

## Bit-Interleaved Turbo-Coded over Wireless Channels

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**Abstract:** This paper presents an improved version of bit-interleaved turbo-coded modulation (BITCM) scheme designed for bandwidth efficient transmission over wireless channels. The proposed scheme consists to apply signal space diversity (SSD) to conventional BITCM and a rotated modulation. At the receiver side, an iterative demapping and decoding is proposed in order to optimize the error performance. Simulation results carried out on 2 bit/s/Hz 64-QAM BITCM indicate that is possible to obtain a gain exceeding 0.5 dB at a BER =  $10^{-7}$  compared to the classical 64-QAM BITCM scheme. It is also shown that, the error floor can be significantly lowered using SSD technique at a little cost in terms of system's complexity.

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**Keywords:** BICM, DVB-RCS turbo code, signal space diversity, iterative demapping, error floor, wireless channels.

### 1. Introduction

Bit-interleaved turbo-coded modulation (BITCM) is bandwidth-efficient coding designed using the bit-interleaved coded modulation (BICM) approach. It consists on a serial concatenation of turbo coding, bit-by-bit interleaving and higher-order modulation according to Gray mapping. BITCM scheme have been shown to achieve bit error rate (BER) performance close to the capacity limit over additive white Gaussian noise (AWGN) channels. The mapping used in , the opposite mapping strategy is used, where the most protected bit positions are dedicated to the systematic bits. Due to the systematic information bits in the turbo decoding algorithm, the latter mapping strategy outperforms the former at only of what is known as the waterfall region. Recently, it is shown that, the performance of bit-interleaved coded modulation (BICM) can be greatly improved through iterative information exchange between the soft-input soft-output (SISO) damper/demodulator and the (SISO) convolution decoder at the receiver. This system, introduced in is usually referred to as BICM with iterative decoding (BICM-ID). The purpose of this paper is to apply the BICM-ID concept to BITCM scheme. An initial exploration of this concept was presented in which proposed an additional feedback from the (SISO) turbo decoder to the damper. Furthermore, only indicate a performance gain of 0.1 dB compared to classical BITCM. In this contribution, we propose to modify the BITCM structure so as to improve their error performance for Rayleigh wireless channels, with respect to low error floor and early convergence threshold. For that, an iterative decoding process is implemented in the receiver side which incorporates the soft-input soft-output (SISO) demodulator into the iterative turbo decoding loop. Moreover, the

diversity order can be maximized by optimally rotating the constellation and separately interleaving the signals in each coordinate such technique is also known as signal space diversity (SSD). As far communications on wireless channels are considered, we show that the diversity order can be increased using (SSD) technique. Throughout this work, we focus on the design of a 2 bit/s/Hz BITCM system employing the well known DVB-RCS turbo code and a 64-ary quadrature amplitude modulation (QAM) constellation. The DVB- RCS turbo code is the double binary 8-state turbo code which is adopted in the digital video broadcasting (DVB) standards for return channel via satellite (DVB-RCS) (DVB-RCS 2000) and the terrestrial distribution system (DVB-RCT) (DVB-RCT 2000). The proposed system improves the iterative process convergence and lowers the error floor. The paper is organized as follows: in Section 2, we provide a brief review of BITCM schemes focusing on the reason behind the degradation in performance for Rayleigh channel. The proposed BITCM scheme with signal space diversity is presented in Section 3. Simulation results and performance comparisons are shown in Section 4. Finally, Section 5 presents conclusions of this contribution.

### 2. Review of BITCM

#### 2.1. BITCM Structure

Conventional BITCM can be modeled as a serial concatenation of a turbo encoder which may stand for binary turbo encoder or duo-binary turbo encoder, a bit interleaver  $\pi$  and an  $M$ -ary memory less modulator (where  $M = 2^m$ ) as shown in Figure 1. At the transmitter, the sequence of information bits  $u$  is first encoded by a turbo encoder to produce the output coded sequence  $c$  before being bitwise

interleaved to  $v$ . The purpose of the bit interleaver  $\pi$  (required only for the Rayleigh wireless channel) is to break the fading correlation and increase the diversity order to the minimum Hamming distance. After the interleaver  $\pi$ ,  $m$  consecutive bits of the interleaved sequence  $v$  can be grouped as a channel symbol  $v_t = (v_t^{(1)}, \dots, v_t^{(m)})$ , where  $v_t^{(i)}$  denotes the  $i$ th bit in the bit pattern at time index  $t, t = 1, \dots, T$ . The complex transmitted signal  $s_t = \mu(v_t)$ , is then chosen from the  $M$ -ary constellation  $\chi$  to carry  $m$  coded bits over each symbol duration. Here,  $\mu$  denotes the mapping scheme from the bit patterns to constellation points. We assume a frequency nonselective Rayleigh wireless channel and coherent detection, the received discrete-time base band signal can be written as

$$r_t = \rho_t s_t + n_t \tag{1}$$

Where  $\rho_t$  is the Rayleigh-distributed fading coefficient with  $E(\rho_t^2) = 1$  and  $n_t$  is a complex white Gaussian noise sample with independent in-phase and quadrature components having two-sided power density the variance  $\sigma^2 = N_0/2$ . For AWGN channel,  $\rho_t = 1$ . Throughout this work, we assume perfect channel state information (CSI) so that  $\rho_t$  is perfectly estimated and available to the receiver.

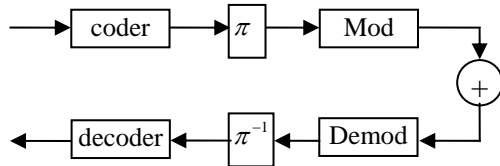


Figure 1. Structure of the classical BITCM.

The receiver of BITCM system depicted in Figure 1 includes three elements: the demodulator, the de-interleaver  $\pi^{-1}$ , and the turbo decoder. For each received  $r_t$ , the Logarithm of Likelihood Ratio (LLR)  $\Lambda(v_t^i)$  associated with each bit  $v_t^i, i \in \{1, \dots, m\}$ , is computed by demapper  $\mu^{-1}$  and used as relevant soft decision by the turbo decoder. The LLRs  $\Lambda(v_t^i)$  are obtained as

$$\Lambda(v_t^i) = \log \frac{\sum_{s_t \in \chi_t^1} P(r_t | s_t, \rho_t)}{\sum_{s_t \in \chi_t^0} P(r_t | s_t, \rho_t)} \tag{2}$$

Where  $\chi_b^i$  denotes the subset of symbols  $s_t \in \chi$  whose bits labels have the binary value

$b \in \{0, 1\}$  at  $i$  th bit position. The function  $p(r_t | s_t, \rho_t)$  is the probability density function of the revived signal  $r$  given the fading amplitude  $\rho_t$  and signal  $s_t$  was transmitted. With  $M$ -ary signal constellation  $\chi$ ,  $p(r_t | s_t, \rho_t)$  is given as

$$p(r_t | s_t, \rho_t) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{d_{rs}^2}{2\sigma^2}\right), \tag{3}$$

With

$$d_{rs}^2 = \|r_t - \rho_t s_t\|^2 + \|r_{Q_t} - \rho_t s_{Q_t}\|^2 \tag{4}$$

For practical implementation, the computational complexity of the demapper can be considerably reduced by approximating (2) with the following expression:

$$\Lambda(v_t^i) \approx \min_{s_t \in \chi_t^0} \|r_t - \rho_t s_t\|^2 - \min_{s_t \in \chi_t^1} \|r_t - \rho_t s_t\|^2 \tag{5}$$

For a square  $M$ -ary QAM constellation with Gray mapping, LLRs are correlated over  $m/2$  values.

Thus, it necessary to use the de-interleaver  $\pi^{-1}$  to ensure efficient turbo decoding.

## 2.2. Performance Evaluation

A comparison of the performance of BITCM scheme over AWGN and Rayleigh wireless channels shows a higher gap to the channel capacity limit over Rayleigh wireless channels. Moreover, an error floor is observed at higher value of BER over Rayleigh wireless channels. To confirm these results, we have represented in Figure 2 and 3, the BER performance achieved over AWGN and Rayleigh channels respectively, for 2 bit/s/Hz 64-QAM BITCM scheme using DVB-RCS turbo code operating on either large (51200 bits) or small (1504 bits) frames.

DVB-RCS turbo code. MAP decoding with 8 iteration. From Figure 2 and 3, it is seen that, at a BER= $10^{-5}$ , the gap ( $\approx 0.5$  dB) between the capacity limit and the performance of 64-QAM BITCM using scheme A on Rayleigh channel is higher to that obtained on AWGN channel. However, in the case of Rayleigh channel, we can notice that 64-QAM BITCM scheme suffers from an early error floor occurring at a BER level equal to  $10^{-5}$ .

This degradation in performance could explained by the fact that the transmitted QAM constellation signal is subject to the same fading coefficient on both component axes. The proposed

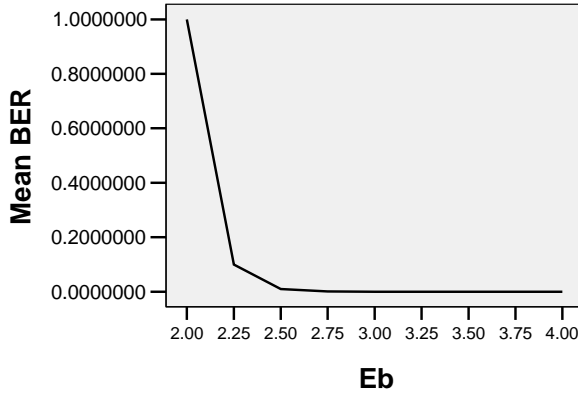


Figure 2. BER performance over AWGN channel of a 2 bit/s/Hz 64-QAM BITCM using a 1/2-rate DVB-RCS turbo code.

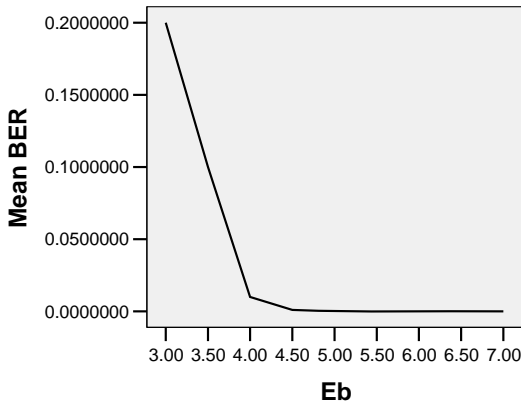


Figure 3. BER performance over Rayleigh channel of a 2 bit/s/Hz 64-QAM BITCM using a 1/2-rate

BITCM In this section, we start by presenting the (SSD) technique which is suitable for wireless channels. It is followed by the structure of the proposed BITCM system for Rayleigh channel.

### 3.1 Diversity Improvement

Signal space diversity can provide performance improvement over wireless channels by increasing the diversity order of communication system it combats wireless channel by rotating the constellation and adding independent interleavers for the in-phase (I) and quadrature (Q) components of a transmitted symbol chosen from a properly rotated constellation after a modulator. The purpose of coordinate interleaving is to make the I and Q component of a modulated symbol uncorrelated during the same fading interval. To maximize the diversity order, the constellation should be properly rotated such that all distinct signal symbols are separable on every coordinate. Figure 4 shows the system model of  $\pi/8$ -rotated encoded 64-QAM with

an IQ interleaver, which is adopted in our system. It can be seen as a serial concatenation of a memory less modulator and an IQ interleaver ( $\pi$  IQ). Therefore, the diversity order of BITCM can be achieved by adding independent interleavers for the I and Q components of a transmitted signal  $s_t$  chosen from a  $\pi/8$ -rotated 64-QAM constellation after a modulator. In practice, it is possible to replace the IQ interleaver with a time delay in only one of the quadrature (Q) components. For a memory less wireless channel, a delay of at least one symbol period is sufficient.

### 3.2 Structure of the Proposed BITCM Scheme

The block diagram of the BITCM system using a rate-1/2 DVB-RCS turbo code and 64-QAM constellation is shown in Figure 5. A modulator maps each channel symbol  $v_t$  according to Gray mapping to a complex symbol  $s_t = \mu(v_t) = s_{I_t} + js_{Q_t}$ , chosen from a  $\pi/8$ -rotated 64-QAM constellation  $\tilde{\chi}$ . With IQ interleaver, the complex transmitted signal is given by

$$\tilde{s}_t = \begin{cases} s_{I_t} + js_{Q_{t-1}} & t \in [2, t] \\ s_{I_t} + js_{Q_t} & t = 1 \end{cases} \quad (6)$$

Thus, the received channel symbols  $\tilde{r}_t$  can be written as

$$\tilde{r}_t = \rho_t \tilde{s}_t + n_t \quad (7)$$

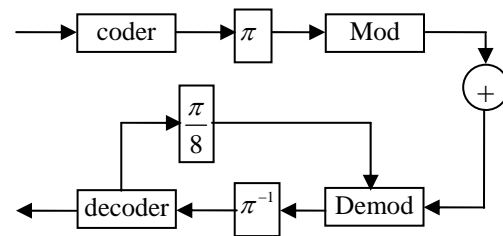


Figure 5. Structure of the proposed BITCM.

In order to optimize the error performance of the receiver, demapping and turbo decoding are performed according to an iterative algorithm. This receiver is made up of three (SISO) modules, connected through interleavers/de-interleavers, that exchange information in order to improve the reliability of the LLRs flowing through them. A particular SISO module process LLR sequence, called *a priori* information, produced by the other modules in order to generate updated versions of those input sequences, called *a posteriori* information. An *extrinsic* knowledge is then obtained by subtracting an *a priori* LLR from the

corresponding *a posteriori* LLR. This *extrinsic* information is then passed as an *a priori* knowledge to the other modules for further iterative steps. In our receiver, there are two iterative loops operating in parallel and allowing for information exchange between demapper and turbo decoder, and both constituent decoders inside turbo decoder. Based on sample  $r_t$  and the corresponding *a priori* LLRs  $\Lambda_{\mu^{-1}}^a(v_t^i)$ , generated by the turbo decoder, demapper  $\mu^{-1}$  calculates an *extrinsic* LLR  $\Lambda_{\mu^{-1}}^e(v_t^i)$  associated with bit  $v_t^i, i \in \{1, \dots, m\}$ , by using a calculation similar to that as follows

$$\Lambda_{\mu^{-1}}^e(v_t^i) = \ln \frac{\sum_{\tilde{s}_i \in \tilde{\chi}_1^i} P(r_t | \tilde{s}_i, \rho_t) \prod_{l=1, l \neq i}^m e^{\Lambda_{\mu^{-1}}^a(v_t^l) \tilde{s}_l}}{\sum_{\tilde{s}_i \in \tilde{\chi}_0^i} P(r_t | \tilde{s}_i, \rho_t) \prod_{l=1, l \neq i}^m e^{\Lambda_{\mu^{-1}}^a(v_t^l) \tilde{s}_l}} \quad (8)$$

where  $\tilde{\chi}_b^i$  denotes the subset of symbols  $\tilde{s}_i \in \tilde{\chi}$  whose labels have the binary value  $b \in \{0, 1\}$  at  $i$  th bit position, and  $\tilde{s}_i$  is the value of the label of symbol  $\tilde{s}$  in position  $l$ . The LLR  $\Lambda_{\mu^{-1}}^a(v_t^i)$  represents the extrinsic LLR generated by the turbo decoder at the previous iteration. In case of IQ interleaver, the probability density  $p(r_t | \tilde{s}_i, \rho_t)$  describing the channel model is given by

$$p(r_t | \tilde{s}_i, \rho_t) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{d_{r\tilde{s}}^2}{2\sigma^2}\right) \quad (9)$$

$$\text{, with } d_{r\tilde{s}}^2 = \|r_{I_t} - \rho_{I_t} \tilde{s}_{I_t}\|^2 + \|r_{Q_t} - \rho_{Q_t} \tilde{s}_{Q_t}\|^2 \quad (10)$$

Without IQ interleaver (10) simplifies to  $d_{r\tilde{s}}^2 = \|r_{I_t} - \rho_{I_t} \tilde{s}_t\|^2$ . After appropriately de-interleaving the LLRs  $\Lambda_{\mu^{-1}}^e(v)$  by  $\pi^{-1}$  to  $\Lambda_{dec}^a(c)$ . The LLRs  $\Lambda_{dec}^a(c)$  are fed into the turbo decoder (as *a priori* information), which computes extrinsic LLRs  $\Lambda_{dec}^e(c)$  on both parity and systematic bits. For the next iteration, the LLRs  $\Lambda_{dec}^e(c)$  are interleaved again by  $\pi^{-1}$  to LLRs  $\Lambda_{\mu^{-1}}^a(v)$  in order to be fed into the demodulator. The number of metrics to be computed at the demapper in the case of a classical BITCM with non-rotated  $2^m$ -QAM is  $2 \times 2^m$ . Each metric is computed over one dimension according to the in-phase I or quadrature Q component. This is due to the independence induced by the Gray mapping. Whereas, in the case of the proposed BITCM with a rotated  $2^m$ -QAM, the number of metrics to be computed at the demapper becomes  $2^m$ .

Owing to correlation between the I and Q components introduced by the rotation, each metric is now computed over two dimension corresponding to the I and Q components of the rotated constellation signal. The computational is multiplied by a factor of  $\sqrt{2^m}$  with respect to the classical demapper case. 4. Simulation results We now consider a 2 bit/s/Hz BITCM system using a rate-1/2 DVB-RCS turbo code and 64-QAM constellation. The iterative decoding at the receiver side is performed in 8 iterations (with a single local iteration of turbo decoding for each global iteration of BITCM), and the MAP algorithm is used for the decoding of the component decoders. Interleaver  $\pi/8$  is designed. Figure 6 illustrates the bit error rate (BER) performance versus signal-to-noise ratio  $E_b/N_0$  with and without iterative demapping, obtained with the classical 2 bit/s/Hz 64-QAM BITCM schemes using two different types of bit allocation, scheme A and Y

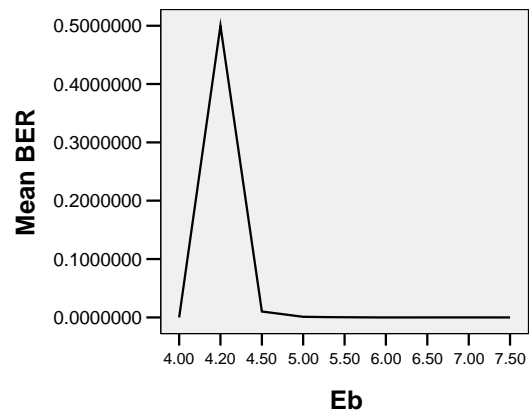


Figure 6. BER Performance achieved over Rayleigh channel with and without iterative demapping,

BITCM schemes operating on small (1504 bits) frames. All schemes employ a 1/2-rate DVB-RCS turbo code. Turbo decoding is performed in 8 iterations. From Figure 6, it is seen that, the iterative demapping can be only beneficial for BITCM systems operating at sufficiently low SNR values, i.e., at the asymptotic region. For instance, the scheme Y outperforms the scheme A, at a BER below to  $5 \times 10^{-7}$ . Figure 7 shows that, the proposed BITCM scheme results in significant improvement performance compared to all classical BITCM schemes with a better convergence threshold and a lower error floor. At a BER= $10^{-7}$ , our scheme outperforms the classical 64-QAM BITCM schemes by more than 0.5 dB. We believe that this significant performance gain is mainly due to the application of

the signal space diversity technique. However, we point out that the convergence threshold, above the iterative decoding becomes effective, shifts by more than 0.5 dB to a higher  $E_b/N_0$  for the proposed 64-QAM BITCM scheme without iterative demapping. Nevertheless, such scheme can still be of interest in some applications for which the asymptotic error-rate performance is the target.

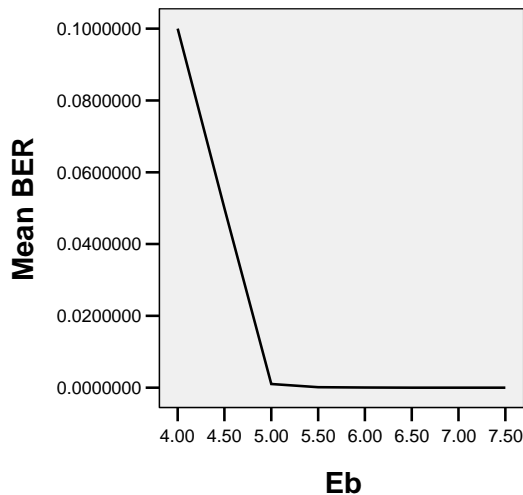


Figure 7. BER Performance comparison over Rayleigh channel between several 2 bit/s/Hz 16-QAM

## 5. Conclusion

In this paper, we investigated the gain which can be obtained by applying signal space diversity and implementing an iterative decoding algorithm at the receiver side, to the design of BITCM over Rayleigh wireless channels. We showed that the error performance of a 2 bit/s/Hz 64-QAM turbo coding scheme can be significantly improved, especially in what is known the asymptotic region, without loss in the convergence threshold. The proposed modifications lead only to a slight increase in the system complexity. Finally, note that the method introduced here can be applied in straightforward manner to BITCM systems employing high-order modulations in place of 64-QAM. We also think that further error performance improvement of BITCM schemes on wireless channel is possible by using constellation shaping and an iterative receiver. This will be the subject of future research.

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