^2\left|H_{bk}(\omega)\right|^2 \quad (44)$$

The quantity of the complex frequency response function of k^{th} and j^{th} vibration modes of building **b** can be obtained from the multiplying of conjugate complex frequency response function to complex frequency response function, here are:

$$\left|H_{bj}(\omega)\right|^2=\frac{1}{M^2_{bj}\{(\omega^2-\omega_{bj}{}^2)^2+4\omega^2\xi_{bj}{}^2\omega_{bj}{}^2\}} \quad (45)$$

$$\left|H_{bk}(\omega)\right|^2=\frac{1}{M^2_{bk}\{(\omega^2-\omega_{bk}{}^2)^2+4\omega^2\xi_{bk}{}^2\omega_{bk}{}^2\}} \quad (46)$$

Finally by substituting eq.44 to 46 in eq.38 we have;

$$I=2M_{bj}M_{bk}\int_{-\infty}^{\infty}N(\omega)S_0\left|H_{bj}(\omega)\right|^2\left|H_{bk}(\omega)\right|^2d\omega\}$$

(47)

The equation under integral in eq.47 can be simplified as;

$$\frac{A_{jk}+B_{jk}\omega^2}{[(\omega^2-\omega_{bj}{}^2)^2+4\omega^2\xi_{bj}{}^2\omega_{bj}{}^2]}+\frac{C_{jk}+D_{jk}\omega^2}{[(\omega^2-\omega_{bk}{}^2)^2+4\omega^2\xi_{bk}{}^2\omega_{bk}{}^2]}$$

(49)

Where the coefficients A_{jk}, B_{jk}, C_{jk} and D_{jk} are obtained from the following equations :

$$B_{jk}+D_{jk}=w_1$$
$$A_{jk}+B_{jk}(4\xi_{bk}{}^2\omega_{bk}{}^2-2\omega_{bk}{}^2)+C_{jk}+D_{jk}(4\xi_{bj}{}^2\omega_{bj}{}^2-2\omega_{bj}{}^2)=w_2$$
$$A_{jk}(4\xi_{bk}{}^2\omega_{bk}{}^2-2\omega_{bk}{}^2)+B_{jk}\omega_{bk}{}^4+C_{jk}(4\xi_{bj}{}^2\omega_{bj}{}^2-2\omega_{bj}{}^2)+D_{jk}\omega_{bj}{}^4=w_3$$
$$A_{jk}\omega_{bk}{}^4+C_{jk}\omega_{bj}{}^4=w_4$$

(50)

In which:

$$u=-2\omega_k^2(1-2\xi_k^2),\qquad v=\omega_k^4$$

$$s=-2\omega_j^2(1-2\xi_j^2)\qquad t=\omega_j^4$$

$$w_1=0, w_2=1, w_3=-(\omega_{bj}{}^2+\omega_{bk}{}^2-4\xi_{bk}\omega_{bk}\xi_{bj}\omega_{bj}), w_4=\omega_{bj}{}^2\omega_{bk}{}^2$$

Finally the close form of the solution of these equations is obtained as below:

$$A_{jk}=\frac{(u-s)\times\{tw_3-(stw_4+t^2w_1)\}-(v-t)\times\{tw_2-(stw_1+w_4)\}}{(tu-sv)\times(u-s)+(t-v)^2}$$

$$B_{jk}=\frac{tw_2-(stw_1+w_4)+(v-t)\times A_{jk}}{t\times(u-s)}$$

$$C_{jk}=\frac{1}{t}(w_4-A_{jk}\times v)$$

$$D_{jk}=w_1-B_{jk}$$

Hence the integral part of eq.47 becomes:

$$II=A_{jk}\int_{-\infty}^{\infty}S_0\left|H_{bj}(\omega)\right|^2d\omega+B_{jk}\int_{-\infty}^{\infty}S_0\omega^2\left|H_{bj}(\omega)\right|^2d\omega+$$
$$C_{jk}\int_{-\infty}^{\infty}S_0\left|H_{bk}(\omega)\right|^2d\omega+D_{jk}\int_{-\infty}^{\infty}S_0\omega^2\left|H_{bk}(\omega)\right|^2d\omega$$

(51)

In order to determine the integral terms in eq.51 the following relation is used:

$$\int_{-\infty}^{\infty}\left|\frac{B_0+i\omega B_1}{A_0+i\omega A_1-\omega^2 A_2}\right|^2 d\omega=\frac{\pi\left(A_0 B_1^2+A_2 B_0^2\right)}{A_0 A_1 A_2}$$

(52)

This leads to:

$$II=\frac{S_o\pi}{2}\{\frac{1}{\omega_{bj}\xi_{bj}}(A_{jk}\frac{1}{\omega_{bj}^2}+B_{jk})+\frac{1}{\omega_{bk}\xi_{bk}}(C_{jk}\frac{1}{\omega_{bk}^2}+D_{jk})\}$$

(53)

Finally equation (37) reduced to:

$$\int_{-\infty}^{\infty} S_{u_{b,Na}u_{b,Na}}d\omega=$$

$$\sum_{j=1}^{2Nb}\varphi_b^2(N_a,j)\times\frac{1}{M_{bj}^2}\{\sum_{r=1}^{Nb}\sum_{s=1}^{Nb}\varphi_b(r,j)\varphi_b(s,j)m_{br}m_{bs}\}\times\frac{S_o\pi}{2\omega_{bj}^3\xi_{bj}}+$$

$$\frac{2}{M_{bj}M_{bk}}\sum_{j=1}^{2Nb-1}\sum_{k=j+1}^{2Nb}\varphi_b(N_a,j)\varphi_b(N_a,k)\times\{\sum_{r=1}^{Nb}\sum_{s=1}^{Nb}\varphi_b(r,j)\varphi_b(s,k)m_{br}m_{bs}\}\times$$

$$\frac{S_o\pi}{2}\{\frac{1}{\omega_{bj}\xi_{bj}}(A_{jk}\frac{1}{\omega_{bj}^2}+B_{jk})+\frac{1}{\omega_{bk}\xi_{bk}}(C_{jk}\frac{1}{\omega_{bk}^2}+D_{jk})\qquad(54)$$

The other terms in the equation (27) can be calculated from similar procedure.

A method implemented for obtaining of the Mean Square function of relative displacement is also applied for determining the elements required for the Mean square of relative velocity function; the only difference is that the constant coefficient for the equation (50) are :

$$w_1=1, w_2=-(\omega_{bj}^2+\omega_{bk}^2-4\xi_{bk}\omega_{bk}\xi_{bj}\omega_{bj}), w_3=\omega_{bj}^2\omega_{bk}^2, w_4=0$$

It should be noted that the relation between Displacement PSDF and velocity PSDF of an arbitrary process **z** can be stated as:

$$S_{\dot{z}\dot{z}}(\omega)=\omega^2 S_{zz}(\omega)\qquad(55)$$

6.2. The Required Separation Distance and the Calculation of the Standard Deviation

Using a Gaussian process, Davenport, showed that the Mean and Standard Deviation of the maximum values of the system is obtained [11]:

$$\bar{\delta}\cong\left(\sqrt{2\ln(\upsilon T)}+\frac{0.5772}{\sqrt{2\ln(\upsilon T)}}\right)\sigma_Z\qquad(56)$$

$$\sigma_\delta\cong\left(\frac{\pi}{\sqrt{12\ln(\upsilon T)}}\right)\sigma_Z\qquad(57)$$

$$\upsilon=\frac{\sigma_{\dot{Z}}}{2\pi\sigma_Z}\qquad(58)$$

In this equation σ_Z is the relative displacement function, $\sigma_{\dot{Z}}$ is the relative velocity function, T is the time duration of the random vibrations process and υ is the Euler constant equal to 0.5772. Using these equations, the Mean and Standard deviation of the required separation distance of adjacent buildings can be determined.

3. Results and Discussions

A Computer program developed in this study (2009) to calculate the mean and standard deviation of the separation distance .In order to study the effect of various parameters on this distance, some building with torsional behavior and with different stories, eccentricities and damping ratios have been analyzed and the mean and standard deviation of their separation distances have been determined. The dynamic properties of the buildings are summarized in table 1 to 3. The stiffness of buildings is selected in such a way that their periods are nearly equal to that obtained by empirical relations in seismic codes.

3.1. The Effects of the Natural Period

In order to study the effects of the natural period on the separation distance, various buildings with different periods have been considered. Building **a** and **b** have 2,4,6,8,10,12,14,16,18,20 stories so, 10 different cases have been considered. In figures 1, 2 and 3, three cases of 10 cases have been shown. For example, in figure 1 the building **a** has 4 stories and stories of building **b** varies from 2 to 20. So, the natural period of building **a** is considered constant and that of building **b** is varing. From these graphs it can be seen that as the difference between natural periods of both buildings increases, the more separation distance between them is required. If the natural period of adjacent buildings is similar to each other there would be no need for separation distance theoretically. However, standard building codes require a minimum distance for the buildings.

Table 1: Dynamic Characteristic of Building **a** & **b** with 5% Eccentricity

Number of stories	Mass (ton)	Building Height (m)	Stiffness of story 1 –x direction (KN/mm)	Stiffness of story 1 –y direction (KN/mm)	experimental Natural period (sec)	Analytical Natural period (sec)	Buildin g length (m)	Building wide (m)
2	454.5	6	380	400	0.2684	0.2692		
4	454.5	12	430	450	0.451	0.4507		
6	454.5	18	480	510	0.612	0.6132		
8	454.5	24	520	600	0.759	0.7615		
10	454.5	30	570	680	0.897	0.8950	9	6
12	454.5	36	610	770	1.029	1.0247		
14	454.5	42	640	820	1.155	1.1589		
16	454.5	48	690	880	1.277	1.2703		
18	454.5	54	720	920	1.394	1.394		
20	454.5	60	760	1000	1.509	1.500		

Eccentricity of x-Direction: 0
Eccentricity of y-Direction: 5%

Table 2: Dynamic Characteristic of Building **a** & **b** with 10% Eccentricity

Number of stories	Mass (ton)	Building Height (m)	Stiffness of story 1 –x direction (KN/mm)	Stiffness of story 1 –y direction (KN/mm)	experimental Natural period (sec)	Analytical Natural period (sec)	Buildin g length (m)	Building wide(m)
2	454.5	6	410	460	0.2684	0.2688		
4	454.5	12	470	510	0.451	0.4498		
6	454.5	18	520	560	0.612	0.6171		
8	454.5	24	570	660	0.759	0.7586		
10	454.5	30	620	720	0.897	0.8976	9	6
12	454.5	36	670	770	1.029	1.085		
14	454.5	42	720	820	1.155	1.1541		
16	454.5	48	770	870	1.277	1.2718		
18	454.5	54	800	920	1.394	1.394		
20	454.5	60	850	970	1.509	1.500		

Eccentricity of x-Direction: 0
Eccentricity of y-Direction: 10%

Table 3: Dynamic Characteristic of Building **a** & **b** with 20% Eccentricity

Number of stories	Mass (ton)	Building Height (m)	Stiffness of story 1 –x direction (KN/mm)	Stiffness of story 1 –y direction (KN/mm)	experimental Natural period (sec)	Analytical Natural period (sec)	Buildin g length (m)	Building wide(m)
2	454.5	6	460	520	0.2684	0.2707		
4	454.5	12	520	580	0.451	0.4548		
6	454.5	18	590	660	0.612	0.6146		
8	454.5	24	650	750	0.759	0.7579		
10	454.5	30	710	830	0.897	0.8920	9	6
12	454.5	36	760	880	1.029	1.0289		
14	454.5	42	790	960	1.155	1.1540		
16	454.5	48	840	1020	1.277	1.2737		
18	454.5	54	890	1070	1.394	1.3913		
20	454.5	60	940	1130	1.509	1.500		

Eccentricity of x-Direction: 0
Eccentricity of y-Direction: 20%

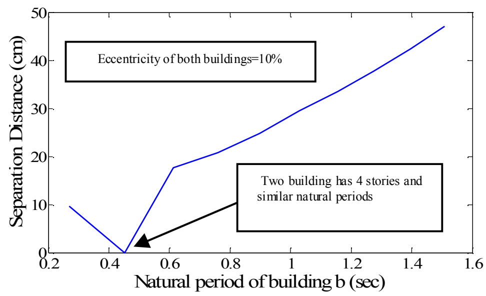

Fig. 1-Natural Period Effects, Building **a** 4 story

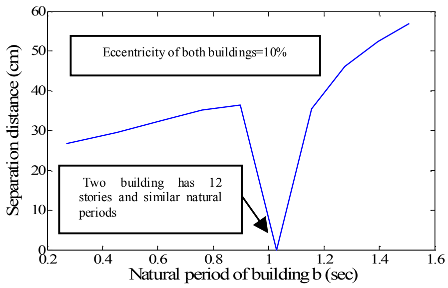

Fig .2-Natural Period Effects, Building **a** 12 story

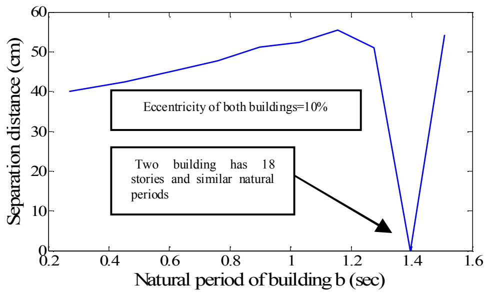

Fig.3 -Natural Period Effects, Building **a** 18 story

3.2. The Effects of Eccentricity

In order to determine the effects of eccentricity (the relative distance between center of mass and center of rigidity of story) on separation distance of adjacent buildings, the 2 cases were studied:

Case 1:

The eccentricity of building **a** considered as constant and of building **b** varies as below:

1) Building **a** has an eccentricity equal to %5 and building **b** has corresponding value of %5, %10, and %20.
2) Building **a** has an eccentricity of %10 and building **b** has eccentricities equal to %5, %10, and %20 respectively.
3) Building **a** has an eccentricity of %20 and building **b** has eccentricities of %5, %10, and %20 respectively.

It is worth to note that building **a** has *8* , *16* and *20* stories respectively and the number of stories in building **b** varies from *2* to *20* floors. The obtained results have been presented in figures 4 to 7.

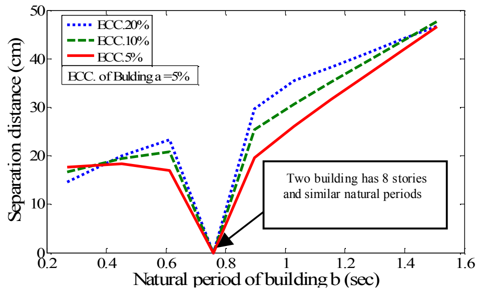

Fig.4 - Case1, Building **a** 8 story – ECC : 5% ,

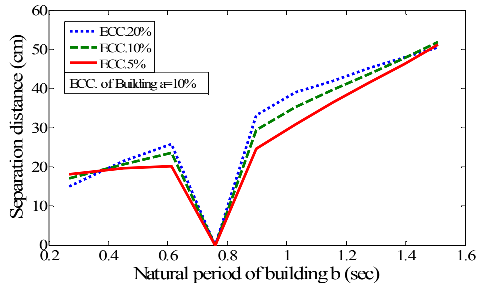

Fig. 5- Case 1, Building **a** 8 stories -ECC : 10%

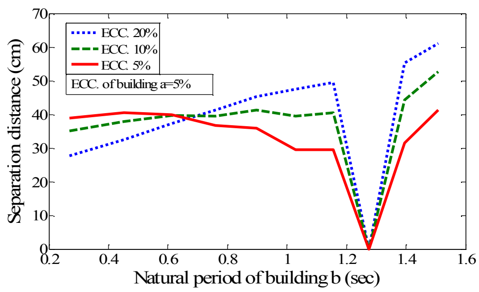

Fig .6- Case 1, Building **a** 16 stories- ECC : 5%

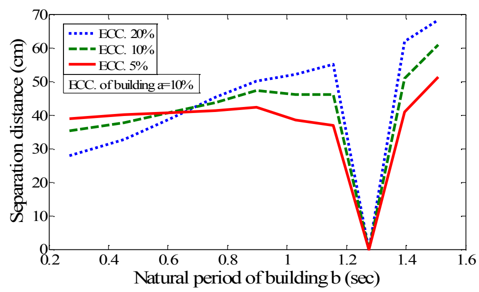

Fig .7- Case 1, Building **a** 16 stories- ECC :10%

Case 2:

In this case the eccentricity of both building is taken similar and equal to 5, 10 and 20%. Furthermore, the number of stories for building **a** have fixed and for building **b** is set to be varied from *2* to *20* stories. The obtained results have been shown in figures 8 to 10.

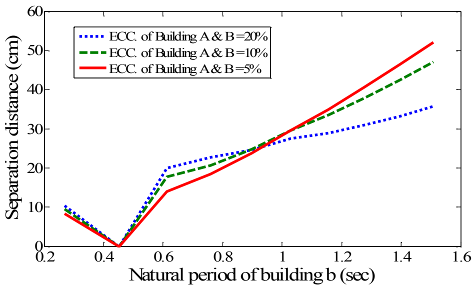

Fig.8- Case2, Building **a** 4story

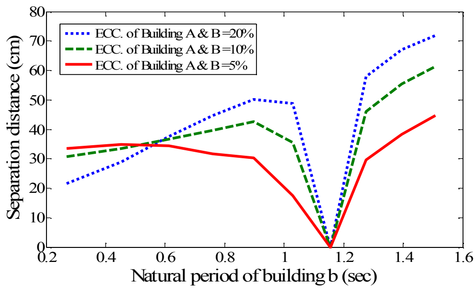

Fig.9- Case2, Building **a** 14 story

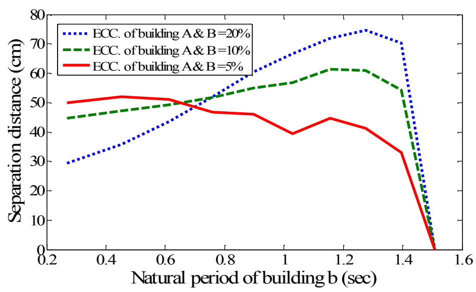

Fig.10- Case2, Building **a** 20story

From figures 4 to 10 we can obtain that when the difference between natural period of building **a** and **b** becomes large , the eccentricity would be increased and subsequently lead to increase in required separation distance. But if the difference closes to zero, the eccentricity would be decreased and cause to decrease in the required separation distance value. So, we can say that the effect of eccentricity is greatly influenced by the natural period factor and its role is not important like as the natural period in separation distance calculations.

3.3. The Effect of Damping Ratio

In order to investigate the effect of the damping ratio on the separation distance of two adjacent buildings, the values of damping coefficients were considered as 2%, 5% and 10%. The number of the stories of building **a** assumed to be constant and for building **b** were considered *2, 4, 6, 8, 10, 12, 14, 16, 18* and *20*, respectively. In order to calculate the separation distance, data of Table (2) were used. The results of the calculations for a case in which the building **a** has 4 and 12 stories are demonstrated in figures (11) to (12).

From figure 11 and 12 it could be found that as the damping ratios becomes larger the required separation distance in all cases with different eccentricities decreases.

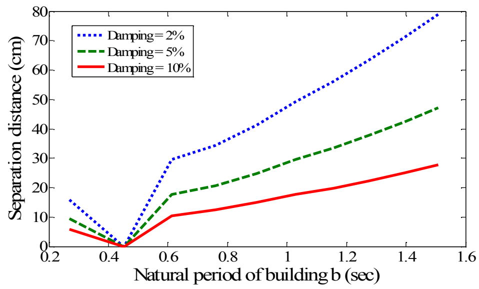

Fig.11- Effect of damping ratio, Building **a** 4story – ECC: 10%

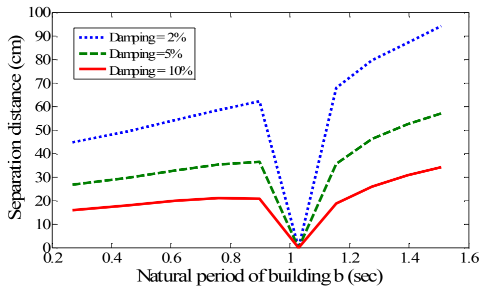

Fig.12- Effect of damping ratio, Building **a** 12story – ECC: 10%

3.4. Comparison with Seismic Codes

In this section, obtained results in previous paragraphs calculated based on random vibration theory have been compared with corresponding values in mentioned code relations. Figures 13 to 15 have shown this comparison for 4, 12 and 20 stories buildings.

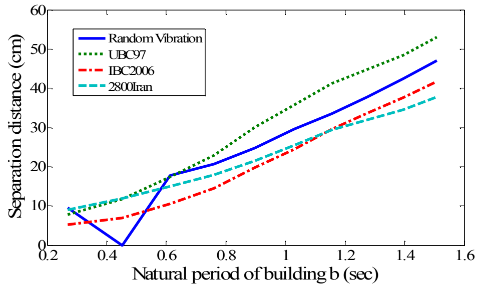

Fig .13- Building **a** with 4 story

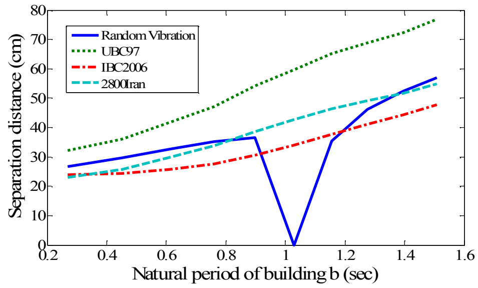

Fig.14- Building **a** with 12 story

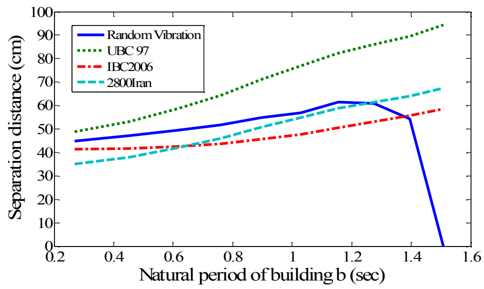

Fig .15- Building **a** with 20 story

From above figures, it can be seen that in all cases UBC 97 require much more separation distance than the other codes and random vibration theory. Furthermore, when the height of the buildings becomes large, in most range of natural period, the required separation distance calculated by random vibration theory is close to Standard 2800 of Iran approximately.

4. Conclusions

In this paper relations for calculating required separation distance between torsional adjacent buildings have developed using random vibration method. Some building has been analyzed and effects of various parameters have been investigated. Furthermore, the results have been compared with seismic codes. The main results are summarized as below:

1. If the difference between natural periods of adjacent structures increases the required separation distance increases too. It is evident that if the natural periods of the two structures become equal, i.e all the dynamic characteristics of the adjacent buildings are the same; the theoretical separation distance between the two buildings becomes negligible.
2. As shown in the Figures 11 and 12 with the increase of the damping ratio of the two adjacent buildings the required separation distance would be reduced. So for retrofitting of the coherent buildings, one can use dampers to increase damping. This is used as a method in order to prevent the pounding of the adjacent old buildings in order to avoid pounding, during earthquake
3. When the difference between natural periods of buildings becomes large, any increase of the eccentricity will reduce the required separation distance between them. It can be justified that the torsion mode of the both buildings becomes more similar. This case, in fact, is similar to a case which two shear buildings have a similar natural period and vibration mode.
4. Fig 13 to 15 shows that the calculated separation distances of the UBC97 code in comparison to IBC2006, 2800Iran and random vibration theory are overestimated.
5. The random vibration method gives high values of separation distance in compared to IBC2006 while it has no meaningful difference from that estimated by 2800Iran.

Corresponding Author:
Dr. Mojtaba Labibzadeh
Head of Civil Department,
Faculty of Engineering, Shahid Chamran University,
Ahvaz, Iran.
E-mail:
labibzadeh_m@scu.ac.ir

References

1. Uniform Building Code, UBC97, 1997.
2. International Building Code, IBC 2006.
3. Design of Resistance Building for earthquake (2800Iran), 3rd Edition, Building Research Center publication, Tehran, Iran, 2007.

4. Chopra, A, K. Dynamic of Structures, 2nd Edition, Prentice hall Publisher, New Delhi, India, 2004.
5. Kasai K. and Maison B.F. Observation of structural pounding damage from the 1989 Loma Prieta earthquake, Proc-6th Canadian conf. on Earthquake Engineering, Toronto: 735-742.
6. Chen, W.F. and Lui, E. M. Earthquake engineering for structural design. CRC press publication, 2006.
7. Ohta, R., Wijeyewickrema, C., Farahani, A. End building pounding during earthquakes. Symposium on infrastructure development and the environment 2006.
8. Mahmoud, S. and Jankowski, R. Elastic and inelastic multi-story buildings under earthquake excitation with the effect of pounding. Journal of Applied Sciences. 9(18):3250-3262, 2009.
9. Shehata, E. Seismic pounding between adjacent Buildings. Electronic Journal of Structural Engineering. 6:66-74, 2006.
10. Jankowski, R. Non-linear viscoelastic modeling of earthquake-induced structural pounding. Earthquake Engineering and Structural Dynamics. 34:595-611, 2005.
11. Lopez-Garcia, D and Soong, T.T. Evaluation of current criteria in predicting the separation necessary to prevent seismic pounding between nonlinear hysteretic structural systems. Engineering Structures, 31:1217-1229, 2009.
12. Lopez-Garcia, D and Soong, T.T. Assessment of the separation necessary to prevent seismic pounding between linear structural systems. Probabilistic Engineering Mechanics. 24:210-223, 2009.

5/9/2010