An Investigation on Fuzzy Numbers

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Abstract: Ranking fuzzy numbers plays an important role in a fuzzy decision making process. However, fuzzy numbers may not be easily ordered into one sequence due to the overlap between fuzzy numbers. A new approach is introduced to detect the overlapped fuzzy numbers based on the concept of similarity measure incorporating the preference of the decision maker into the fuzzy ranking process. Numerical examples and comparisons with other method are straight forward and are practically capable of comparing similar fuzzy numbers. The proposed method is an absolute Ranking and no pair wise comparison of fuzzy numbers is necessary. Furthermore, through some examples discussed in this work, it is proved that the proposed method possesses several good characteristics as compared to the other comparable methods examined in this work.
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1. Introduction

Fuzzy set theory (Zadeh, 1965) has been extensively applied to solve decision– making problems in a fuzzy environment where the measurements of alternatives are imprecise in nature. The imprecise numerical measurements of alternatives are often represented by fuzzy numbers. Thus, comparing the alternatives is based on the comparison of their corresponding fuzzy numbers (Chen, 2001).

Fuzzy ranking is used to deal with the ordering of fuzzy numbers. Fuzzy numbers may be similar to each other in the problem; thus, the ranking process must be capable of distinguishing the similarity of fuzzy numbers, In addition, the need for comparing the similar fuzzy numbers is likely to grow when the problem size increases (Tseng, 1989). This reflects that efficiency should be of priority concern in the ranking process. In summary, the selection of a good fuzzy ranking method should satisfy the following criteria (Nojavan, 2006):

- Rationality of preference ordering– the consistency of ranking results with the decision marker's intuition
- Robustness- the ability to rank the fuzzy numbers with different shapes and using all information represented by the whole possibility distribution of fuzzy numbers
- Efficiency– the simplicity of computational process
- Fuzzy preference presentation– the ability to facilitate the representation of decision maker's viewpoint

Many fuzzy ranking techniques have been proposed in the literature. (Bortolan, 1985), (Chen, 1992), (Lee, 1988) thoroughly reviewed the existing methods and pointed out some illogical conditions embedded in these methods, such as producing counter-intuitive ranking orders, lack of discriminative ability, complex and considerable computational efforts. In recent studies, (Chen, 2001) used the left and right dominance to mark fuzzy numbers. (Chen, 2002) proposed a new method for ranking fuzzy numbers using a-cuts and signal/ noise ratio. (Deng, 2006) presented a modified area method to rank fuzzy numbers. Fuzzy ranking can be achieved by calculating the similarity between two fuzzy sets (Wang, 1997). Measure of similarity between two fuzzy numbers depends on the subjective preference from different weighting members in the fuzzy numbers (Wang, 1997). Preference reveals the view and interest of the decision maker about the ordering of the fuzzy numbers ad is always considered important to handle decision problems. With regard to the similar concept, (Lee, 1988) suggested that the fuzzy numbers with larger mean and smaller spread are ranked at higher position. Among the existing ranking methods, however, most of these measures are limited to incorporate the preference of the decision maker into the ranking process.

Thus, a new method will be proposed based on this concept. The remaining sections of this paper are organized as follows. The concept of using the preference to solve the fuzzy ranking problem will be described in the next section. In section 3, the proposed model will be developed and analyzed to compare a variety of fuzzy numbers. The proposed algorithm for ranking fuzzy numbers will be demonstrated in section 4. Then, the new algorithm will be verified by testing it through some previously reported examples. The last section is devoted to certain concluding observations.

2. Fuzzy Ranking Using Preference

Before the preference model is discussed, some basic concepts of fuzzy numbers are briefly reviewed. Le A_i be any one of n normal fuzzy numbers to be compared and is represented as $A_i = \{x, \mu_{Ai}(x), x \in \Re\}$, where $\Re$ is the universe of discourse and $\mu_{A_i}(x)$, $0 \le \mu_{A_i}(x) \le 1$, indicating the degree of membership of $_x$ in A_i, can be defined as:

$$\mu_{A_i}(x) = \begin{cases} \mu_{A_i}^L(x) & a \le x \le b \\ 1 & b \le x \le c \\ \mu_{A_i}^R(x) & c \le x \le d \\ 0 & \text{other wise} \end{cases}$$

where $\mu_{A_i}^L(x)$ is the left membership function that is an increasing function and $\mu_{A_i}^R(x)$ is real numbers? A normal trapezoidal fuzzy number is denoted by $A_i = (a, b, c, d)$, If $b = c$, then A_i is called a triangular fuzzy number.

Based on the relative position of fuzzy numbers on the real line, there exist two views, the indifference and the dominance, between fuzzy numbers *A* and *B* means the overlap area in which *A* and *B* intersect (i.e., fuzzy numbers *A* and *B* are indifferent to each other in the area); while the dominance means that if there exist one or more no overlap areas between fuzzy numbers, then for each no overlap area either *A* dominates *B* or *B* dominates *A*. As pointed out in (Tseng, 1989), *A* dominates *B*.

Therefore, the fuzzy ranking between fuzzy numbers in the nonoverlap case is very straightforward; the ordering of the fuzzy number in the right-hand side is preferred to the ordering of the fuzzy number in the left-hand side. However, it is more difficult to rank fuzzy numbers in the overlap case if there exist both dominance and indifference between *A* and *B*. The more Overlap areas between *A* and *B* (either *A* dominates *B* or *B* dominance *A*) the more difficult to compare fuzzy numbers. Figure 1 illustrates the dominance situation in the no overlap case and the situations of dominance and indifference in overlap case of three fuzzy numbers. In the Figure, fuzzy numbers *B* and *A* and fuzzy numbers *C* and *A* are the nonoverlap cases where both fuzzy numbers *B* and *C* dominate *A* and are on the right-hand side of *A*, thus, fuzzy numbers *B* and *C* dominate *A* and are on the right – hand side of *A*, thus, fuzzy numbers *B* and *C* are preferred to *A*. On the other hand, it is more difficult to compare fuzzy numbers *B* and *C* are since situations of dominance and indifference (the shaded area between B and C in the Figure) exist in this overlap case.

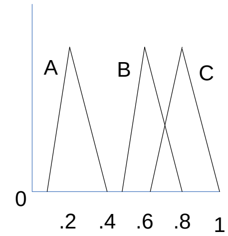

Figure 1. Dominance and indifference between fuzzy numbers

(Kang, 2006) incorporated the user preference to calculate the similarity between two fuzzy sets. The similarity measure $S(D,Q)$ Computes the degree of overlap between two fuzzy sets *D* (a document) and *Q* (a query) at each membership degree and is defined as: $S(D,Q) = \sum_{\mu} f(\mu : D,Q)p(\mu)$

Where $f(\mu : D,Q)$ represents the overlap function at a membership degree $\mu = [0,1]$ between fuzzy sets D and Q and is defined as:

$$f(\mu :\ D,Q) = \sum_{i=1}^{n} \delta(t_i, \mu : D,Q)$$

Where

$$\delta = (t_i, \mu : D,Q) = \begin{cases} 1 & \text{if } \mu_D(t_i), \mu_Q(t_i) \ge \mu \\ 0 & \text{otherwise} \end{cases}$$

Where t_i is a term in the index set I, $\mu_D(t_i)$ and $\mu_Q(t_i)$ represent a measure of degree to which D and Q are characterized by each index term t_i. The value of $\delta(t_i, \mu : D,Q)$ determines whether two fuzzy sets are overlapped at the membership degree μ for index term t_i. Here, $p(\mu)$ is a membership preference function. When the ranking results yield the same degree of similarity between two fuzzy numbers, the preference function is able to discern the two fuzzy numbers by focusing on the higher range of membership degree. The preference function is given a value of 1 if $\mu_D(t_i) \ge \mu_p$ and a value between 0 and 1 otherwise. The symbol μ_p is a preference threshold determined by the user to verify the degree of significance for the compared fuzzy numbers. The larger the value of $S(D,Q)$ the more the similarity for two fuzzy sets.

3. Proposed Method

A new algorithm for ranking fuzzy numbers will be introduced in this section. The algorithm is developed based on the concept of similarity measure incorporating the preference of the decision maker into the fuzzy ranking process.

To compare the similarity between fuzzy numbers, a fuzzy reference set is applied for this purpose in this paper. The fuzzy reference set is used here since it is found an efficient way in comparing fuzzy numbers in some approaches among the existing ranking methods (Chen, 1985), (Yager, 1980). It can be applied to compare fuzzy numbers in a straightforward manner and provides a common comparison base for the absolute position of each fuzzy number. The fuzzy maximum and fuzzy minimum, representing the fuzzy reference set, will be utilized to calculate the similarity between the fuzzy numbers. The fuzzy maximum and fuzzy minimum, representing the fuzzy reference set, will be utilized to calculate the similarity between the fuzzy numbers. The idea is that a fuzzy number is ranked firs if its similarity to the fuzzy maximum is large and its similarity to the number is ranked first if tits similarity to the fuzzy maximum is large and its similarity to the fuzzy minimum is small. If the condition is satisfied by some fuzzy numbers at the same time, a fuzzy number might be outranked the other fuzzy numbers depending on the preference of the DM.

For the proposed method, the fuzzy maximum ($M = (x, \mu_M(x), x \in \Re)$) and fuzzy minimum ($N = (x, \mu_N(x), x \in \Re)$) are given respectively by

$$\mu_M(x) = \begin{cases} x & 0 \le x \le 1 \\ 0 & other\ wise \end{cases}$$

$$\mu_N(x) = \begin{cases} 1-x & 0 \le x \le 1 \\ 0 & other\ wise \end{cases}$$

It is clear that the fuzzy maximum is the same as Yager's definition (Yager, 1980). The use of Yager's fuzzy maximum is because the absolute locations of fuzzy numbers can be incarnated automatically in the comparison process, resulting in comparable ranking values. The fuzzy minimum represents the set of small reference values with higher membership grades and is undesired by the decision maker. Based on the definitions of fuzzy maximum and fuzzy minimum, each fuzzy number is compared with the two fuzzy reference sets and two scores, the right score and the left score, are formed for the fuzzy number. Based on the similarity measure concept, the right score that compares the similarity of fuzzy number A to the fuzzy maximum is defined as $S_M = (A, M) = \sum_{\mu} f(\mu : A, M)\, p(\mu)$,

where

$$f(\mu : A, M) = \sum_{i=1}^{n} \delta(x_i, \mu : A, M), \text{ where}$$

$$\delta(x_i, \mu : A, M) = \begin{cases} 1 & \begin{cases} \min\limits_{\substack{x_i \in R \\ D(A \cap M, 0)}} (\mu_A(x_i), \mu_M(x_i)) \ge \mu \\ \mu_A(x_i), \mu_M(x_i) > 0 \end{cases} \\ 0 & other\ wise \end{cases}$$

$\min(\mu_A(x_i), \mu_M(x_i))$ represents intersection between A, M where $\mu_A(x_i) = \mu_M(x_i) > 0$. $D(A \cap M, 0)$ is the Hamming distance measure, representing the area where A and B are indifferent. (Tseng, 1989) pointed out that the Hamming distance is the best way to express the difference concept in the fuzzy ranking process. That claim is therefore utilized in the formulation of the fuzzy ranking process. That claim is therefore utilized in the formulation of the fuzzy ranking index.

The Hamming distance between fuzzy numbers *A* and *B* on the interval in the real line is defined by:

$$D(A, B \mid S) = \int_{u \in S} |\mu_A(u) - \mu_B(u)|\, du$$

Where $S = \Re$ $D(A, B \mid \Re) = D(A, B)$ that is $D(A, B)$ represents the non overlap area where A dominates B while $D(A \cap B, 0)$ means the area where A and B are indifferent.

The areas where A dominance B or B dominates A measure the Hamming distance between fuzzy numbers A and B on the interval in the real line. It is east to identify the interval of dominance. However, finding this interval can be difficult for no convex fuzzy numbers. In addition, the significance of comparing no normal fuzzy numbers is unclear (Bortolan, 1985). Therefore, consider only the normal fuzzy numbers in this study.

The value of $\delta(x_i, \mu : A, M)$ determines the overlap situation between fuzzy numbers A and M at μ for a given support x_i. The decision maker provides a mechanism to reflect the preference of decision maker. Like, the left score that measures the similarity of the fuzzy number A to the fuzzy minimum is determined by

$S_N(A, N) = \sum_{\mu} f(\mu : A, N)\rho(\mu)$ Where

$$\delta(x_i, \mu : A, N) = \begin{cases} 1 & \begin{cases} \min\limits_{\substack{x_i \in R \\ D(A \cap N, 0)}} (\mu_A(x_i), \mu_N(x_i)) \ge \mu \\ \mu_A(x_i), \mu_N(x_i) > 0 \end{cases} \\ 0 & other\ wise \end{cases}$$

The use of both SM and SN guarantee the full utilization of the information in the fuzzy number. The SM and SN indicate the indifference of each fuzzy number with respect to the fuzzy maximum and fuzzy minimum, respectively. A rational DM would prefer a larger S_M and a smaller S_N.

Finally, combine both scores obtained from the similarity measures between the fuzzy number and the fuzzy reference sets as a ranking index, representing the overall similarity measure of each fuzzy number, given by $O_A = \frac{S_N}{S_M}$.

That is, the smaller the overall similarity measure, implying the fuzzy reference sets. Based on the concept of overlap function, indifference between two fuzzy numbers can be measured by the number of intersections in the overlap area. Suppose $A = (a,\ b,\ c)$ is a normal triangular fuzzy number and fuzzy number *B* represents the fuzzy maximum M or fuzzy minimum N in the proposed model. To determine the areas of overlap on the real line for fuzzy min $(\mu_A(x), \mu_B(x)), \forall x \in \Re$, i.e., The number of times the fuzzy numbers intersect in the overlapped area, where $\mu_A(x) > 0$ *and* $\mu_B(x) > 0$. The possible cases include two points of intersection and one point of intersection.

Case I. Two points of intersection. When comparing the fuzzy numbers A and M A, two points of intersection can only occur if the support of A is included in the support of M, that is when $0 < a \le b < c \le 1$ *or* $0 < a < b \le c < 1$. Based on the overlap area between A and M, the overlap function can be found as follows:

$0 \le a < b \le c \le 1$ *or* $0 < a \le b < c \le 1$. On the other hand, when the fuzzy number A is compared with the fuzzy minimum N, two points of intersection may exist if the support of A is included in support of N, that is when $0 \le a < b \le c < 1$ or $0 < a \le b < c < 1$. Thus, the overlap function is given by

$$\delta(x_i, \mu : A, N) = \begin{cases} 1 & \text{if } \min(\mu_A(x_i), \mu_N(x_i)) \ge \mu, \mu_A(x_i) \\ & \qquad \mu_N(x_i) > 0 \\ 0 & \textit{otherwise} \end{cases}$$

Case II. One point of intersection

Let *x* be the point of intersection. One point of intersection is possible for the fuzzy number A and M if the support of A is contained in the fuzzy set M and when $0 \le a < n = v = 1, 0 = a \le b < c \le 1$ or $0 = a = b < c \le 1, 0 \le a < b \le c < 1$. Therefore,

$$\delta(x_i, \mu : A, M) = \begin{cases} 1 & \begin{cases} \text{if } \min(\mu_A(x_i), \mu_M(x_i)) \ge \mu, 0 \le a < b = c = 1 \\ 0 = a \le b < c = 1 \text{ or } 0 = a = b < c \le 1 \end{cases} \\ 0 & \text{otherwise} \end{cases}$$

For a fuzzy number *A*, the algorithm of the practical approach for obtaining the overall similarity index can be summarized as follows:

Step1. Calculate the intersection points in the area where *A* and *M* are indifferent.

Step 2. Calculate the overlap function $f(\mu)$ based on comparing the intersection points with the membership degree μ between fuzzy sets *A* and *M*.

Step 3. Determine the preference function $\rho(\mu)$.

Step 4. Calculate the similarity measure $S_M(A, M)$ between fuzzy sets *A* and *M*.

Step 5. Repeat Step 1 to Step 3 to calculate the similarity measure $S_N(A, N)$ between *A* and *N*.

Step6. Calculate the overall similarity index for fuzzy number A by $OA = {S_N}/{S_M}$.

4. A descriptive example

To describe the proposed method briefly, suppose there are five fuzzy numbers $\{A_1, A_2, ..., A_5\}$, to be raked, where $A_1 = (0.6, 0.7, 0.7, 0.8)$, $A_2 = (0.4, 0.5, 0.6, 0.7)$, $A_3 = (0.2, 0.5, 0.5, 0.8)$, $A_4 = (0.3, 0.4, 0.4, 0.9)$ and $A_5 = (0.1, 0.2, 0.2, 0.3)$ as shown in Figure 2. By intuition, A_1 is preferred to A_5 or $A_1 > A_5$ since they are nonoverlapped and $A_1$is on the right hand-side of A_5. Likewise, $A_2 > A_5$ and $A_4 > A_5$. However, it is more difficult to compare fuzzy numbers A_1, A_2, A_3 and A_4 since they are overlapped to each other. Even some of them are too close to be distinguished, the ordering of fuzzy numbers can be decided by the proposed algorithm by comparing the similarity between each fuzzy number and the fuzzy reference sets.

The first step shows that there exist two intersection points in the overlapped area between A_1 and M, which are 0.67 and 0.73, respectively. Suppose that the increment of membership degree used to calculate the overlap function $f(\mu)$ is 0.1. Given that the value 1 is assigned to the preference function if $f(\mu)_A(x) \ge 0.5$ otherwise. Then the right score for $A_1$is found by the following calculation:

$$S_M(A_1, M) = \sum_{\mu=0}^{1.0} f(\mu : A_1, M)p(\mu) =$$
$$f(0)p(0) + f(0.1)p(0.1) + ... + f(1.0)p(1.0) = 15$$

Likewise, two intersection points present in the overlapped area between A_1 is 3.5, which can be obtained by applying the same calculation procedures as those for the right score Thus, the overall similarity index for fuzzy number A_1 is determined from both scores as 0.233. Based on the proposed algorithm, it is clear that the final ranking is $A_1 > A_2 > A_4 > A_3 > A_5$, which is the same as in (Tseng, 1989). However, in the study of (Chen, 2001), the ranking order gives non-discrimination of two or three this case, a fuzzy number in this example, either $A_1 > A_2 > A_3 = A_4 > A_5$ or $A_1 > A_2 = A_3 = A_4 > A_5$. In this case, a fuzzy number may be preferable or equal to the other, depending on the preference of the decision maker.

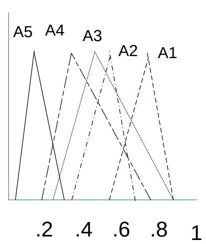

Figure 2. Fuzzy numbers for the descriptive example

When the decision maker increases the preference threshold from 0.5 to 0.7, the proposed algorithm indicates that the ordering for the five fuzzy numbers is changed to be $A_1 > A_4 > A_2 > A_3 > A_5$. Therefore, the proposed similarity measure can clarify the difference between fuzzy numbers I the ranking process in terms of the decision maker's preference. See Table 1.

5. Comparative examples

In this section, several typical examples are displayed to illustrate the validity of the proposed method. These examples are selected since the features they have can thoroughly test the ability of a ranking algorithm to differentiate between fuzzy numbers.

Example 1. Two triangular fuzzy numbers adapted from (Tseng, 1989) are ranked as shown in Figure 3, where the two competing fuzzy numbers share the right side and are different on the left side. This example challenges Jain's method, which only considers the partial information of fuzzy numbers being compared, especially those on the right- hand side. Since the lower values of the supports re ignored, this results in counter-intuitive

In addition, (Chen, 1985) noted that if some of the fuzzy numbers contain negative support values, then Jain's membership function becomes negative if the value of k is an odd integer. That contradicts the definition of membership function.

Although intuition would yield $B > A$, it is not surprising that no-discriminative results are seen using Jain's three indices. Bass and Kwakernaak's method gives a same result for this example while Yager's Hamming Distance method presents a counter-intuitive result. Chen's three indices give only one answer for this example.

Kerre's and the proposed methods give a fair ranking, in accord with human intuition.

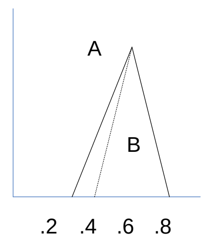

Figure 3. Fuzzy numbers for example 1

Example2. Cited from (Chen, 1992), two fuzzy numbers and a crisp number are ranked as shown in Figure 4. For Bass and Kwakernaak's method, fuzzy number A is equal to fuzzy number B. The ranking order is $A > B > C$ based on Yager's index, which would go against intuition. Note that the mixed comparison of fuzzy numbers and crisp, cannot obtain a consistent result using Jain's three indices. However, Chen's three indices give only one answer in this example l the consequence, $A < B < C$, obtained by Kerre's and the proposed methods, complies with human intuition as suggested by (Chen, 1992).

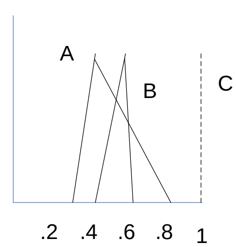

Figure 4. Fuzzy numbers for example 2

Example 3. Using the example from Chen (Chen, 1985), two triangular fuzzy numbers are illustrated in Figure 5. This example challenges Chen's method. A non-discriminative result is seen using Chen's method when $k = 1$. Chen's method ignored the absolute locations of the fuzzy numbers on the horizontal axis (Chen, 1992). As a result, Chen's method may not provide adequate discrimination ability for some fuzzy numbers that have the same relative position (Gonzalez, 1990). Since there contain situations f dominance and indifference in overlap between the fuzzy numbers, intuition in this example is not as obvious as in the previous examples. (Lee, 1988) pointed out that people would prefer a fuzzy number if it provides characteristics of showing higher mean value with smaller spread.

Based on the explanation from (Lee, 1988) pointed out that people would prefer a fuzzy number if it provides characteristics of showing higher mean value with smaller spread, fuzzy number A shows more satisfying characteristics than that of fuzzy number B in this example. However, this explanation is not seen from Bass and Kwakernaak's method and Yager's index. For Jain's method, as not all indices are identical, the DM needs to select which k to use in determining ranking order. Kerre's and the proposed methods favor fuzzy number a over B, complying with the result obtained by (Tseng, 1989), (Kolodziejczyk, 1986).

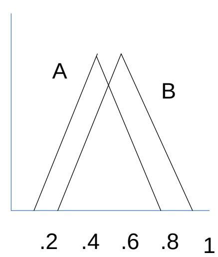

Figure 5. Fuzzy numbers for example 3

Example 4. Three triangular fuzzy numbers which have the same spread, cited from (Chen, 2002), are compared in this example as shown in Figure 6. A reasonable ranking $A<B<C$ is given by all methods, complying with the results suggested by (Deng, 2006).

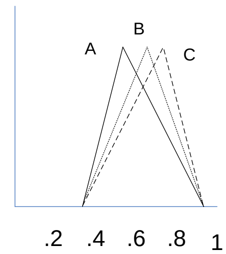

Figure 6. Fuzzy numbers for example 4

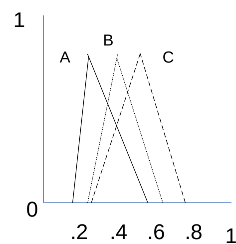

Figure 7. Fuzzy numbers for example 5

Example 5. Consider three triangular fuzzy numbers adapted from (Chen, 2002) are ranked as shown in Figure 7. All methods obtain the same ranking order $A < B < C$ in this example. This conclusion correlates with that in (Chen, 2002).

In general, the proposed ranking method possesses good characteristics and advantages from the results in all cases as compared to other existing methods. It shows clearly from the cases tested above that the proposed method enables one to explain them move effectively than would be possible with the other methods.

Table 1. The overall similarity measures with varying μ_i for $A = \{A_1, A_2, \cdots, A_5\}$ of Figure 1

Fuzzy Numbers	$\mu_p = 0.1$	$\mu_p = 0.3$	$\mu_p = 0.5$	$\mu_p = 0.7$	$\mu_p = 0.9$
A_1	0.467	0.367	0.233	0.304	0.467
A_2	0.833	0.833	0.842	0.833	0.833
A_3	1.00	1.00	1.00	1.00	1.00
A_4	0.818	0.727	0.889	0.818	0.818
A_5	3.40	6.80	6.80	6.80	3.40

6. Conclusion

Ranking fuzzy numbers has been realized as one of important topics in fuzzy set theory since it is a base of decision –making applications. However, fuzzy numbers may not be easily ordered in one sequence because their magnitudes involve uncertain values. To ensure a reliable decision outcome, a rational ordering method becomes necessary.

Accuracy and effectiveness in determining proper outcomes are also considered as important characteristics. When ranking a large quantity of fuzzy numbers with limited information about them, an efficient fuzzy ranking method is tremendously significant. The preference based algorithm is developed in this paper to deal with the ranking of fuzzy numbers. Then new algorithm measures the similarity between the competing fuzzy number and the fuzzy reference sets by allowing the decision marker to assign the preference to the index calculation. The fuzzy reference set is used in the new model to determine the absolute location of fuzzy numbers. In addition, the full information contained in fuzzy numbers is used in the ranking process to obtain the overall ranking index for each competing fuzzy number. Through some examples discussed in this work, it is proved that the proposed method possesses several good characteristics as compared to the other comparable methods examined in this work. The computational process of the proposed method is straight forward and is practically capable of comparing similar fuzzy numbers. Furthermore, the proposed method is an absolute ranking and no pair wise comparison of fuzzy numbers is necessary, saving the computicipational time. The new algorithm also provides flexibility allowing the participation of he decision maker.

In general, the proposed method is an effective practical aspect which is not seen in several other methods. It is transitive in giving a consistent conclusion in the comparison of more than more than two fuzzy numbers, robust in providing a mixed comparison of fuzzy numbers and crisp numbers, and simple in the computational process. These features of the proposed method can be a valuable tool for comparing fuzzy numbers and used in many applications same as fuzzy control.

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