H_2/H_∞ Controller Design for Singular Perturbation Systems

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Abstract: In this paper the synthesis of logic-based switching H_2/H_∞ state-feedback controller for singular perturbation systems is considered that achieves a minimum bound on the H_2 performance level, while satisfying the prescribed H_∞ performance. The proposed hybrid control scheme is based on a fuzzy supervisor which manages the combination of two controllers. A convex LMI- based formulation of the two fast and slow subsystem controllers leads to a structure that ensures a good performance in both the transient and the steady state phase. It is shown that the system with the proposed controller remains globally stable despite the configuration (controller) changing.

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1. Introduction

There has been increasing interest in hybrid control in recent years, due to its potential to overcome limitations of adaptive control and benefits in controlling of systems that cannot achieve the desired performance by a single controller. Indeed, hybrid control scheme provides an effective mechanism when facing large modelling uncertainty and highly complex systems. Even for simple linear time invariant systems, controllers switching can be utilized in improving the performance (Sun, 2005, Feuer, 1997, and McClamroch, 2000). To date, Morse, Hespanha and Liberzon have established a theoretical backbone for hybrid controllers (Morse, 1997, Hespanha, 1999, and Liberzon, 2003). By now, stabilizing a continuous system via hybrid output feedback has attracted a number of authors, such as (Santarelli, 2008) where a comparison between the responses of the switching controller and two other forms of LTI control have been made. An experimental assessment of controller switching with state and control magnitude constraints is carried out in Kogiso, 2004. In Zheng, 2006, the multi-objective robust control of an induction motor with tracking and disturbance rejection specifications is proposed via switching. In Essounbouli, 2006, DeCarlo, 1988, and Jamshidi, 2010 controller switching has been proposed to improve the trade-offs in design multi objectives.

Supervisory control employs logic-based switching for adaptation, instead of continuous tuning of parameters as in conventional adaptive control. This type of switching-based supervisory control scheme consists of the following subsystems: a plant to be controlled, a bank of controllers, and a switching logic. Dwell-time method is representative of the trajectory independent switching logic for supervisory control (see Yoon, 2007 and its references). On the other hand, Lyapunov functions are employed in such trajectory dependent switching methods as in Yoon, 2007.

Systems with slow and fast dynamics, described mathematically by singular perturbations, are studied extensively in numerous papers and books; see for examples (Kokotovic, 1986, Tan, 1998). For robust control of singular perturbation systems, the controller is usually derived through indirect mathematical programming approaches (e.g. solving Riccati equations), which encounter serious numerical problem linked with the stiffness of the equations involved in the design. To avoid this difficulty, several approaches (Oliveira, 1999, Pan, 1993) have been developed to transform the original problem into ε-independent sub-problems, among which, the time-scale decomposition (Oliveira, 1999) is commonly adopted. As an alternative to Riccati equation solution, LMI formulation has been attracting more and more attention of robust control researchers. However, up to the present, it remains an open area solving mixed H_2/H_∞ control problems for singular perturbation systems through LMI approach. Garcia et al. Garcia, 1998 proposed a solution to the infinite time near optimal regulator problem (H_2 control) for singular perturbation systems through an LMI formulation. A time scale-decomposition was employed on the overall system as well. In (Li, 2001) the problem is formulated into a set of inequalities independent of ε. An algorithm is given to solve this set of inequalities through LMI formulation. But extension of this method to mixed H_2/H_∞ control is very difficult. In (Li, 2007) a same approach is used for solving problem with static output feedback instead of state feedback. Combination of different techniques to obtain the different performances is widely used today (Essounbouli, 2006, Pan, 1993, and Peres, 1994). This method results in hybrid dynamical systems which include continuous and discrete dynamics and a mechanism (supervisor) managing the interaction between these dynamics. In the present paper, the switching mixed H_2/H_∞ state feedback control problems for continuous-time linear singular perturbation systems are solved. The simple design methods of Garcia, 1998 are applied to derive the state-feedback gains, separately for two fast and slow sub-systems. A fuzzy supervisor is proposed for hybrid combination of these controllers to use their advantages and to ensure the required performances and the stability of the closed loop system.

The contribution of the presented work is combining fast and slow sub-system controllers using a supervisor, which manages the gradual transition from one controller to another. This method is applied to use the advantages of each controller. The control signal is obtained via a weighted sum of the two signals given by the slow and fast sub-system controllers. This weighted sum is managed thanks to a fuzzy supervisor, which is adapted to obtain the desired closed loop system performances. So, the fast sub-system controller mainly acts in the transient phase providing a fast dynamic response and enlarging the stability limits of the system, while the slow sub-system controller acts mainly in the steady state to reduce chattering and to maintain the tracking performances. Furthermore, the global stability of the system even if the system switches from one configuration to another (transient to steady state and vice versa) is guaranteed.

The structure of the paper is as follows. Section 2 presents the system definition and the controllers used. In Section 3, the fuzzy supervisor and the proposed control law are described. Stability analysis is demonstrated in Section 4. The design procedure is explained in Section 5 and an example is given to illustrate the efficiency of the proposed method, followed by conclusions in Section 6.

2. Problem Statement

Consider the following linear singularly perturbated system Σ with slow and fast dynamics described in the "singularly perturbated" form:

$$\Sigma:\begin{cases}\dot{x}_{slow} = A_1 x_{slow} + A_2\, x_{fast} + B_1 u + B_{w_1} w \\ \varepsilon\dot{x}_{fast} = A_3 x_{slow} + A_4\, x_{fast} + B_2 u + B_{w_2} w \\ z = C_{z_1} x_{slow} + C_{z_2} x_{fast} + D_z u\end{cases} \qquad (1)$$

where x_{slow}, x_{fast} are the states; $u \in R^{m_1}$ is the control input; $w \in R^{m_2}$ is the disturbance input; $z \in R^{l_z}$ is the output to be regulated; and ε is a small positive parameter. By introducing the following notations:

$$x = \begin{bmatrix} x_{slow} \\ x_{fast}\end{bmatrix}, A_\varepsilon = \begin{bmatrix} A_1 & A_2 \\ \frac{1}{\varepsilon}A_3 & \frac{1}{\varepsilon}A_4\end{bmatrix}$$

$$B_\varepsilon = \begin{bmatrix} B_1 \\ \frac{1}{\varepsilon}B_2\end{bmatrix}, B_{w_\varepsilon} = \begin{bmatrix} B_{w_1} \\ \frac{1}{\varepsilon}B_{w_2}\end{bmatrix}, C_z = \begin{bmatrix} C_{z_1} & C_{z_2}\end{bmatrix}$$

(2)

The system Σ can be rewritten into the following compact form:

$$\Sigma:\begin{cases}\dot{x} = A_\varepsilon x + B_\varepsilon u + B_{w_\varepsilon} w \\ z = C_z x + D_z u\end{cases} \qquad (3)$$

Applying a static state feedback control:

$$u = Kx \qquad (4)$$

leads to the following closed-loop system:

$$\Sigma_{cl}:\begin{cases}\dot{x}_{cl} = A_{cl} x_{cl} + B_{cl} w \\ z = C_{cl} x_{cl}\end{cases} \qquad (5)$$

where $A_{cl} = A_\varepsilon + B_\varepsilon K$, $B_{cl} = B_{w_s}$, $C_{cl} = C_z + D_z K$.

Denote the transfer function of the closed-loop system Σ_{cl} from w to z as: $T(s,K) = C_{cl}(sI - A_{cl})^{-1} B_{cl}$. The H_2 norm of $T(s,K)$ is defined by:

$$\|T(s,K)\|_2 = \frac{\|z\|_\infty}{\|w\|_2} \qquad (6)$$

and the H_∞ norm of $T(s,K)$ is defined by:

$$\|T(s,K)\|_\infty = \frac{\|z\|_2}{\|w\|_2} \qquad (7)$$

2.1. Slow and fast sub-systems

If A_4 be a non- singular matrix, we can decompos original singularly perturbated system (1) to two slow and fast subsystems. The slow subsystem defined letting $\varepsilon = 0$ in second equation of (1) and computing x_{fast} in terms of x_{slow} , u and w , then substituting it in the first equation. Therefore, slow subsystem obtained as follows:

$$\begin{aligned}\dot{x}_{slow} &= A_s x_{slow} + B_s u + B_{w_s} w \\ z_{slow} &= C_{z_s} x_{slow} + D_s u + D_{w_s} w\end{aligned} \qquad (8)$$

where [6]

$$A_s = \left(A_1 - A_2 A_4^{-1} A_3\right), B_s = \left(B_1 - A_2 A_4^{-1} B_2\right)$$

$$B_{w_s} = \left(B_{w_1} - A_2 A_4^{-1} B_{w_2}\right), C_{z_s} = \left(C_{z_1} - C_{z_2} A_4^{-1} A_3\right)$$

$$D_s = \left(D_z - C_{z_2} A_4^{-1} B_2\right), D_{w_s} = -C_{z_{fast}} A_4^{-1} B_{w_2}$$

The fast subsystem of (1) is defined by [6]:

$$\dot{x}_{fast} = A_f x_{fast} + B_f u + B_{w_f} w \qquad (11)$$
$$z = C_{z_f} x_{fast} + D_f u$$

Therefore, the overall system is decomposed into slow and fast subsystems. In sequel, these subsystems are used to design slow and fast controller and then are mixed using a fuzzy supervisor to produce a controller for the overall system. In this paper we focus on the suboptimal mixed H_2/H_∞ static state feedback control problem in terms of linear matrix inequalities (LMI).

Lemma 2. 1. [1] (H_2 control problem): Consider overall system (1). The static state feedback control law (4) stabilize closed loop system (5) and achieves a prescribed H2-norm bound $0 < \nu$ for it, if and only if there exists $Q = Q^T > 0, T, Z$ with appropriate dimensions such that:

$$\begin{bmatrix} A_{11} & QC_z^T + T^T D_z^T \\ C_z Q + D_z T & -I \end{bmatrix} < 0$$
$$A_{11} = A_\varepsilon Q + QA_\varepsilon^T + B_\varepsilon T + T^T B_\varepsilon^T \qquad (10)$$
$$\begin{bmatrix} Q & B_{w_\varepsilon} \\ B_{w_\varepsilon}^T & Z \end{bmatrix} > 0, trace(Z) < \nu$$

By solving mentioned LMI's, Q, T and Z will be found and control law (4) is calculated as:

$$K = TQ^{-1} \qquad (11)$$

It guarantees that closed loop system is asymptotically stable and H_2-norm (6) is less than ν.

Lemma 2. 2. [1] (H_∞ control problem): The control law (4) stabilize closed loop system (5) and achieves a prescribed H∞- norm bound $0 < \gamma$ for it,

if and only if there exists $Q = Q^T > 0$ and T with appropriate dimension such that:

$$\begin{bmatrix} A^{11} & B_{w_\varepsilon} & QC_z^T + T^T D_z^T \\ B_{w_\varepsilon}^T & -I & D_{cl}^T \\ C_z Q + D_z T & D_{cl} & -\gamma^2 I \end{bmatrix} < 0 \qquad (12)$$
$$A^{11} = AQ + QA^T + B_\varepsilon T + T^T B_\varepsilon^T$$

By solving LMI (12), Q and T will be found and control law (4) is calculated from (11).

Lemma 2. 3. [1] (Mixed H_2/H_∞ control problem): The control law (4) satisfies mixed H_2/H_∞ control problem if and only if the following LMI's for $Q = Q^T > 0, T, Z$ and a given positive scalar $\gamma > 0$ are satisfied:

$$\min \quad \nu$$
$$\text{subject to} \quad (12) \text{ and } (10) \qquad (13)$$

By solving (13), Q, T, Z and ν are found and the control law (4) is computed from (11).

3. Fuzzy Supervisor

The approach used in this paper for solving mixed H_2/H_∞ control problem for linear singular perturbation system is different from former approaches. We start with an overall linear singular perturbation system and decompose it to slow and fast subsystems. Then we solve mixed H_2/H_∞ control problem for each slow and fast subsystems and find K_{slow}, K_{fast} by solving corresponding LMI's. It is well known that fast subsystem can be a good approximation for transient time of overall system response and slow subsystem can be a good model for steady state time of overall system response. Therefore, fast subsystem controller K_{fast} can be used during the transient time and slow subsystem controller K_{slow} can be used during the steady state, their control actions are combined by means of a weighting factor, $\alpha \in [0 \quad 1]$, representing the output of a fuzzy logic supervisor that takes the tracking error e and its time derivatives $\dot{e}, \ddot{e}, \ldots, e^{n-1}$ as inputs.

The fuzzy system is constructed from a collection of fuzzy rules whose jth component can be given in the form:

If e is H_1^j And … And e^{n-1} is H_n^j Then $\alpha = \alpha_j$. Where H_i^j is a fuzzy set and α_j is a singleton.

The fuzzy implication uses the product operation rule. The connective AND is implemented by the minimum operation, whereas fuzzy rules are combined by algebraic addition. Defuzzification is performed using the centroid method. Since the membership functions that define the linguistic terms of the output variable are singletons, the output of the fuzzy system is given by

$$\alpha = \frac{\sum_{i=1}^{m} \alpha_i \prod_{j=1}^{n} \mu_i^j}{\sum_{i=1}^{m} \prod_{j=1}^{n} \mu_i^j}$$

where μ_i^j is the degree of membership of H_i^j and m is the number of fuzzy rules used. The objective of the fuzzy supervisor is to determine the weighting factor, α which gives the participation rate of each control signal. Indeed, when the norm of the tracking error e and its time derivatives $\dot{e}, \ddot{e}, \ldots, e^{n-1}$ are small, the plant is governed by the slow subsystem controller K_{slow} ($\alpha = 1$). Conversely, if the error and its derivatives are large, the plant is governed by the fast subsystem controller K_{fast} ($\alpha = 0$). The control action u, is determined by:

$$u = (1-\alpha)u_{fast} + \alpha u_{slow} \qquad (14)$$

where

$$u_{slow} = K_{slow} x_{slow}, u_{fast} = K_{fast} x_{fast} \qquad (15)$$

Structure of proposed controller with a fuzzy supervisor has been shown in Figure 1.

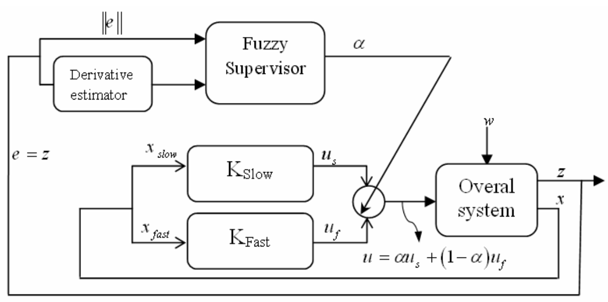

Figure 1. The structure of the proposed controller

4. Stability Analysis

The theorem of Essounbouli et al. [7] is used to prove the global stability of the system. Similar to [7], this theorem is rewritten as follows:

Theorem 4.1. Consider a combined fuzzy logic control system as described in this work. If:

1. There exists a continuously differentiable and radially unbounded scalar function $V > 0$ for each subsystem,
2. Every subsystem gives $\dot{V} < 0$ in its active region,
3. The weighted sum defuzzification method is used, such that for any control input u

$\min(u_{slow}, u_{fast}) \le u \le \max(u_{slow}, u_{fast})$

Then the resulting control u, given by (14), guarantees the global stability of the closed loop system.

Proof: Satisfying two first conditions guarantees the existence of a Lyapunov function in the active region which is a sufficient condition for ensuring the asymptotic stability of the system during the transition from the fast subsystem controller to the slow subsystem controller. Consider the Lyapunov function $V_{fast} = \zeta^T P_{fast} \zeta$ where P_{fast} is a positive definite matrix and the solution of (13) for fast subsystem and the Lyapunov function $V_{slow} = \zeta^T P_{slow} \zeta$ where P_{slow} is a positive definite matrix and the solution of (13) for slow subsystem. To satisfy the second condition it is enough to choose P_{slow}, P_{fast} such that:

$$P_{slow} \le P_{fast} \tag{20}$$

This condition guarantees that in the neighbourhoods of the steady state, the value of the Lyapunov function V_{fast} is greater than that of V_{slow}. To guarantee the third condition, the balancing term α takes its values in the interval [0 1]. Consequently, the three conditions of the above theorem are satisfied and the global stability of the system is guaranteed.

So, The Problem formulation (switching H_2/H_∞ control) will be as:

$$\begin{cases} \min & \left(\|T(s,K)\|_2\right)_{slow} \\ \text{subject to} & \left(\|T(s,K)\|_\infty\right)_{slow} < \gamma_{slow} \end{cases} \quad \text{and}$$

$$\begin{cases} \min & \left(\|T(s,K)\|_2\right)_{fast} \\ \text{subject to} & \left(\|T(s,K)\|_\infty\right)_{fast} < \gamma_{fast} \end{cases}$$

$$\text{while}: P_{slow} \le P_{fast}$$

5. Design Procedure

The design procedure can be summarizing as follows:

Compute slow and fast subsystems of overall system from (8) and (9). Solve control problem (16) for each subsystem with given positive scalars γ_{slow} and γ_{fast} to find K_{slow} and K_{fast} from (11). Compute u_{slow} and u_{fast} from (15). Calculate overall control signal u from (14) that α is governed by fuzzy supervisor according to error and its derivatives. Apply this control signal to (1) and construct closed loop system (5). To construct the fuzzy supervisor, firstly, the fuzzy sets are defined for each input (the error and its derivatives) and output; then, the rule base is elaborated. The error vector is computed and then is injected in the supervisor to determine the value of α to apply to the global control signal.

6. Simulation

To demonstrate the solvability of the various LMIs, simplicity and low conservatives of the proposed method, The formulation of the switching H_2/H_∞ control of the singularly perturbed system is now applied to control the longitudinal flight dynamics of F-8 aircraft model. The longitudinal dynamics of the aircraft exhibits two- time scale properties identifiable by the phugoid (slow) and the short period (fast) mode. The H_2/H_∞ controllers are designed for the longitudinal axis dynamics of the aircraft for the cases of full- order control, fast control and slow control.

The linearized small-perturbation longitudinal equations of the motion and aerodynamics stability derivations are provided in [8]. The longitudinal F-8 aircraft model is for a flight condition of Mach 0.6 ($V_0 = 620\,ft\,s^{-1}$) altitude of 20000 feet, and angle of attack of 0.078 rad. The state variables are v : velocity ($ft\,s^{-1}$); α : angle of attack (rad); q : pitch rate ($rads^{-1}$); θ : pitch angle (rad); and the input variable is: σ stabilator deflection (rad). Here, the slow states are the forward air speed and pitch angle, while the fast states are the angle of attack and pitch rate. The space model is obtained as [6]:

$$A_1 = \begin{bmatrix} -3 & 1 \\ 1 & -1 \end{bmatrix}, A_2 = \begin{bmatrix} 1.5 & 1 \\ 1 & 1 \end{bmatrix}, A_3 = \begin{bmatrix} 0.5 & 1 \\ 1 & 2 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} -5 & 1 \\ 3 & -4 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B_{w_1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, B_{w_2} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C_{z_1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}^T, C_{z2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$D_Z = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T, \varepsilon = 0.1$$

The input disturbance w is zero mean white noise process with $E\left(w\left(t\right)w\left(\tau\right)\right) = \delta\left(t-\tau\right)$ is injected into the system in the interval $t \in \left[10 \quad 30\right]$.

Following the proposed design method in section 5, the following results are obtained:

$$K_{H_2/H_\infty} = \begin{bmatrix} -1.8265 & -3.9832 & 1.0093 & -0.8187 \\ -1.5087 & -3.2663 & -0.7951 & 0.7210 \end{bmatrix}$$

$$K_{fast} = 10^{-7} \times \begin{bmatrix} 0.4230 & 0.3809 \\ -0.3759 & 0.2860 \end{bmatrix}$$

$$K_{slow} = \begin{bmatrix} -2.1457 & -4.6675 \\ -1.7237 & -3.7761 \end{bmatrix}$$

$$K_{switching} = \left[\left(1\text{-}\alpha\right)K_{fast} \quad \alpha K_{slow}\right]$$

Only first time derivative of tracking error is used because in practical system, it is difficult to measure the higher order time derivatives of the tracking error. The fuzzy supervisor is constructed by using three fuzzy sets zero, medium and large for the norms of the tracking error and its time derivative. The corresponding membership functions are triangular. For the output, five singletons are selected; very large (VL), large (L), medium (M), small (S) and zero (Z), corresponding to 1, 0.75, 0.5, 0.25 and 0, respectively. Rules are defined in Table 1., for example, a rule in the table can be stated as follows: "IF the norm of the error is medium AND the norm of the error derivative is large, THEN α is zero".

From obtained simulation results in Table 2., it is clear that the proposed method gives better response than conventional overall design method for full order system. In our proposed switching method, with a smaller γ for H_∞ constraint, the H_2 norm is smaller. But both of H_2 and H_∞ norms are increased in conventional overall method. From Figure 2, it is clear that output regulation in our proposed controller is better related to conventional overall controller.

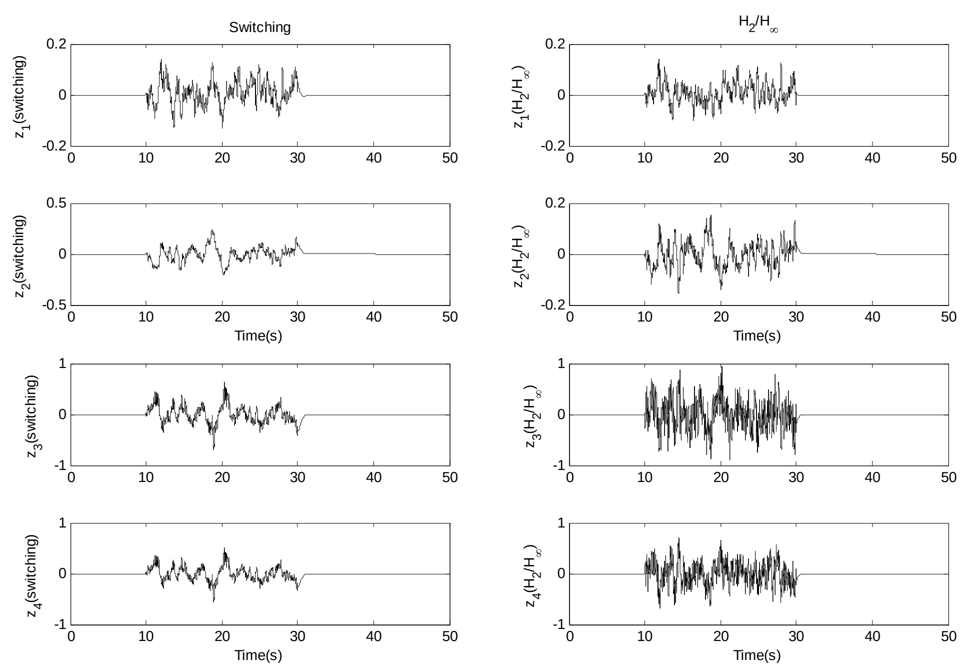

Figure 2. The State Response of simulation

7. Conclusion

In this paper, convex optimization method is used to design the logic based switching H_2/H_∞ controller for a linear singular perturbation system. Proposed controller guarantees the stability of the closed loop system and satisfies the prescribed level of performance indexes for both of H_2 and H_∞ norms. Using the reduced-order fast and slow mode controllers instead of one full-order overall controller with higher order is the main contribution of this paper. A fuzzy supervisor manages both of fast and slow controller performance efficiently such that in spite of switching nature of control scheme, stability of closed loop system is guaranteed and the performance criterion is satisfied. In reality, fast mode controller has a good performance in transient mode (low energy impulse response) and slow mode controller affects the steady mode section and attenuates the low frequency disturbances. Simulation results show that the proposed controller causes the considerable improvement in the overal performance of the closed loop system.

Table 1. The rules of the proposed Supervisor

	|de/dt|			
$\|e\|$	Z	VL	L	M
	M	S	S	Z
	L	Z	Z	Z

Table 2. The results of the simulation

	$\sup_w \frac{\|z\|_\infty}{\|w\|_2}$	$\sup_w \frac{\|z\|_2}{\|w\|_2}$
K_{H_2/H_∞}	10.2309	0.3062
$K_{switching}$	6.341	0.2361

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References

1. Sun Z, Ge SS. Analysis and synthesis of switched linear control systems. Automatica. 2005;41:181-195.
2. Feuer A, Goodween GC, Saldago M. Potential benefits of hybrid control for linear time invariant plants. Proceedings of American Control Conference. New Mexico. 1997:2790-2794.
3. McClamroch NH, Kalmanovsky I. Performance benefits of hybrid control design for linear and nonlinear systems. Proceedings of IEEE, 2000:1083-1096.
4. Morse AS. Control using logic-based switching. Springer-Verlag New York, Secaucus, New Jersy, USA, 1997.
5. Hespanha JP. Stabilization of nonholonomic integrators via logic-based switching, Automatica. 1999;35:385-393.
6. Liberzon D. Switching in systems and control. Systems and control: Foundations and applications. MA, Birkh, Boston, 2003.
7. Santarelli KR, Dahleh MA. Comparison of a switching controller to two LTI controllers for a class of LTI plants. Proceedings of American Control Conference. Seattle, Washington. 2008:4640-4646.
8. Kogiso K, Hirata K. Controller switching strategies for constrained mechanical systems with application to the remote control over networks. Proceedings of IEEE International Conference on Control Applications. 2004:480-484.
9. Zheng K, Lee AH, Bentsman J, Krein PT. High performance robust linear controller synthesis for an induction motor using a multi-objective hybrid control strategy. Nonlinear Analysis. 2006;65:2061-2081.
10. Essounbouli N, Manamanni N, Hamzaoui A, Zaytoon J. Synthesis of switching controllers: A fuzzy supervisor approach. Nonlinear Analysis. 2006;65:1689–1704.
11. DeCarlo RA, Zak SH, Matthews GP. Variable structure control of non-linear multivariable systems: A tutorial. Proceedings of the Decision and control Conference. 1988:212–232.
12. Jamshidi F, Beheshti MTH, Fakharian A. Fuzzy supervisor approach on logic based switching H_2/H_∞. Proceedings of the Institution of Mechanical Engineers, Part I, Journal of Systems and Control Engineering. 2010;11-19.
13. Yoon TW, Kim JS, Morse AS. Supervisory control using a new control relevant switching. Automatica. 2007;43:1791-1798.

1. Scherer C, Gahinet P, Chilali M. Multiobjective output feedback control via LMI optimization, IEEE Transaction on Automatic Control. 1997;42(7):896-911.
2. Chen CY. Hybrid controller design for a mechanical transmission system with variable compliance and uncertainties, International Journal of Innovative Computing, Information and Control. 2008;4(8):1821-1834.

3. Gershona E, Shaked U. Static H_2 and H_∞ output-feedback of discrete-time LTI systems with state multiplicative noise, Systems and Control Letters. 2006;55:232-239.
4. Garcia G, Daafouz J, Bernussou J. A LMI solution in the H_2 optimal problem for singularly perturbated systems. Proceedings of American Control Conference. 1998:550-554.
5. Garcia G, Daafouz J, Bernussou J. H_2 guaranteed cost control for singularly perturbated uncertain systems, IEEE Transaction on Automatic Control. 1998; 43(9):1323-1328.
6. Kokotovic PV, Khalil HK, O'Reilly J. Singular Perturbation Methods in Control: Analysis and Design. Academic, New York, 1986.
7. Oliveira MC, Geromel JC, Bernussou J. An LMI optimization approach to multiobjective controller design for discrete- time system. Proceedings of Decision and Control Conference. 1999:3611-3616.
8. Essounbouli N, Manamanni N, Hamzaoui A, Zaytoon J. Synthesis of switching controllers: a fuzzy supervisor approach, Nonlinear Analysis. 2006;65:1689-1704.
9. Khargonekar P, Rotea MA. Mixed H_2/H_∞ control: a convex optimization approach, IEEE Transaction on Automatic Control. 1991;39:824-837.
10. Pan Z, Basar T. H_∞-Optimal control for singularly perturbated systems-Part I: Perfect state measurement, Automatica.1993;29(2):401-403.
11. Peres PLD, Geromel JC. An alternate numerical solution to the linear quadrautic problem, IEEE Transaction on Automatic Control.1994;39(1):198-202.
12. DeCarlo RA, Zak SH, Matthews GP. Variable structure control of non-linear multivariable systems: A tutorial. Proceedings of Decision and Control Conference. 1988:212-232.
13. Tan W, Leung T, Tu Q. H_∞ control for singularly perturbated systems, Automatica.1998;34(2): 255-260.
14. Dragan V, Morozan T. The linear quadratic optimization problem for a class of discrete-time stochastic linear systems, International Journal of Innovative, Computing, Information and Control. 2008;4(9):2127-2137.
15. Li Y, Wang JL, Yang GH. Sub-optimal linear quadrautic control for singularly perturbated systems. Proceedings of Decision and Control Conference.2001:3698-3703.
16. Li Y, Jiao Y, Wang X. Suboptimal H_2 static output feedback control for singularly perturbated systems. Proceedings of Automation and Logistics Conference. 2006:2796-2800.

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