Dynamic Optimization of LQ Objective Loss Function: Application in Economic Planning

[1] Amir Mansour Tehranchian, [2] Rezvaneh Poorhabib, [3] Nayere Karegar and [4] Masoud Behravesh

[1] Assistant Professor of Economics, Faculty of Economics, Mazandaran University, Babolsar, Iran. E-mail: a.tehranchian@umz.ac.ir
[2] M.Sc Student of Economics, Faculty of Economics and Administrative Sciences, Institute of Higher Education Allameh Mohaddes Noori, Noor City, Mazandaran, Iran-mail: rg_poorhabib@yahoo.com.
[3] M.Sc Student of Economics, Faculty of Economics and Administrative Sciences, Institute of Higher Education Allameh Mohaddes Noori, Noor City, Mazandaran, Iran. E-mail: nayere.karegar@yahoo.com.
[4] (Corresponding Author), Economics Researcher, Department of Economics and Management, Bonab Branch, Islamic Azad University, Bonab, Iran. Address: No.370, Alemi Alley, Imam St, Shabestar City, East Azerbaijan, Iran. P.O Box: 53817465833. Cell Phone: +989192227434. E-Mail: behravesh@bonabiau.ac.ir, . behrawesh@gmail.com

Abstract
In this paper, the stochastic dynamic optimization is used to calculate optimal values of liquidity and government budgetary expenditures for the Iran's forth five year development plan (2005-2009). For this purpose, we minimized quadratic variations of inflation rate and the rate of economic growth from their plan target subject to a nonlinear dynamic system. The results show that, the optimal values of the above control variables are greater than those proposed in forth development plan whereas the optimal values are less than the occurred quantities. Based on obtained results, using the optimal macroeconomic policies will improve the rate of economic growth and inflation rate in comparison with their occurred values. [Amir Mansour Tehranchian, Rezvaneh Poorhabib, Nayere Karegar and Masoud Behravesh. Dynamic Optimization of LQ Objective Loss Function: Application in Economic Planning. [Amir Mansour Tehranchian, Rezvaneh Poorhabib, Nayere Karegar and Masoud Behravesh. **Dynamic Optimization of LQ Objective Loss Function: Application in Economic Planning.** Journal of American Science 2011;7(5):656-660]. (ISSN: 1545-1003). http://www.americanscience.org.

Keywords: Macroeconomic Policies, Optimal Control Theory, Nonlinear Macro econometric system.
JEL Code: C02, C13, C32, C61, E21.

1- Introduction

After the great depression (1929-32), Keynes criticized the fundamentals of classical school in wage-price flexibility, full employment, inherent equilibrium and so unnecessary policy-making decisions. However, according to "all or nothing" demand for money theory; he attempted to justify government intervention in economy to improve economic situations (Hicks, J. R., 1937; Mankiw, Gregory, 1988). After three decades, Friedman (1968) showed how monetary policies lead to incentive output in the short run and inflation in the long run. Due to Friedman's idea that "inflation is anywhere and always a monetary phenomenon", conducting monetary policy got more attention in economic literatures. Peterson and Lerner (1971) pointed out that increasing money supply lead to increase in the capital accumulation and economic growth. In a new classical school framework, unanticipated policies will have real effects, even in a very short period. Some recent developments in macroeconomic theories such as "central bank independence" as well inflation targeting indicate that the role of central banker in monetary control of inflation and economic growth (Cukierman, Alex, 1992; Taylor, John. B, 1993). Thus, until initially of 2000 decade intermediation and policy-making in economy, voided from visionary and tentative battles. However, due to the trade-off between the effects of macroeconomic policies, these policies must be optimal and coordinated. So, on the base of recent development in macroeconomics, determining optimal monetary and fiscal policies as the satisfy macro goals is very important in economic planning. In real world, policy makers in decision-making process should determine the objectives and constraints. Then, he (or she) should choose the alternative that gives the nearest outputs to objectives. For this reason, the application of optimal control theory has been widely developed in economic studies. During the last few decades, Iran's economy has witnessed high inflation and high fluctuations in economic growth. Combating the above problems has been one of the important goals of policy making in Iran. The present paper deals with the quantities determination of optimal monetary and fiscal policies in Iran in order to achieve the goals of the Iran 4th development plan (2005-2009) in terms of economic growth and inflation with minimum disturbance. To do so, using the stochastic dynamic programming, the quadratic deviation of inflation and economic growth are minimized from the 4th plan targets subject to a Keynesian nonlinear macro econometric system. Using the optimum control algorithm "OPTCON", approximately optimal monetary policy and its effects on the mentioned macroeconomic indexes are determined. The layout of present paper is as following. In the next section, the "OPTCON" algorithm is reviewed. In section 3, the findings are shown. Section 4 concludes the paper with as summary.

2- The "OPTCON" Algorithm

In decision-making process, first, the objectives and constraints should be determined. Then, the alternative that gives the nearest outputs to objectives is chosen as an optimal policy. For this reason, during the last three decades, the application of optimal control theory has been widely developed in economic studies. Hence, the structure of dynamic optimization models is constructed from an objective function (, i.e. functional) and a dynamic nonlinear system of equations as a constraints. The objective function is a quadratic loss function in which penalized on deviation of objective variables from their desired values. The constraints of optimization are determined by the econometric models specified on the base of economic theories. Because of **the nonlinearly of the economic** optimization models, approximate solutions are applied in the studies. For this purpose, some stochastic optimal control algorithms such as: The open-loop feedback (OLF) algorithm derived by Kendrick (1984), the optimal control algorithm for nonlinear model (OPTNL) designed by chow (1981) and the "OPTCON" algorithms developed by Matulka & Neck (1992) are used. Among the algorithms, the "OPTCON" provides the most facilities for economic planning in comparison with the others. In this algorithm a quadratic loss function is minimized subject to a stochastic nonlinear dynamic system. In this intertemporal objective loss function, the policy-maker penalizes on quadratic deviations the vector of control and state variables from their target values. So:

$$L = \frac{1}{2}\begin{bmatrix} X_t - \overline{X_t} \\ U_t - \overline{U_t} \end{bmatrix}' . W_t . \begin{bmatrix} X_t - \overline{X_t} \\ U_t - \overline{U_t} \end{bmatrix} \qquad (1)$$

Where X_t , U_t , $\overline{X_t}$ and $\overline{U_t}$ are vector of state variables, vector of control variables and vector of desired (target) levels of the state and control variables respectively. W_t Denotes the symmetric positive semi definite matrix, so:

$$W_t = \alpha^t . W \quad ; \quad t = 1, 2\ldots T \qquad (2)$$

Where α is a discount factor, W denotes a constant value matrix and T denotes the terminal period of the finite planning horizon.

The dynamic stochastic nonlinear system is defined as:

$$X_t = F(X_{t-1}, U_t, \hat{\theta}, Z_t) + \varepsilon_t \qquad (3)$$

In this system $\hat{\theta}$, Z_t and ε_t are the expected value of the stochastic parameter vector, exogenous variables vector and the matrix of the additive system noise respectively. As inputs of the algorithm, the user has to supply the following: The nonlinear system function, the initial value of the state vector, a tentative path for the control variables, the expected value and the covariance matrix of the stochastic parameter vector, the covariance matrix of the additive system noise, the weight matrices of the objective function, the planning horizon, the desired paths for the state and control variables and a discount rate of the objective function. This algorithm is executable in "GAUSS" programming system. In this paper, the nonlinear system of equations is taken from a macro econometric model based on Keynesian macroeconomic theory. The system of equations contains two group equations: behavioral equations and identities. The behavioral equations include goods and services market and money market from the aggregate demand side. The goods and services market contains private consumption function, private investment function, government total expenditures function, total tax revenues function, imports and non oil exports functions. Hence, regarding Wagner law (Lamartina, S. & Andrea Z, 2008; Afxentiou, P. C. & Apostolos S, 1996) government total expenditures variable was considered endogenous. Also, the model include an exchange rate equation, an interest rate function (a reduced form of money market equilibrium), consumer price function and GDP deflator function. The list of variable is shown in appendix. Table 1 shows the estimated behavioral equations and identities. The stochastic nonlinear system by "3SLS" method was estimated using time series data for the period 1959-2004 (CBI, 2009). So, full covariance matrix of the parameters is available. The software package PC Eviews, version 10 has been used for estimating the system. The purpose is determining the optimal government budgetary expenditures and optimal money stock M3 (total liquidity) in such a way as to satisfy the 4th Iranian development plan goals, i.e., the rate of economic growth and inflation rate. So, the planning horizon is the period from 2005 to 2009. Among the variables whose deviations from the desired values are to be penalized in objective function, two categories are distinguished:

First, the "main variables" are those which are more important in assessing the performance of government. They are the rate of inflation and the rate of GDP growth.

Second, the "minor variables" are those which include the consumer price index and gross domestic product. It was assumed that the desired (or target) values of the main and minor variables are those determined by Iran's 4th development plan (2005-2009).

After several experiments sensitivity analysis, a discount factor $\alpha = 0.25$, the weight 100 for main and 10 for minor objective variables have chosen. Then, in the weight matrix of the objective function, off diagonal elements were all set to zero. In addition, all state variables in the model not mentioned above, got the weight zero. In this paper, the tentative path to state variables is calculated by simulation of the model. Also, projections for the exogenous variables as well as control variable over the optimization horizon were required too. The values of these variables were the same which occurred during the period of 2005-2009.

3- The optimization results

Table 2 shows the values of optimal, 4th plan and occurred control variables. A comparison of the results shows that the optimal values liquidity and government budgetary expenditures are greater than the amounts proposed in 4th plan but less than the occurred quantities. The optimal values of main variables are shown in table 3. Hence, as show, using the optimal macro policies will improve the rate of economic growth and inflation rate in comparison with their occurred values. Based on obtained results, the actual (occurred) inflation rates are bigger than the plan targets, since there was not monetary and fiscal discipline over the period regarding table 2 and table 3. The table results show that under optimal monetary policy, the fluctuation in the rate of inflation and economic growth will be reduced. In fact, optimal monetary policies act as a stabilizing program.

Table 1: The System Equations

Number of Equation	Behavioral
1	CRR=0.54 CRR(-1) + 0.13 YDR + 28 M3R $\bar{R}^2 = 99\%$ t: (4.86) (4.11) (3.1) DW=2.08
2	INVPR=0.91 INVPR(-1) + 0.2 TGEN – 543.19 LTIRR t: (14.46) (2.45) (-0.82) DW=1.93 $\bar{R}^2 = 68\%$
3	TGEN=2420.9 +0.2 GDPN t: (1.39) (36.61) DW=2.03 $\bar{R}^2 = 96\%$
4	NOILEXPR=0.96 NOILEXPR(-1) + 0.34 ERR t: (16.1) (1.51) DW=2.09 $\bar{R}^2 = 87\%$
5	IMPR=0.33 GDPR – 6.03 ERR t: (5.65) (-1.72) DW=1.77 $\bar{R}^2 = 84\%$
6	CPI=0.58 CPI(-1) + 0.00006 M3N + 0.003 ERN t: (9.06) (6.8) (11.72) DW=2.1 $\bar{R}^2 = 99\%$
7	LTIRN=0.94 LTIRN(-1) + 0.000003 GDPR – 0.000003 M3N t: (9.98) (0.91) (-0.62) DW=1.65 $\bar{R}^2 = 94\%$
8	ERN=1.06 ERN(-1) – 0.00013 NX t: (3.63) (2.87) DW=1.19 $\bar{R}^2 = 98\%$
9	TAXRN=45.79 + 0.06 GDPN t: (1.91) (49.17) DW=1.19 $\bar{R}^2 = 98\%$
10	GDPDEF=1.015 CPI t: (17.41) DW=2.19 $\bar{R}^2 = 99\%$
	Identities
11	GDPR= CPR + INVPR + TEGR + EXPR – IMPR
12	TGER=(TGEN * GDPDEF)/100
13	EXPR=OILEXPR+NOILEXPR
14	GRGDPR=((GDPR-GDPR(-1))/GDPR(-1))*100
15	GRCPI=((CPI-CPI(-1))/CPI(-1))*100
16	LTIRR=LTIRN-GRCPI
17	M3R= (M3N/CPI).ERN
18	ERR= (CPIF/CPI) * ERN
19	NX=EXPN- IMPN
20	EXPN= (EXPR*EXPDEF) /100
21	IMPN= (IMPR*IMPDEF) /100
22	TAXPR= (TAXRN/GDPDEF)*100
23	YDR= GDPR- TAXRR

*t, $\bar{R}^2$ and DW are the T statistic, adjusted $\bar{R}^2$ and Durbin Watson statistic respectively.

Table 2. The values of optimal, 4th plan and occurred control variables

Milliard Rial (IRR)

Control variables		2005	2006	2007	2008	2009
M3N	Proposed in the 4th plan	814408	993577	1192293	1406906	1634280
	Optimal	850273	1041724	1279447	1563158	1930937
	Occurred	921019	1284199	1640293	1901366	1974851
GBEN	Proposed in the 4th plan	325055	379204	439974	510167	601214
	Optimal	439502	551511	752221	1036958	1410535
	Occurred	448523	561359	578550	777786	….

Source: Authors Calculations.

Table 3: The Optimization Results

Percentage

Main variables		2005	2006	2007	2008	2009
GRGDPR	Targets	7.1	7.4	7.8	8.4	9.3
	Optimal	10.4	7.3	7.6	8	8.2
	Occurred	6.9	6.6	6.7	2.3	1.9*
GRCPI	Targets	14.6	11.5	9.1	7.9	6.8
	Optimal	16.4	16.5	17	17.2	18.1
	Occurred	10.4	11.9	18.4	25.4	10.8

Source: authors calculations.

* Based on IMF estimation (IMF, 2009).

4- Concluding remark

In this paper the optimal macroeconomic policies were determined to achieve the goals of the 4th five year Iranian plan (2005-2009). This includes economic growth and inflation rate. To do so, an intertemporal quadratic objective loss function was minimized subject to a dynamic nonlinear macro econometric system using statistic optimal control theory. The obtained results show that the optimal values of liquidity government currency as well as capital expenditures are greater than those proposed in forth development plan. Also, the optimal values of the above mentioned control variables are different from these executed during the 4th plan years. The comparison between the effects of the optimal macroeconomic policies on goal variables with these occurred show that using the optimal policies improve the goals in the 4th plan with respect to rate of economic growth and inflation rate. Therefore, in the absent of active tax instrument, there must be "big push" in fiscal and monetary policies in order to achieve macroeconomic goals in Iran. Furthermore, the study shows that the optimum macroeconomic policies could lead to a considerable stabilization of the time path of the rate of economic growth and inflation rate. Based on the obtained results, it is recommended to apply optimal control theory for actual political decisions.

Acknowledgement

The authors wish to thank professor **Reinhard Neck**, professor **Klaus Weyerstrass** and professor **Gottfried Haber** faculty members of Austria Klagenfurt University and also professor **Ahmad Jafari Samimi** faculty member of Mazandaran University for their useful guidance and their helpful constructive comments regarding "OPTCON" algorithm.

Appendix: List of variables

State (or endogenous) variables:	
CPI	Consumer price index
CPR	Private consumption expenditures, real
Demand	Total final demand, real
ERN	Exchange rate, nominal
ERR	Exchange rate, real
EXPN	Total export, nominal
EXPR	Total export, real
GDPR	Gross domestic product, real
GDPDEF	GDP deflator
GBER	Government budgetary expenditure, real
GRCPI	Annual growth rate of CPI (inflation rate)
GRGDPR	Annual growth rate of real GDP
IMPR	Total imports, real
IMPN	Total imports, nominal
INVPR	Private investment, real
LTIRN	Long-term interest rate, nominal
LTIRR	Long-term interest rate, real
M3R	Money stock M3, real
NOILEXPR	Non-oil export, real
NX	Net export, nominal
TAXRN	Government tax revenue, nominal
TAXRR	Government tax revenue, real
GER	Government expenditure, real
YDR	Personal disposable income, real
Non-Control Exogenous variables:	
CPIUSA	USA CPI Consumer price index
IMPDEF	Import price level (import deflator)
EXPDEF	Export price level (export deflator)
OILEXPR	Oil exports, real
Control Exogenous variables:	
M3N	Money stock M3, nominal
GBEN	Government budgetary expenditures, nominal

References:

1. Afxentiou P.C., Apostolos S., (1996) Government expenditures in the European Union: Do they converge or follow Wagner's low? International Economic journal. 10, (3), pp. 33.47.
2. Central Bank Islamic Republic of Iran. (2004) Economic Report and Yearbook of Central Bank for years 1959-2004. Office of Economic Investigation.
3. Chow, G. C. (1981). Econometric analysis by control method. New York, Wiley Press.
4. Cukierman, Alex. (1992). Central Bank strategy, Credibility, and Independence: Theory and Evidence. USA, MIT Press.
5. Friedman, M. (1968). The role of monetary policy. American economic review,.58, pp.1-77.
6. Hicks, J. R. (1937). "Mr. Keynes and the “Classics” A Suggested Interpretation". Econometrica. 5, pp. 147-159.
7. Kendrick, P., Coomes, P. (1984). DUAL: A program for quadratic-linear stochastic control problems. In: *Discussion paper 84-15*, Center for Economic Research, University of Texas, Austin, TX.
8. Lamartina S., Andrea Z. (2011) Increasing Public Expenditures: Wagner's law in OECD Countries. German Economic Review. 12 (2), pp. 149-164.
9. Mankiw, Gregory. (1988). Recent Developments in Macroeconomics: A Very Quick Refresher Course. Journal of Monetary, Credit and Banking. 20 (3), pp.436-449.
10. Josef, M., Neck, R. (1992). Opticon: An algorithm for the optimal control of nonlinear stochastic models. Annals of Operation Research. 37, pp. 375-401.
11. Peterson, D. & Eugene, L. (1971). Optimal Control and Monetary policy. International Economic Review, 12 (2), pp.186-208.
12. Taylor, J., (1993). Discretion versus policy rules in practice. Carnegie-Rochester Conference series on public policy, North-Holland, 39 (1993), pp. 195-214.
13. www.imf.org

4/13/2011