Mathematical Model for Locating Input and Output Points Considering Time Value of Money

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Abstract: For the first time, we consider time value of money in determining location of input and output points. We present a new mixed integer programming formulation and compare it with the conventional model in the literature. Computational results show significant effects of considering time value of money. [Journal of American Science 2010;6(10):351-354]. (ISSN: 1545-1003).

Keywords: Facility design, I/O points location, time value of money, mathematical modeling

1. Introduction

One of the oldest activities done by industrial engineers is facilities planning. The term facilities planning can be divided into two parts: facility location and facility layout (Tompkins et al., 2003). Determining the most efficient arrangement of physical departments within a facility is defined as a facility layout problem (FLP) (Garey and Johnson, 1979). Tompkins and White (1984) stated that 8% of the United States gross national product (GNP) has been spent on new facilities annually since 1955. Layout problems are known to be complex and are generally NP-Hard (Garey and Johnson, 1979). There are many review papers in the literature that they can be studied for comprehending more details (see Loiola et al., 2007, Kulturel-Konak, 2007, Drira et al., 2007, Gu et al., 2007 and Liang and Chao, 2008).

In a basic layout design, each cell is represented by a rectilinear, but not necessarily convex polygon. The set of fully packed adjacent polygons is known as a block layout (Asef-Vaziri and Laporte, 2005). The two most general mechanisms in the literature for constructing such layouts are the flexible bay and the slicing tree (Arapoglu et al., 2001). A slicing structure can be represented by a binary tree whose leaves denote modules, and internal nodes specify horizontal or vertical cut lines (Wu et al., 2003). The bay-structured layout is a continuous layout representation allowing the departments to be located only in parallel bays with varying widths. The width of each bay depends on the total area of the departments in the bay (Konak et al., 2006). We focus on bay structure layout.

There are three principal decisions for industrial designers as follows: Designing block layout, determining the location of input and output (I/O) points and designing material handling flow paths. There are few works on determining location of I/O points. In bay layout environment, Arapoglu et al. (2001) developed a genetic algorithm (GA), simulated annealing (SA) algorithm and three heuristic algorithms to determine location of I/O points and Norman et al. (2001) integrated determination of block layout and location of I/O points. They embed a heuristic algorithm to determine location I/O points in a GA algorithm that it design block layout.

One of the important issues that it has not considered is time value of money. This subject can be considered in facility layout when each I/O stations have a limited capacity for transporting material handling flows. We have to trade between present costs, cost of I/O stations installment, and annually cost, material handling cost. In this paper we propose a mixed integer programming (MP) formulation for considering time value of money in determining location of I/O points. Remainder of paper is as follows: in section II, an MIP formulation is proposed, in section III, computational results are illustrated, conclusions and future research are discussed in section IV.

2. Mathematical model

We present mathematical model for determination of I/O points location without considering time value of money as follows:

Sets and Indices:

\( N \): Set of departments \((i, j \in N, |N| = n)\).

\( Node \): Set of possible location of I/O points \((k \in L, |Node| = p)\).

Variables:

\[ x_{ik} = \begin{cases} 
1, & \text{If I/O point of department } i \text{ is located in the point } k \\
0, & \text{Otherwise} 
\end{cases} \]

Parameters:

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\[ x_{ik} = \begin{cases} 1, & \text{If I/O point of department } i \text{ can be located in the point } k \\ 0, & \text{Otherwise} \end{cases} \]

\[ f_{ij} : \text{Material flow between departments } i \text{ and } j, \]
\[ d_{kl} : \text{Distance between potential points } k \text{ and } l. \]

**Formulation:**

\[
\min \sum_{i=1}^{p} \sum_{k=1}^{n} \sum_{j=1}^{n} f_{ij} d_{kl} x_{ik} x_{jk} a_{ik}
\]
\[
\sum_{k=1}^{n} x_{ik} a_{ik} = 1 \quad \forall i
\]

Statement (1) is to minimize material handling cost between I/O points, constraint (2) state that each department has a single I/O point. Here, we present an MIP formulation for determining I/O location points considering time value of money. We introduce some additional notations as follows:

**Sets and Indices:**

\[ T : \text{Type of I/O stations } (r \in T, |T| = R), \]

**Variables:**

\[ y_{rk} = \begin{cases} 1, & \text{If I/O station with type } r \text{ is located in the point } k \\ 0, & \text{Otherwise} \end{cases} \]

**Parameters**

\[ V_r : \text{Maximum capacity of } r^{th} \text{ type of I/O station}, \]
\[ M_r : \text{Variable cost of maintenance for } r^{th} \text{ type of I/O station}, \]
\[ Q_r : \text{Installment cost for } r^{th} \text{ type of I/O station}, \]
\[ S_r : \text{Salvage value for } r^{th} \text{ type of I/O station}. \]

Year: Number of days in a year,

\[ i : \text{Rate of return}, \]

\[ C : \text{Lifecycle} \]

**Formulation**

\[
\min \text{ Year.}(P / A, i \%, C)
\]
\[
\left( \sum_{i=1}^{p} \sum_{k=1}^{n} \sum_{j=1}^{n} f_{ij} d_{kl} x_{ik} x_{jk} a_{ik} \right) + \sum_{r=1}^{R} \sum_{k=1}^{n} \sum_{l=1}^{n} f_{ij} M_r y_{rk} x_{rk}
\]

\[
- \sum_{r=1}^{R} \sum_{k=1}^{n} S_r y_{rk} (P / F, i \%, C)
\]

\[
\sum_{k=1}^{n} x_{ik} a_{ik} = 1 \quad \forall i
\]

\[
\sum_{k=1}^{n} y_{ik} \leq 1 \quad \forall i
\]

\[
\sum_{r=1}^{R} y_{ik} V_r \leq \sum_{j=1}^{n} \sum_{l=1}^{n} x_{ik} f_{ij} \quad \forall k
\]

Statement (3) is to minimize material handling cost and cost of maintenance minus salvage value considering time value of money, constraint (4) state that each department has a single I/O point, constraint (5) state that only one type of I/O station can be installed in each point, constraint (6) consider capacity limitation of each type of I/O station.

**3. Computational results**

In this paper, we compare two approach, conventional approach without time value of money and proposed approach considering time value of money. We use block layout for the well-known data sets that are developed by Konak et al. (2006) (see Table 1). We determine parameters as follows:

\[ R = 3, i = 5\% \quad \text{and} \quad 10\%, C = 3, \quad \text{and} \quad 5, \text{Year=250} \]

days and for \( V_1, M_1, Q_1, S_1 \) we have:

\[ (V_1, V_2, V_3) = (0.1, 0.2, 1) \sum_{j=1}^{n} \sum_{i=1}^{n} f_{ij}, \]

\[ M_1 = 0.5, M_2 = 0.5, M_3 = 0.75, \]
\[ Q_1 = 7500, Q_2 = 10000, Q_3 = 15000, \]
\[ S_1 = 0, S_2 = 5000, S_3 = 10000. \]

**Table 1. Summary of test problems**

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<thead>
<tr>
<th>Problem name</th>
<th>Problem data Reference</th>
<th>Layout</th>
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<tbody>
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<td>4</td>
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<td>Meller et al. (1998)</td>
<td>4</td>
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<tr>
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<td>Meller et al. (1998)</td>
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<td>O72</td>
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<td>2 – 1</td>
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4. Conclusions

For the first time, we consider time value of money in determining location of input and output points. We present a new mixed integer programming formulation and compare it with the conventional model in the literature. Computational results show significant effects of considering time value of money. Average improvement is about 8.5% for all test problems. It is shown this improvement is not sensitive respect to different and , whereas for , , , and , average improvements are 9.2%, 7.7%, 9.4%, 8% and 8% respectively. For future research, it is suggested to integrate design of block layout and determination of I/O points location considering time value of money.

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References


<table>
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