Precipitation of Suspended Particles on Tube Walls

Bedier B. EL-Naggar

Department of Engineering Mathematics and Physics, Faculty of Engineering, Cairo Universty Giza, Egypt
bbnaggar@hotmail.com

Abstract: In this article, the steady state convective diffusion equation for the suspended particles in a suspension is solved for tube flow. Linear concentration drop and uniform axial velocity are assumed. An experiment is designed to measure the concentration at exit and the rate of precipitation on a wall is also measured experimentally after a sufficient time of flow. Accordingly, the diffusion constant is determined and the resulting of area of contraction due to this precipitation is calculated and hence the complete blocking time. This model is suggested for fat precipitation on walls of blood vessels in vivo and the precipitation of sault on walls of water tubes in boilers.

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Introduction

Theoretical and experimental investigations are required for the problem of precipitation of suspended particles on the inner walls of tubes carrying rich suspensions of fine particles. Here, we list two important applications of this investigation. The first is the precipitation of fat on the tubes of blood vessels in living humans. The second is the precipitation of saults on the walls of water tubes in boilers. Theoretical derivations are based on steady state conditions, which imply constant values of different parameters in the required experiments, which necessarily extend for long times since the precipitation process is rather slow. Two experiments are needed in this case, determining the diffusion constant and the measurement of the precipitation rate.

As we accept molecular diffusion as the mechanism behind these two phenomena as explained in the Appendix, we look for theoretical estimates. Aside from limitations on viscosity and temperature, the only listed estimate of the diffusion constant is due to Einestien [1] in which he defined the diffusion constant D as the ratio the thermal activation of particles to its mobility in the medium, i. e.

$$D = \frac{KT}{6\pi\mu a} \qquad (1)$$

Where, $K = 1.3807 \times 10^{-23}\, J\, ^{\circ}K^{-1}$ is the Boltzman constant in Joules per degree Kelvin,

T is the absolute temperature in degrees Kelvin,

μ is the viscosity of the suspending of the suspending medium in poise, a is the diameter of the particles in meters, KT is the thermal activation and $6\pi\mu a$ is the mobility of spherical particles and is due to Stoke's [2].

Literature review:

As far as could be surveyed in literature, only the study of Wang et al. [3]for the computation fluid dynamics approach to the effect of mixing and draft tube on the precipitation Barium sulfate in a continues stirred tanks could be cited as an analytical study. Next we mention the paper by Chen et al. [4] on the interaction of macro and micro mixing on particle size distribution in reactive precipitation. Manth et al. [5] made experimental investigation of precipitation reactions under homogeneous mixing conditions. At last, we refer to the work of Pohorecki et al. [6] on the use of new model of micro mixing for determination of crystal size in precipitation. Specifically speaking precipitation in narrow tubes could not be cited in literature.

Formulation of the problem

Now we proceed to design the experiment for determining the diffusion constant and the time of complete blocking of the tube carrying a rich suspension.

Consider a long narrow tube of radius r_0 before precipitation and $r < r_0$ after precipitation at time t. The length of the tube is L and the concentration at inlet is c_0 while due to precipitation this concentration will be $c < c_0$, the uniform steady velocity in the tube is u and the yet unknown diffusion constant is D.

Since the tube is long relative to its inner radius, we can assume linear concentration gradient with the tube length as

$$\frac{\partial c}{\partial z}=\frac{c_1-c_0}{L}<0 \qquad (c_1-c_0)<0 \quad (2)$$

The equation describing the variation of concentration inside a tube is given by [7]

$$\frac{\partial c}{\partial t}= D\nabla^2 c + \nabla\cdot\left(c\underline{u}\right) \quad (3)$$

This is the general equation for convective diffusion. For our case three simplifications are valid:

1. Steady state distribution is time independent , $\frac{\partial c}{\partial t}=0$
2. Velocity is uniform inside the tube u along z , $\nabla\cdot\left(c\underline{u}\right)=u\frac{\partial c}{\partial z}$ since velocity is also axial.
3. Linear concentration drop with z , $\nabla^2 c=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c}{\partial r}\right)$ and in this expression $\frac{\partial^2 c}{\partial z^2}$ vanished for linear drop with z .

The reduced equation for the concentration distribution is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c}{\partial r}\right)=\frac{u}{D}\frac{c_0-c_1}{L}=\alpha^2>0 \quad (4)$$

The first integration of this equation yields $\frac{\partial c}{\partial r}=\alpha^2\frac{r}{2}+\frac{k_1}{r}$, k_1 must be set zero for bounded $\frac{\partial c}{\partial r}$ at $r=0$, the second integration gives $c(r)=\alpha^2\frac{r^2}{4}$ for $c=0$ at $r=0$, which is almost the case. The r average of c is given by:

$$\bar{c}=\frac{\alpha^2}{4}\frac{1}{\pi a^2}\int_0^a 2\pi r^3\,dr=\frac{a^2\alpha^2}{8}=c_0.$$

Since we considered the area a , which is the case at entry before any precipitation. Accordingly

$$c_0-c_1=\frac{8c_0 DL}{a^2 u} \quad (5)$$

Which, is the value of the drop in concentration; the product $a^2u\left(c_0-c_1\right)=8c_0DL$ is equal to the precipitation rate and can be measured experimentally measuring the two concentrations c_0 and c_1 at the entry and the exit of the tube and if divided by $8\,c_0\,L$, the coefficient diffusion D can be estimated as

$$D=\frac{a^2u}{8L}\left(1-\frac{c_1}{c_0}\right) \quad (6)$$

We proceed now to calculate the variation in radius as a drop due to precipitation as $8\pi c_0 DL=-L\pi\frac{dr^2}{dt}$, which is integrated $r^2=r_0^2-8\,c_0\,D\,t$; t is the observation time. Complete blocking occurs when $r\rightarrow 0$ at t^* as $t^*=\frac{r_0^2}{8Dc_0}$

The results in Equations (6) and (7) are very useful for the applications.

Comment

In this paper, we succeeded in designing two experiments to measure the precipitation rate and the diffusion constant. Measurements must be made after a sufficient time to ensure the steady state. Initially rich suspensions will help reduce the observation time also long narrow tubes.

The precipitation rate can be obtained by measuring the tube weight before and after the precipitation. Division by the specific weight is necessary if the volume concentration is considered.

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References

1. Gardon, M. B., “Physical Chemistry” 4th ed. McGraw Hill, New York, (1988), pp. 748
2. Weinstein, N. “Handbook of Physics “, Mir Pub., Moscow, (1975), pp. 643.
3. Wang, Z., Mao, Z., Yang, C. and Shen, X. “ Computational Fluid Dynamics approach to the Effect of Mixing and Draft Tube of the Precipitation of Barium sulfate in a continuous

Stirred Tank", Chinese J. Chem. Eng., Vol. 14,No. 6, pp. 713-722, (2006)
4. Chen, J. F., Zheng, C. and Chen, G. T., " Interaction of Macro and Micro- Mixing on Particle Size Distribution In Reactive precipitation", Chem. Eng. Sci., Vol. 51, No. 10, pp. 1957-1966,(1996).
5. Manth, T., Mignon, D. and Ottermam, H. " Experimental investigations of precipitation reactions under homogeneous mixing conditions "Chem. Eng. Sci., Vol. 51, No. 11, pp. 2571-2576,(1996).
6. Pohorecki, R. and Baldyga, J.," The Use of a New model of Micro-Mixing for Determination of Crystal size in Precipitation" Chem. Eng. Sci., Vol. 38, No. 1, pp. 79-83,(1985).
7. Arrora, C. P., " Heat and Mass Transfer", Khanna Pub., Delhi, (1989)
8. Fox and McDonald,(1985) " Introduction to Fluid Mechanics", 3rd Edition, John Wiley and Sons, New York.

Appendix

Analytical verification of Fick's law:

If the relation $\dot{q} \propto -\underline{\nabla} c$ between the flux vector $\dot{q}$ over a unit area and the concentration gradient $\underline{\nabla} c$ exists, The constant D (the diffusion constant) in Fick' s law $\dot{q} = -D \ \underline{\nabla} c$, D is physical. For this purpose, the following model is made.

Consider a cylindrical vessel of unit area and unit height; initially filled with uniform suspension of concentration $\frac{1}{2}$. In the absence of any external force field acting on particle (Buoyancy neutralizes gravity) [8], the conditions on the bottom $c\big|_{z=0} = 1$ and $\left.\frac{\partial c}{\partial z}\right|_{z=0} = 0$ will give the solution of $\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial z^2}$, with $c(0,z) = \frac{1}{2}$ as [7] : $c(t,z) = 1 - \frac{1}{2} erf(\frac{z}{2\sqrt{Dt}})$ by similarity, now let us consider the level $z = \frac{1}{2}$ for all times, we have $c(t,\frac{1}{2}) = 1 - \frac{1}{2} erf(\frac{1}{4\sqrt{Dt}})$, this implies

$$2\,[1 - c(t,\frac{1}{2})] = \frac{2}{\sqrt{\pi}} \int_0^{\frac{1}{4\sqrt{Dt}}} e^{-u^2}\, du.$$

Differentiate both sides with respect to. t then

$$8\sqrt{\pi D}\, t^{3/2} \frac{d\,C(t,1/2)}{dt} = e^{-1/(16Dt)}$$

Both sides are positive and as $t \to \infty$, the $L.H.S. = 1 = \infty \times 0 = R.H.S.$ as $C(t,1/2)$ is constant and therefore D is physical and we attempt an evaluation for it.

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