

## Transmission Fixed Cost Allocation in Deregulated Environment based on Cooperative Game Theory

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**Abstract:** The cooperative game theory is proposed to the transmission fixed cost allocation incurred to accommodate all the players. This method dominates the difficulties of conventionally used methods, such as postage stamp method and MW miles method, and encouraging the economically optimal usage of the transmission facilities. Under the deregulated environment, the cost needs to be allocated to the loads as well as generators fairly and unbiased so as to provide a locational signal to both types of players for optimal setting. This paper proposes game theoretic models based on the Shapley value approaches for transmission cost allocation problems under the deregulated environment. The obtained results are compared with the conventionally adopted methodologies to defend easy implementation and effectiveness of the proposed methodologies.

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### 1. Introduction

The term fixed costs, generally, embraces the capital invested to build the network as well as the network maintenance costs. In a monopoly market, the utility covers those costs through the tariff policy. In the deregulated electricity markets, the network operation is the responsibility of the Independent System Operator (ISO). However, the company which is the network owner must still be compensated for those fixed costs. Hence, the ISO has to charge the market participants so as to collect the necessary amount.

In the deregulated power markets, the issue of charging the participants, regarding the fixed costs, is of great significance. The reason is that the fixed costs make up the largest part of transmission charges. Hence, it is easy to explain the demand for a fair and effective allocation of those costs to the market participants.

The problem therefore becomes how to allocate these costs among the players, who pays what portion of the overall cost [1].

The need to charge all players on an unbiased basis for transmission services made it an open research issue. Conclusion of the transmission cost should be simple and transparent. It is difficult to attain an efficient transmission pricing scheme that could fit all market structures in different countries. The continuous research on transmission pricing indicates that there is no generalized agreement on pricing methodology. In practice, each deregulation market has chosen a method that is based on the particular characteristic of its network. Measuring

whether or not a certain transmission pricing scheme is technically and economically adequate would require additional standards.

Various methods for allocation of transmission cost have been reported in the literature. The most common and simplest approach is the postage stamp method, which depends only on the amount of power moved and the duration of its use, irrespective of the supply and delivery points, distance of transmission usage. Contract path method proposed for minimizing transmission charges does not reflect the actual flows through the transmission grid [2- 3].

As another, MW Mile method was introduced in which different users charged in proportion to their utilization of the grid [4]. The main key in MW Mile method is to find the contribution or share of each generator and each demand in each of the line flows.

Various methods reported for finding the share and contribution of generators and demands is flow based. J. Bialek has proposed a tracing method based on topological approach resulting in positive generation and load distribution factors [5]. D. Kirschen et al proposed a method to find the contributions of generators and loads by forming an acyclic state graph of the system making use of the concepts of domains, commons and links [6].

A. J. Conejo et al proposed a method to find the share of participants to transmission cost allocation by forming  $Z_{bus}$  that makes generator-load use the lines electrically close to it. The

$Z_{bus}$  present's numerical behavior model based on circuit theory and relates the nodal currents to line power flows [7].

Other methods that use generation shift distribution factors are dependent on the selection of the slack bus and lead to eristic results [8-11].

The usage-based method reported in [12] uses the so-called equivalent bilateral exchanges (EBEs). To build the EBEs, each demand is proportionally assigned a fraction of each generation, and conversely, each generation is proportionally assigned a fraction of each demand, in such a way as both Kirchhoff's laws are satisfied.

This paper presents a new method based on Game Theory for transmission cost allocation. Game theory is the study of multi player decision problems. In these problems there is conflict of interests between players. The term game corresponds to the theoretical models that describe such conflicts of interests.

## 2. Cooperative Game Theory Concepts

Several methods have been proposed aiming at a proper allocation of fixed costs. These methods are well established from an engineering point of view but some of them may fail to send the right economical signals. The allocation of the fixed costs is a typical case where the cooperation between some agents produces economies of scale. Consequently, the resulting benefits have to be shared among the participating agents. The cooperative game theory concepts, taking into account the economies of scale, suggest reasonable allocations that may be economically efficient. The analysis in this paper will illustrate the use of game theory in the fixed cost allocation.

Let  $N = \{1, 2, 3, \dots, n\}$  define the set of all the players in the game. A coalition  $S$  is defined as a subset of  $N$  that  $S \subset N$ . The null set is called the empty coalition and the set  $N$  is called the grand coalition. The game on  $N$  is a real valued function  $v: 2^N \rightarrow \mathbb{R}$  that assigns a worth to each coalition and satisfies  $v(\emptyset) = 0$ . The characteristic value  $v(S)$  gives the maximum gain the coalition  $S$  can guarantee itself by coordination or cooperation between its members, irrespective of what other players and coalitions do [11].

The application of cooperative game theory is to suggest an optimal or a fair allocation of the cost among its different players.

The cost allocation is represented in terms of a pay off vector denoted as  $\{\varphi_1, \varphi_2, \varphi_3, \dots, \varphi_n\}$  such that  $\sum_{i=1}^n \varphi_i = v(N)$ . If the allocation needs to be

optimal and fair for all the players, three conditions, as given below, namely, individual, group and global rationalities need to be satisfied.

$$\varphi(i) \leq v(i) \quad i \in N \quad (1)$$

$$\varphi(S) \leq v(S) \quad S \subset N \quad (2)$$

$$\varphi(N) = v(N) \quad (3)$$

Any pay off vector satisfying the individual and global rationalities is called an imputation. There are numerous methods for allocation of costs among the players of a cooperative game. This paper is widely based on one Cooperative Game methods, namely Shapley Value (SV) for obtaining a particular solution.

### 2.1 Shapley Value Approach

The Shapley Value is calculated as follows. Let  $v$  be the characteristic function and  $i$  be any player in the game. The cost of serving none is assumed to be zero, that is,  $v(0) = 0$ .

The variable  $S$  represents the number of players in the coalition containing  $i$ , and  $n$  is the total number of players in the game. Therefore, the allocation  $\varphi_i$  to player  $i$  by the Shapley Value is determined by:

$$\varphi_i(v) = \sum_{S \subseteq N-i} \frac{|S|(n-|S|-1)!}{(n-1)!} [v(S \cup i) - v(S)] \quad (4)$$

where

$S$  is the coalition excluding  $i$

$(S \cup i)$  is the coalition obtained by including  $i$

$|S|$  is the number of entities in coalition  $S$

$n/N$  is the total number of players

$v(S)$  is the characteristic value associated with coalition  $S$ .

In Equation (4), the first part of the expression gives the probability of a particular player joining that coalition and the second part gives the contribution that any particular player makes to the coalition by his joining.

The characteristic function  $v(S)$  of the proposed cooperative game is calculated as follows:

$$v(S) = \sum_{l \in S} P_l \times C_l \quad (5)$$

in which,  $v(S)$  is the fixed cost of providing transmission service to coalition  $S$ .  $P_l$  is the active power flowing through the line  $l$  that can be obtained from the Optimal Power Flow which it is explained in the next section,  $C_l$  is the transmission cost of active power through line  $l$ .

### 3. Optimal Power Flow

Optimal power flow (OPF) is performed on the case studies to obtain the different line flows passing for various possible coalitions between the generators and loads. OPF is performed supposing

peak load on all load buses, as peak loads justify the design of any transmission network. In all possible combinations, at least one generator and one load have always been taken to represent realistic coalitions.

In this paper, the problem has been formulated so as to transmission fixed cost allocation over the set of generators and loads using game theory. Both the generators as well as the loads are supposed to be using the transmission system, and so the cost is allocated between both types of players. This provides a locational signal to players to set at optimized locations. The loads are obliged to set at power surplus centers and generators at load centers.

This optimizes the overall cost of supplying power for a given set of loads. The game theory approach of the Shapley value is used to solve and obtain the cost allocation. The Shapley value was calculated using TuGames Package, an extension of Cooperative Games, a Mathematica Package [13].

The percentage cost allocation for each individual line is calculated and used with the line lengths to obtain allocation of the complete system cost between the different players.

**4. Test Case**

To determine the allocation for players, the methods have been tested on three conditions, cases A, B modeling pool market and C modeling the bilateral transaction. Note that the cost of each line is considered to be proportional to its series reactance. Thus,

$$C_l = 1000 \times X_l (\$/h) \tag{6}$$

**A. First Case 5 Bus Power System**

Figure 1 depicts a 5 bus test system, which is modeling the pool market that composed of three loads and two generators. The seven lines in the system have the same values of series resistance and reactance: 0.02 and 0.10 [pu] respectively. With considering the cost of each line  $C_l = 1000 \times X_l (\$/h)$ , total transmission cost is equal 700 \$/h. The generators and load data is given in the Tables 1 and 2. It is supposed that two generators, G1 and G2, seller their production power to three loads in an open access transmission environment that a Transco responsible for providing the required transmission service and allocating the cost incurred to the participants involved in the service.

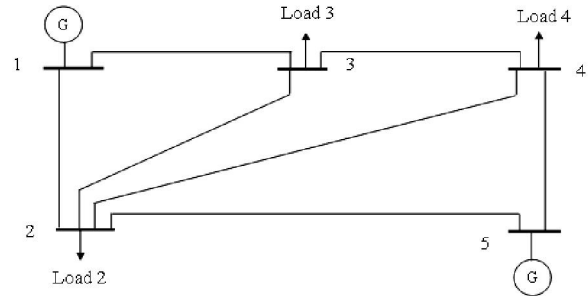


Figure 1. The single-line diagram of the 5 bus system

Table 1. Load data

Bus	P (MW)	Q (MW)
2	30	10
3	40	20
4	40	20

Table 2. Generators data

Coefficients of polynomial cost function ( $C_2P^2 + C_1P + C_0$ )				
Gen	Max Power (MW)	$C_0$	$C_1$	$C_2$
G1	250	150	5	0.09
G5	250	300	5	0.15

Let  $N = \{1, 2, 3\}$  represent the set of players in the game, which elements 1, 2 and 3 represent load 2, load 3 and load 4 respectively. Then  $S = (\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\})$  denote all possible coalition between these three players. The optimal power flow is then simulated to determine the power flow ( $P_l$ ) through the network while taking the physical constraints into account. When there is no cooperation, that is the transmission network is used exclusively by each player, the value of the characteristic function in eqn 4 mentioned above for specific coalition  $\{1\}, \{2\}$  and  $\{3\}$  is as follows:

$$v(\{1\}) = 4383$$

$$v(\{2\}) = 6999$$

$$v(\{3\}) = 7283$$

However, if more than one player agrees to use the transmission network simultaneously, the power flow through some lines would drop due to the possible counter flow which relieves the congestion. In this condition, the characteristic function and its value for coalition  $\{1, 2\}, \{1, 3\}, \{2, 3\}$  should be as follows:

$$v(\{1, 2\}) = 9643$$

$$v(\{1, 3\}) = 10622$$

$$v(\{2, 3\}) = 12102$$

Further more the cost function of the grand coalition  $\{1, 2, 3\}$  would be as follows:

$$v(\{1, 2, 3\}) = 14370$$

It is obvious that the total characteristic function in cooperation is much less than when the network is employed monopoly by each load. Now the problem is how to distribute the total value according to each player's incremental effect to the coalition. Let  $\phi_i$  denote the value allocated to player  $i$  by the shapley value. Thus  $\phi_1$  was calculated as:

$$\phi_1 = \frac{0! \times 2!}{3!} [v(\{1\}) - v(\{1\} - \{1\})] +$$

$$\frac{1! \times 1!}{3!} [v(\{1, 2\}) - v(\{1, 2\} - \{1\})] +$$

$$\frac{1! \times 1!}{3!} [v(\{1, 3\}) - v(\{1, 3\} - \{1\})] +$$

$$\frac{2! \times 0!}{3!} [v(\{1, 2, 3\}) - v(\{1, 2, 3\} - \{1\})] = 3214.2$$

Similarly, the value allocated to player 2 and 3 is calculated as:

$$\phi_2 = 5262.2$$

$$\phi_3 = 5893.6$$

It could be observed that the value allocation using the Shapley value met the rationality conditions.

Coalition rationality, which requires that no player would be allocated a value that is greater than it would value to that player alone.

$$\phi_1 = 3214.2 \leq v(\{1\}) = 4383$$

$$\phi_2 = 5262.2 \leq v(\{2\}) = 6999$$

$$\phi_3 = 5893.6 \leq v(\{3\}) = 7283$$

Table 3. Transmission cost allocation (\$/h)

Player	L2	L3	L4	Total
Shapley value	156.57	256.33	287.10	700
MW- Mile	164.36	262.5	273.14	700
Postage stamp	190.90	254.55	254.55	700

It is assumed that customers in the market paid the total transmission cost. Postage stamp method does not consider the physical location and share the total cost among the players versus active

power. In MW-Mile method to determine the cost allocation, the network operator runs a power flow program. Therefore calculates the power flow over each system line and share the cost among the player with linear equation, but equations of power flow is non linear, thus result is approximately.

Despite these facts, postage stamp and MW- Mile methods are widely implemented because of its simplicity. From the results of Table 3 can be seen, the total cost of transmission system in three methods is equal but in the shapley value method cost allocation between the players is fairly than the postage stamp and MW- Mile method.

### B. Second Case 9 Bus IEEE System

The IEEE 9 bus test system is analyzed to illustrate the proposed technique. The system contains 3 generating units and 3 load points that shown in Figure 2 The system configuration data can be found in [13].

With considering the cost of each line  $C_l = 1000 \times X_l$  (\$/h), total transmission cost is equal 859.5 \$/h. The flow of each transmission line from optimal power flow solution can be calculated that is performed using MATPOWER software [14].

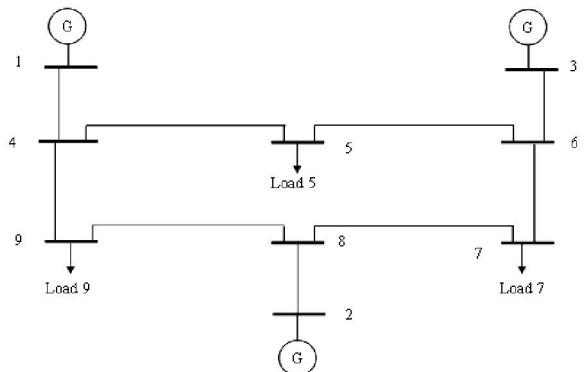


Figure 2. Single-line diagram of the IEEE 9 bus system

Let generators 1, 2, 3 and loads 5, 7, 9 be the players 1, 2, 3, 4, 5 and 6 respectively in the cooperative game  $N = \{1, 2, 3, 4, 5, 6\}$ . Thus there are 63 types of coalition in total. Similar to the calculation process of the first case, we calculate the value of the characteristic function in the case of each coalition. Then, the following results are obtained by using the Shapley Value:

$$\phi_1 = 303282\$, \phi_2 = 490392\$, \phi_3 = 2563.26\$$$

$$\phi_4 = 2907.66\$, \phi_5 = 3155.67\$, \phi_6 = 4436.67\$$$

It is assumed that generators and customers in the market evenly share the total transmission cost. The results obtained are compared with the traditional

methods adopted to allocate the cost allocation. These are reflected in Table 4 for the 9 bus system.

Table 4. Comparative cost allocations for different methods (\$/h)

Player	$G_3$	$G_2$	$G_1$
Postage stamp	101.21	209.17	119.35
MW- mile	102.73	198.04	128.97
Shapley value	104.91	200.71	124.13
Player	$D_9$	$D_7$	$D_5$
Postage stamp	170.53	136.43	122.81
MW- mile	185.19	132.87	111.7
Shapley value	181.58	129.16	119.01

As can be seen from the results, it is not only the generation or load quantity that decides the cost allocation, but it is also affected by the location of the corresponding player and cost of each line. Thus, this method is capable of providing proper locational signals for the players to locate. As postulated in game theory, it can be proved that no player is paying more than the cost it would have to pay if the system was designed for his individual use.

Also, the contribution from any possible combination is less than the sum of individual contributions. Thus, all players are incentives to stay in the coalition.

### C. Third Case Bilateral Transactions

When the electricity market operates in a deregulated environment then each player is responsible to pay a part of the transmission cost. Similarly to the case of pool market, the form of a coalition between some players can be profitable by the existence of counter flows. Considering the IEEE 9 bus network that shown in Figure 2 and assuming the transactions of Table 5.

Table 5. Transaction Contracts

Transaction	Seller	Buyer	Power (MW)
T1	G1	L5	30
T2	G2	L7	30
T3	G3	L9	45

In the beginning the cost allocation of the entire system is investigated by means of an optimal power flow program. In Table 6 the network usage ( $f_S$ ), as well as the characteristic function value for each coalition, regarding the transaction, are presented.

Table 6. Use and characteristic function value for Coalitions

Coalition	$f_S$ (MW)	$v(S)$ (MW)
T1	64.45	0
T2	72.38	0
T3	135.34	0
T1, T2	125.58	11.25
T1, T3	176.82	22.97
T2, T3	188.93	18.79
T1, T2, T3	234.73	37.44

The savings achieved by the grand coalition {T1, T2, T3} is 37.44 MW that should be shared by the three players. A Shapley value approach to this task is given in Table 7 together with the initial ( $f_i$ ) and final ( $f_i'$ ) network usage for each player. An investigation of the game played at each single system branch may follow. Now,  $f_i$  is the optimal power flow over a system branch caused by player  $i$ . In this case it is possible that some coalitions will have negative values at some branches. Actually, a cooperation between a set of transactions results in a superposition of the single transaction patterns. Thus, in the worst case at a branch for a coalition  $S$  it should be  $v(S) = 0$ . This would happen if no counter flows exist. The explanation for the negative  $v(S)$  is that, since an optimal power flow is used, a generator located at the reference bus must cover the losses. However, when the electricity market is organized according to a bilateral transaction model the cost allocation game can be played at each single system branch.

Table 7. Initial and final use for the Transactions

Transaction	$f_i$ [MW]	$\varphi$ [MW]	$f_i' = f_i - \varphi$ [MW]
T1	64.45	11.40	53.05
T2	72.38	10.34	62.04
T3	135.34	15.7	119.64

It is assumed that the contracts in the market paid the transmission cost. The results obtained for transmission cost allocation in different contracts reflected in Table 8.

Table 8. Transmission cost allocations for bilateral transaction (\$/h)

Transaction	T1	T2	T3	Total
Shapley Value	194.25	227.17	438.08	859.5



## 5. Conclusion

The Shapley Value of cooperative game theory is proposed to allocate the transmission fixed cost incurred by the ISO to accommodate all the players while taking physical constraints into account. It offers an alternative solution method based on game theory that can realistically stimulate the practical situation, where the players join together to form a coalition. This method overcomes the difficulty of the conventionally used postage stamp method or MW Mile method by taking the incremental contribution of each player into account, thus encouraging the economically optimal usage of the transmission facilities.

The proposed method with considering active power passing the transmission system provides a stable and unbiased solution to the complex problem of fixed cost allocation in both pool market and the bilateral transaction structure. Thus, it can be seen that in a deregulated environment, where the fixed costs are to be allocated between the players, the game theoretic approaches can be applied in a justified way. These incentives the players to join the coalition at a proper setting to optimize the network cost.

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