Mathematical Model for Locating Input and Output Points Considering Time Value of Money

Amir Ardestani Jaafari [1], Seyed Hossein Hashemi Doulabi [1], Mohsen Akbarpour Shirazi [1], Mostafa Khatibi [2]

[1.] Industrial Engineering department at Amirkabir University of Technology, Tehran, Iran.
[2.] Industrial Engineering department at Mazandaran University of Sceince and Technology, Babol, Iran.
Email: ardestani.amir@aut.ac.ir

Abstract: For the first time, we consider time value of money in determining location of input and output points. We present a new mixed integer programming formulation and compare it with the conventional model in the literature. Computational results show significant effects of considering time value of money. [Journal of American Science 2010;6(10):351-354]. (ISSN: 1545-1003).

Keywords: Facility design, I/O points location, time value of money, mathematical modeling

1. Introduction

One of the oldest activities done by industrial engineers is facilities planning. The term facilities planning can be divided into two parts: facility location and facility layout (Tompkins et al., 2003). Determining the most efficient arrangement of physical departments within a facility is defined as a facility layout problem (FLP) (Garey and Johnson, 1979). Tompkins and White (1984) stated that 8% of the United States gross national product (GNP) has been spent on new facilities annually since 1955. Layout problems are known to be complex and are generally NP-Hard (Garey and Johnson, 1979). There are many review papers in the literature that they can be studied for comprehending more details (see Loiola et al., 2007, Kulturel-Konak, 2007, Drira et al., 2007, Gu et al., 2007 and Liang and Chao, 2008).

In a basic layout design, each cell is represented by a rectilinear, but not necessarily convex polygon. The set of fully packed adjacent polygons is known as a block layout (Asef-Vaziri and Laporte, 2005). The two most general mechanisms in the literature for constructing such layouts are the flexible bay and the slicing tree (Arapoglu et al., 2001). A slicing structure can be represented by a binary tree whose leaves denote modules, and internal nodes specify horizontal or vertical cut lines (Wu et al., 2003). The bay-structured layout is a continuous layout representation allowing the departments to be located only in parallel bays with varying widths. The width of each bay depends on the total area of the departments in the bay (Konak et al., 2006). We focus on bay structure layout.

There are three principal decisions for industrial designers as follows: Designing block layout, determining the location of input and output (I/O) points and designing material handling flow paths. There are few works on determining location of I/O points. In bay layout environment, Arapoglu et al. (2001) developed a genetic algorithm (GA), simulated annealing (SA) algorithm and three heuristic algorithms to determine location of I/O points and Norman et al. (2001) integrated determination of block layout and location of I/O points. They embed a heuristic algorithm to determine location I/O points in a GA algorithm that it design block layout.

One of the important issues that it has not considered is time value of money. This subject can be considered in facility layout when each I/O stations have a limited capacity for transporting material handling flows. We have to trade between present costs, cost of I/O stations installment, and annually cost, material handling cost. In this paper we propose a mixed integer programming (MP) formulation for considering time value of money in determining location of I/O points. Remainder of paper is as follows: in section II, an MIP formulation is proposed, in section III, computational results are illustrated, conclusions and future research are discussed in section IV.

2. Mathematical model

We present mathematical model for determination of I/O points location without considering time value of money as follows:

Sets and Indices:

N : Set of departments $(i, j \in N, |N| = n)$,

$Node$: Set of possible location of I/O points $(k \in L, |Node| = p)$.

Variables:

$$x_{ik} = \begin{cases} 1, & \text{If I/O point of department } i \text{ is located in the point } k \\ 0, & \text{Otherwise} \end{cases}$$

Parameters:

$$x_{ik} = \begin{cases} 1, & \text{If I/O point of department } i \text{ can be located in the point } k \\ 0, & \text{Otherwise} \end{cases}$$

f_{ij} : Material flow between departments *i* and *j*,

d_{kl} : Distance between potential points *k* and *l*.

Formulation:

$$\min \sum_{l=1}^{p}\sum_{k=1}^{p}\sum_{j=1}^{n}\sum_{i=1}^{n} f_{ij} d_{kl} x_{ik} x_{jl} a_{ik} \qquad (1)$$

$$\sum_{k=1}^{p} x_{ik} a_{ik} = 1 \qquad \forall i \qquad (2)$$

Statement (1) is to minimize material handling cost between I/O points, constraint (2) state that each department has a single I/O point. Here, we present an MIP formulation for determining I/O location points considering time value of money. We introduce some additional notations as follows:

Sets and Indices:

T : Type of I/O stations $(r \in T, |T| = R)$,

Variables:

$$y_{rk} = \begin{cases} 1, & \text{If I/O station with type } r \text{ is located in the point } k \\ 0, & \text{Otherwise} \end{cases}$$

Parameters

V_r : Maximum capacity of r^{th} type of I/O station,

M_r : Variable cost of maintenance for r^{th} type of I/O station,

Q_r : Installment cost for r^{th} type of I/O station,

S_r : Salvage value for r^{th} type of I/O station.

Year: Number of days in a year,

i : Rate of return,

C : Lifecycle

Formulation

$$\min \text{Year}.(P/A, i\%, C) \left(\sum_{l=1}^{p}\sum_{k=1}^{p}\sum_{j=1}^{n}\sum_{i=1}^{n} f_{ij} d_{kl} x_{ik} x_{jl} a_{ik} + \sum_{r=1}^{R}\sum_{k=1}^{p}\sum_{j=1}^{n}\sum_{i=1}^{n} f_{ij} M_r y_{rk} x_{rk} \right) \qquad (3)$$

$$+ \sum_{k=1}^{p}\sum_{r=1}^{R} Q_r y_{rk}$$

$$- \sum_{k=1}^{p}\sum_{r=1}^{R} S_r y_{rk} (P/F, i\%, C)$$

$$\sum_{k=1}^{p} x_{ik} a_{ik} = 1 \qquad \forall i \qquad (4)$$

$$\sum_{k=1}^{p} y_{ik} \le 1 \qquad \forall i \qquad (5)$$

$$\sum_{r=1}^{R} y_{ik} V_r \le \sum_{j=1}^{n}\sum_{i=1}^{n} x_{ik} f_{ij} \qquad \forall k \qquad (6)$$

Statement (3) is to minimize material handling cost and cost of maintenance minus salvage value considering time value of money, constraint (4) state that each department has a single I/O point, constraint (5) state that only one type of I/O station can be installed in each point, constraint (6) consider capacity limitation of each type of I/O station.

3. Computational results

In this paper, we compare two approach, conventional approach without time value of money and proposed approach considering time value of money. We use block layout for the well-known data sets that are developed by Konak et al. (2006) (see Table 1). We determine parameters as follows: $R = 3, i = 5\%$ and $10\%, C = 3$, and 5, Year=250 days and for V_i, M_r, Q_r, S_r we have:

$$(V_1, V_2, V_3) = (0.1, 0.2, 1) \sum_{j=1}^{n}\sum_{i=1}^{n} f_{ij},$$

$$M_1 = 0.5, M_2 = 0.5, M_3 = 0.75,$$

$$Q_1 = 7500, Q_2 = 10000, Q_3 = 15000,$$

$$S_1 = 0, S_2 = 5000, S_3 = 10000,$$

Table 1. Summary of test problems

Problem name	Problem data Reference	Layout
FO7	Meller et al. (1998)	4 | 5 − 3 | 6 − 2 | 7 − 1
FO71	Meller et al. (1998)	4 | 3 − 5 | 2 − 6 | 1 − 7
FO72	Meller et al. (1998)	1 | 2 | 3 | 4 | 5 − 6 − 7
O71	Meller et al. (1998)	1 − 4 − 2 | 3 − 6 − 5 − 7
O72	Meller et al. (1998)	2 − 1 − 4 | 5 − 7 − 6 | 3
FO8	Meller et al. (1998)	4 − 3 − 2 − 1 | 5 − 6 − 7 − 8

O9	Meller et al. (1998)	7 − 8 | 4 − 1 − 2 | 3 − 6 − 9 − 5
VC10-s	Van Cam et al. (1992)	5 − 3 | 8 − 10 − 9 | 4 − 2 | 7 − 6 | 1
VC10-a	Gau and Meller (1999)	10 − 8 − 5 − 3 | 7 − 4 − 9 | 6 − 2 | 1
MB11-a	Bozer et al. (1994)	3 − 4 − 8 | 2 − 9 − 10 | 7 − 6 − 5 − 1 − 11

Computational results are presented in Table 2 and Improvement in objective function is depicted in Figure 1.

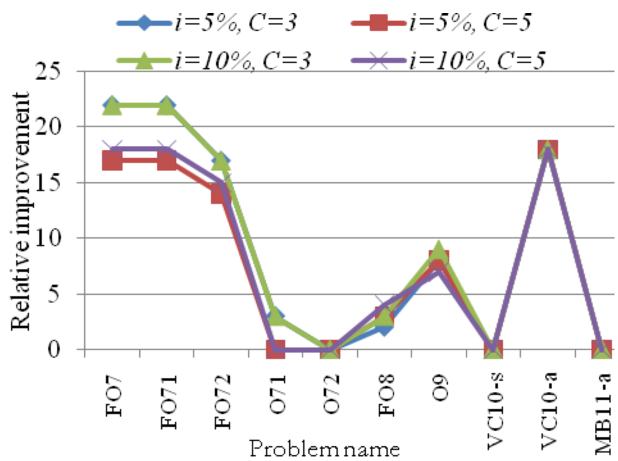

Figure 1. Improvment in objective function

4. Conclusions

For the first time, we consider time value of money in determining location of input and output points. We present a new mixed integer programming formulation and compare it with the conventional model in the literature. Computational results show significant effects of considering time value of money. Average improvement is about 8.5% for all test problems. It is shown this improvement is not sensitive respect to different i and C, whereas for $i = 5\%, C = 3$, $i = 5\%, C = 5$, $i = 10\%, C = 3$, $i = 10\%, C = 5$, average improvements are 9.2%, 7.7%, 9.4, 8% and 8% respectively. For future research, it is suggested to integrate design of block layout and determination of I/O points location considering time value of money.

Corresponding Author:
Mr. Amir Ardestani Jaafari
Department of Industrial Engineering,
Amirkabir University of Technology,
424 Hafez Ave, Tehran, Iran
E-mail: ardestani.amir@aut.ac.ir

References

1. Stirilng G, Wilsey B. Emprical relationships between species richness, eveness and proporational diversity. Am Nat 158 (2001) 286-99.
2. Arapoglu R. A, Norman B. A, Smith A. E, Locating input and output points in facilities design-A comparison of constructive, evolutionary, and exact methods, IEEE Transactions on Evolutionary Computation 3 (2001) 192-203.
3. Asef-Vaziri A, Laporte G, Loop based facility planning and material handling, European Journal of Operational Research 164 (2005) 1–11.
4. Bozer Y. A, Meller R. D, Erlebacher S. J, An improvement type layout algorithm for single and multiple-floor facilities, Management Science 40 (1994) 918–932.
5. Drira A, Pierreval H, Hajri-Gabouj S, Facility layout problems: A survey, Annual Reviews in Control 31 (2007) 255–267.
6. Garey M. R, Johnson D. S, Computers and intractability: A guide to the theory of NP-completeness, WH Freeman, (1979) New York.
7. Gau K. Y, Meller R. D, An iterative facility layout algorithm, International Journal of Production Research 37 (1999) 3739–3758.
8. Gu J, Goetschalckx M, McGinnis L. F, Research on warehouse operation: A comprehensive review, European Journal of Operational Research 177 (2007) 1–21.
9. Konak A, Kulturel-Konak S, Norman B. A. and Smith A. E, A new mixed integer programming formulation for facility layout design using flexible bays, Operations Research Letters 34 (2006) 660–672.
10. Kulturel-Konak S, Approaches to uncertainties in facility layout problems: Perspectives at the beginning of the 21st Century, Journal of Intelligent Manufacturing 18 (2007) 273–284.
11. Liang L. Y, Chao W. C, The strategies of tabu search technique for facility layout optimization, Automation in Construction 17 (2008) 657–669.
12. Loiola E. M, De Abreu N. M. M, Boaventura-Netto P. O, Hahn P, Querido T, A survey for the quadratic assignment problem, European Journal of Operational Research 176 (2007) 657–690.
13. Meller R. D, Narayanan V, Vance P. H, Optimal facility layout design, Operation Research Letter 23 (1998) 117–127.
14. Norman A. B, Arapoglu R. A, Smith A. E, Integrated facilities design using a contour distance metric, IIE Transactions 33 (2001), 337-344.

15. Tompkins J. A, White J. A, (1984) Facilities planning. Wiley, New York.
16. Tompkins J. A, White J. A, Bozer Y. A, Tanchoco J. M. A, (2003) Facilities planning, 3rd edition. Wiley, Chichester.
17. Van Camp D. J, Carter M. W, Vannelli A, A nonlinear optimization approach for solving facility layout problems, European Journal of Operation Research 57 (1992) 174–189.
18. Wu G. M, Chang Y. C, Cheang Y. W, ACM Transactions on Design Automation of Electronic Systems 8 (2003) 188–202.

Table 2. Computational results

Problem name	*i=5%,C=3*		*i=5%,C=5*		*i=10%,C=3*		*i=10%,C=5*	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
FO7	25822.9	20258	34559.8	28699	28141.8	22007	37121.41	30427
FO71	25686.8	20122	34300.3	28440	28014.7	21880	36892.4	30195
FO72	32292.3	26728	42145.9	36285	35121.2	28986	45364	38667
O71	54626.9	52933	84738.7	84739	55967.9	54054	83439.4	83439
O72	63836.6	63837	102302	102302	64564.1	64564	99139.9	99140
FO8	36102.8	35306	49196.8	47893	39285.8	37934	52594.9	50501
O9	194937.1	179307	342812.2	316956	188812.8	172342	317057.8	293944
VC10-s	5832261	5832261	11093355	11093355	5450580	5450580	9927393	9927393
VC10-a	7148163	5832261	13602820	11093355	6678817	5450580	12170700	9927393
MB11-a	543391.9	543392	1007542	1007542	513445.7	513446	910358	910358

7/6/2010