# Simulation Optimization Approach for Facility Layout Problem-A Queuing Theory Based Approach

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**Abstract:** One of the most important issues in facility layout problem is to find the location of the Input/ Output points. We consider single loop path as material flow path for a given layout and find locations of Input/Out points on perimeter of the loop in the uncertain environment. The uncertainty is derived from production time of each department. Our objective is to minimize total time of AGV system after conveying all departmental material flows, we solve an uncertain queuing problem and due to difficulty of the queuing problem, an efficient simulation optimization approach is proposed using simulated annealing algorithm.
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**Keywords:** Facility layout; input/ output points location; queuing theory; simulated annealing

## 1. Introduction

One of the oldest activities done by industrial engineers is facilities planning. The term facilities planning can be divided into two parts: facility location and facility layout (Tompkins et al., 2003). Determining the most efficient arrangement of physical departments within a facility is defined as a facility layout problem (FLP) (Garey and Johnson, 1979). Tompkins (1997) stated that 8% of the United States gross national product (GNP) has been spent on new facilities annually since 1955. Layout problems are known to be complex and are generally NP-Hard (Garey and Johnson, 1979).

If the building size and shape are given, then the three principal and interdependent design decisions in the facility layout design problem are: (1) the determination of the shapes and locations of departments within the facility, which is called the conceptual block layout problem; (2) the determination of the locations of the input and output (I/O) points on the perimeter of each department; and (3) the design of the material flow paths or aisles that connect these I/O points (Kim and Goetschalckx, 2005).

In a basic layout design, each cell is represented by a rectilinear, but not necessarily convex polygon. The set of fully packed adjacent polygons is known as a block layout (Asef-Vaziri and Laporte, 2005). The two most general mechanisms in the literature for constructing such layouts are the flexible bay and the slicing tree (Arapoglu et al., 2001). A slicing structure can be represented by a binary tree whose leaves denote modules, and internal nodes specify horizontal or vertical cut lines (Wu et al., 2003). The bay-structured layout is a continuous layout representation allowing the departments to be located only in parallel bays with varying widths. The width of each bay depends on the total area of the departments in the bay (Konak et al., 2006).

In the design of material flow path, AGV is one of the most common approaches in which a driverless vehicle is used for the transportation of material between departments. Maxwell and Muckstadt (1982) was first introduced the problem of AGV flow system. We focus on single flow path AGV, one of the four well-known general types of design used in production systems (Apple, 1977). It lends itself to both product and production simplicity (Afentakis, 1989).

In determination of I/O points location, there are many research in the literature of facility layout problem. For example, Arapoglu et al., (2001) develop three constructive heuristics, a genetic algorithm (GA) and a simulated annealing (SA) algorithm to find location of I/O points. One of their three heuristic algorithms was deterministic and the other improved the first heuristic using uncertain parameters. They compared results of GA and SA algorithm with those of relaxed formulation and heuristics methods. Norman et al. (2001) and Kim and Goetschalckx (2005) integrate design of block layout and I/O points location problem. Norman et al. developed a heuristic algorithm to find I/O points location embedded in a GA algorithm. GA algorithm determined facility layout problem using bay. Kim and Goetschalkcx (2005) develop a SA algorithm to complete partial solution of layout developed by mixed integer programming (MIP) formulation and three heuristics are embedded in SA algorithm to find I/O points location. Ardestani Jaafari et al. (2010) consider I/O point location problem considering time value of money. They propose an MIP formulation to solve the problem considering I/O stations with different capacity and costs of installment and maintenance.

Conventional approach for encountering with facility design problem was tended to consider all parameters in deterministic environment. This assumption is not appropriate in real world problems. There are two types of uncertain facility layout problem: material flows change in each period and they are deterministic in each period (dynamic layout) and in the latter, material flows change as random variables with known parameters in a single period (stochastic layout). Kulturel-Konak (2007) reviewed the importance of the uncertainty in the future of the facility layout problem.

One of the most important parameters in the facility layout problems is the production time of each department. When an AGV is used for transporting system, material flows have to wait for arriving an empty vehicle due to low capacity of vehicle. Using more than one AGV vehicle or vehicles with more capacity increase the cost of installment and maintenance of AGV systems. Transporting departmental material flows is a type of queuing problem. Finished goods and AGV vehicle are as customers and service provider respectively. There are a few researches in the literature of facility layout problem as the queuing problems.

Raman et al. (2009) discussed the development of a two step analytical approach to determine the quantity of material handling equipment that it is necessary for effective handling of products among facilities. At first, a solution is found by considering the time required for loading and unloading products, loaded travelling, empty travelling and breakdown of material handling equipment. Then a model is proposed to select the best alternatives between those generated from the first part in the stochastic nature. Jain et al. (2008) developed a queuing model for the prediction of flexible manufacturing systems performance using mean value. Smith (2009) presented some topological network design problems for material handling system design considering queuing network models.

In this paper, we discuss I/O points location problem as a queuing problem. Due to difficulty of the problem, we use simulation based optimization approach. A SA algorithm is developed to solve I/O point location problem in stochastic environment based on simulation optimization method. Each I/O point is as customer and AGV is service provider. Our objective is to minimize total time of traveling by AGV. We focus on single loop path; however our approach can be used for tandem. We also consider a single point I/O for each department and it can easily be extended to multiple I/O point with distinct point for input and output points of each department. Our approach can also be useful in routing problem when there multiple suppliers instead of a hub of supplier. The remainder of paper is organized as follows. Scetion 2 develops a SA algorithm. Section 3 gives computational results and efficiency of our approach in comparison with deterministic approach. Finally, section 4 concludes the paper and recommends some future studies.

## 2. SA Algorithm

To solve combinatorial optimization problems, simulated annealing algorithm is first proposed by Kirkpatrick et al. (1983). The name of SA algorithm is attained from the simulation of the annealing of solids. Annealing refers to a process of cooling material gradually to reach a steady state. In this process, a solid is heated until it melts, and then the temperature of the solid is slowly decreased (according to an annealing schedule) until the solid reaches the lowest energy state or the ground state (McKendall et al. 2006). SA algorithm is a well-established stochastic neighborhood search technique has a potential to solve complex combinatorial optimization problems (Gindy and Baykasoğlu 2001).

SA algorithm starts with a solution that is generated randomly. We represent an initial solution as a string that *i*th cell of the string shows the I/O point location of the *i*th department. We change location of I/O point of each randomly selected department. Enhancing moves are always accepted while no enhancing moves are accepted if

$$R = \exp\{-\Delta F / T\}$$

where $R$ is a random number, $\Delta F$ is the increase in objective function and $T$ is the current temperature. Temperature is decreased as follows:

$$T_{new} = \beta . T_{old}$$

We terminate SA algorithm when temperature is decreased to $T_{end}$ . Since our input data are derived from the uncertain sources for each instance, we repeat running the SA algorithm to reach stable results.

## 3. Computational Results

It is important for any meta-heuristic algorithm to optimize their parameters, so we implement several experimental results to tune the parameters of the SA algorithm. The parameters are defined as follows:

| | |
|---|---|
| $T_0$ | Initial temperature |
| $T_{end}$ | Final temperature |
| $\beta$ | Cooling coefficient |
| $M$ | Number of moves in each temperature |

We consider a range of 100 to 150 for initial temperature, 0 to 10 for final temperature, 0.9 to 0.99 for cooling coefficient and 1000 to 1500 for the number of moves in each temperature. After running about 2000 randomly generated test problems with 10 types of size from 10 to 100, we set SA parameters as follows:

| | |
|---|---|
| $T_0$ | 120 |
| $T_{end}$ | 10 |
| $\beta$ | 0.92 |
| $M$ | 1200 |

We propose an approach for generating test problems as follows. We need to generate a block layout with it's shortest path single loop as well as probability density function, PDF, for production time of each department. We assume that AGV velocity is constant and equal to 1m per second. We consider production time of each department as N( $\mu_i, \sigma_i^2$ ) that $\mu_i$ and $\sigma_i^2$ are randomly between (10,15) and (1,3) for each department respectively. We also generate matrix of material flow randomly between (1,10). AGV vehicle moves on the perimeter of the single loop path and loads the first finished goods. We consider single AGV vehicle with a unit capacity. When AGV is occupied, it is impossible to service other finished goods and material flow and they must wait for empty AGV. Moreover, location of I/O point is important in total time of the problem, because an I/O point can support more than one department at the same time. We generate 5 instances for 10 sizes ranging from 10 to 100 departments, merely 50 instances totally. We consider objective function as a probability variable labeled by $X$ . Each instance is solved in several scenarios until the objective function is converged. For each instance, we have $n$ scenario with $x_i$ objective function ($i$=1,2,…, $n$). We show average of objective function of each scenario as follows.

$$\bar{x} = \sum_{i=1}^{n} x_i / n$$

We know that the difference between expected value of $X$ and its' estimator $\bar{x}$ divided by standard deviation of $\bar{x}$ is a random variable with density function of t-student with parameter $n$, namely:

$$(E(X) - \bar{x}) / SD(\bar{x}) \sim t_{n-1}.$$

Using this equation, we find number scenarios for each instance equal to $n$.

Table 1 gives computational results. Objective function is total working time of AGV system and CPU time shows computational time to solve the problem in each instance. SA algorithm is coded in MATLAB software in a PC with 2.3 GHz Core2 Due CPU and 1GB RAM. In the first and second columns, size and number of each department are indicated respectively and in the third column, average total time of each size is shown. In Table 2, a comparison between the proposed approach and mean value approach is stated. We compare two approaches in four situations as follows:

1. Production time of each department is equal to $\mu_i$
2. Production time of each department is equal to $\mu_i + \sigma_i^2$
3. Production time of each department is equal to $\mu_i - \sigma_i^2$
4. Production time of each department is a random number of Normal probability distribution N( $\mu_i, \sigma_i^2$ ), Random Situation (RS).

We show efficiency of our proposed method respect to mean value method in different situations. We use a measure to show the effectiveness as follows:

$$(1 - Z_2 / Z_1) \times 100$$

where $Z_1$ is the objective function of mean value method and $Z_2$ is the objective function of our proposed method. Our proposed method is reasonably better than mean value method except situation 1, however, there is not any significant difference between two methods (less than 4% gap). In other 3 situations, especially in RS, there are a significant difference between two approaches with 13.7%, 13.6% and 16.3% in average for situations 2, 3 and 4 respectively.

## 4. Conclusions and recommendations

In this paper, we consider I/O point location problem with stochastic nature as a queuing model. We consider AGV vehicle as a single channel service provider and each department with uncertain production time. Computational results indicate flexibility of our proposed method in various situations. The proposed approach can be useful in many manufacturing systems. We recommend some extensions as follows. It can be useful to consider several AGV vehicles with multi capacity and investigate material handling cost and installment and maintenance costs. It is also recommended to present an approximation algorithm to estimate effectiveness of the proposed queuing model.

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Table 1. Computational results

| Size | No | Objective | CPU Time | Size | No | Objective | CPU Time |
|---|---|---|---|---|---|---|---|
| 10 | 1 | 1096 | 22 | 60 | 26 | 2855 | 1211 |
| | 2 | 1022 | 16 | | 27 | 2830 | 1256 |
| | 3 | 1010 | 14 | | 28 | 2988 | 1125 |
| | 4 | 1092 | 25 | | 29 | 2942 | 1218 |
| | 5 | 1063 | 23 | | 30 | 3044 | 1334 |
| 20 | 6 | 1548 | 28 | 70 | 31 | 3423 | 2617 |
| | 7 | 1597 | 39 | | 32 | 3220 | 2555 |
| | 8 | 1592 | 28 | | 33 | 3331 | 3654 |
| | 9 | 1596 | 28 | | 34 | 3483 | 3720 |
| | 10 | 1514 | 27 | | 35 | 3680 | 3649 |
| 30 | 11 | 1736 | 158 | 80 | 36 | 3611 | 3379 |
| | 12 | 1728 | 159 | | 37 | 3624 | 4434 |
| | 13 | 1767 | 123 | | 38 | 3534 | 3817 |
| | 14 | 1747 | 104 | | 39 | 3769 | 2758 |
| | 15 | 1721 | 111 | | 40 | 3770 | 3064 |
| 40 | 16 | 2376 | 414 | 90 | 41 | 4578 | 3650 |
| | 17 | 2134 | 246 | | 42 | 4241 | 4208 |
| | 18 | 2214 | 340 | | 43 | 4261 | 3579 |
| | 19 | 2274 | 426 | | 44 | 4561 | 5218 |
| | 20 | 2107 | 311 | | 45 | 4364 | 4226 |
| 50 | 21 | 2519 | 682 | 100 | 46 | 4545 | 4867 |
| | 22 | 2541 | 782 | | 47 | 4489 | 5720 |
| | 23 | 2812 | 765 | | 48 | 4775 | 6541 |
| | 24 | 2841 | 756 | | 49 | 4611 | 7864 |
| | 25 | 2659 | 906 | | 50 | 4252 | 7720 |

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Table 2. Comparing proposed approach with deterministic approach

| Size | No | μ | μ+ σ | μ- σ | RS | Size | No | μ | μ+ σ | μ- σ | RS |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 1 | -0.2 | 7.3 | 7.3 | 9.9 | 60 | 26 | -1.9 | 15.5 | 14.5 | 13.0 |
| | 2 | -0.5 | 9.6 | 9.6 | 9.1 | | 27 | -1.8 | 15.1 | 21.5 | 12.2 |
| | 3 | -0.3 | 5.5 | 5.5 | 8.0 | | 28 | -0.6 | 17.2 | 21.4 | 19.2 |
| | 4 | -0.2 | 9.0 | 9.0 | 8.6 | | 29 | -2.1 | 14.9 | 14.1 | 15.6 |
| | 5 | -0.1 | 9.3 | 9.3 | 6.3 | | 30 | -1.4 | 21.2 | 20.7 | 19.9 |
| 20 | 6 | -0.9 | 14.4 | 15.1 | 8.7 | 70 | 31 | -1.9 | 15.9 | 16.3 | 24.3 |
| | 7 | -1.0 | 14.5 | 8.5 | 8.9 | | 32 | -2.7 | 13.3 | 11.1 | 20.4 |
| | 8 | -0.9 | 11.7 | 12.8 | 10.8 | | 33 | -0.5 | 14.5 | 11.1 | 11.7 |
| | 9 | -1.4 | 11.5 | 8.4 | 9.1 | | 34 | -2.9 | 16.3 | 14.4 | 10.1 |
| | 10 | -0.3 | 8.4 | 8.9 | 10.3 | | 35 | -1.9 | 14.7 | 14.1 | 22.8 |
| 30 | 11 | -0.2 | 10.4 | 16.1 | 14.5 | 80 | 36 | -1.6 | 15.9 | 15.4 | 22.9 |
| | 12 | -1.1 | 15.7 | 9.0 | 13.3 | | 37 | -1.1 | 15.7 | 12.2 | 19.2 |
| | 13 | -0.5 | 16.6 | 14.7 | 13.8 | | 38 | -0.9 | 11.1 | 11.2 | 18.7 |
| | 14 | -1.9 | 12.5 | 10.4 | 14.8 | | 39 | -0.3 | 10.3 | 14.5 | 21.5 |
| | 15 | -1.4 | 16.4 | 13.5 | 9.5 | | 40 | -0.5 | 11.2 | 10.9 | 22.0 |
| 40 | 16 | -0.16 | 15.2 | 15.9 | 23.7 | 90 | 41 | -2.7 | 11.3 | 11.1 | 20.1 |
| | 17 | -0.15 | 16.7 | 16.5 | 16.8 | | 42 | -3.7 | 16.1 | 16.8 | 17.2 |
| | 18 | -1.8 | 15.7 | 12.4 | 18.9 | | 43 | -2.1 | 13.5 | 12.3 | 20.8 |
| | 19 | -0.27 | 13.9 | 21.1 | 20.1 | | 44 | -1.5 | 10.4 | 13.3 | 23.5 |
| | 20 | -1.51 | 14.4 | 13.6 | 21.5 | | 45 | -1.4 | 10.7 | 15.2 | 20.2 |
| 50 | 21 | -2.9 | 15.3 | 20.9 | 22.6 | 100 | 46 | -2.2 | 10.6 | 13.0 | 17.7 |
| | 22 | -2.9 | 15.7 | 12.7 | 16.0 | | 47 | -2.5 | 12.7 | 16.1 | 22.5 |
| | 23 | -2.2 | 20.9 | 14.9 | 19.1 | | 48 | -3.3 | 10.3 | 12.8 | 15.3 |
| | 24 | -1.4 | 21.8 | 18.7 | 10.0 | | 49 | -2.4 | 14.3 | 16.9 | 21.7 |
| | 25 | -1.0 | 19.5 | 13.0 | 22.0 | | 50 | -3.0 | 12.2 | 13.2 | 16.8 |

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