Information Sharing in Designing a Supply Chain Model Considering Demand Forecasting Using Markov Process

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Abstract: In this paper, we incorporate information flow in a supply chain model. Also for decreasing the risk of the supply chain system, we first predict the customers’ demands and then this forecasting is used as an input to the supply chain model. In this paper a markov chain model will be used to forecast the customers’ demands. A simulated annealing (SA) algorithm is developed for solving the supply chain problem. The results indicate that the SA method and proposed markov chain model are efficient for a wide variety of problem sizes.


Keywords: Supply chain model; Demand forecasting; Markov chain; Simulated annealing

1. Introduction

The performance of a supply chain depends critically on how its members coordinate their decisions. Information sharing between supply chain partners is a prerequisite for coordinated supply chain management (Selamat et al., 2010; Saba et al., 2011). Over the last few years, considerable research has been devoted to understanding the role of information in achieving supply chain coordination. The research of information sharing in supply chain grew largely out of two-stage inventory models. The following papers mainly examine what kind of benefit can be gained through demand-oriented information sharing: Gavirneni et al. (1999), Lee et al. (2000). Further studies that analyze the benefit of information sharing with multiple retailers are as follows: Cachon and Fisher (2000), Gavirneni (2001), Aviv and Federgruen (1998). Some other researchers investigated forecasting information sharing. Cachon and Lariviere (2001) studied forecast sharing in a single product, two-level supply chain. Aviv (2001) compared three settings under a two-level supply chain. Zhao et al. (2002) presented the impact of different forecasting models on the value of information sharing in a supply chain.

In this paper, we will establish a supply chain model that includes sources, make and deliver processes to examine the effect of supply information sharing. Also we predict the customers’ demands in the supply chain system. The forecast of future demand forms the basis for all strategic and planning decisions in a supply chain. In the literature several techniques have addressed time series prediction (pourahmadi, 2001). Time series can be modeled by using Markov chains. In many occasions, one has to consider multiple Markov chains (categorical sequences) together at the same time, i.e., to study the chains in a holistic manner rather than individually. The reason is that the chains (data sequences) can be “correlated” and therefore the information of other chains can contribute to explain the captured chain (data sequence). Thus by exploring these relationships, one can develop better models (Haron et al., 2011).

When we predict the customers’ demand, then it is used an input to the proposed supply chain model. For solving the supply chain model, a simulated annealing algorithm is used. The reminder of this paper is organized as follows. Section 2, discusses the proposed markov chain model. In section 3, the mathematical formulation of the supply chain model is presented. Section 4, discusses the solution approach for solving the problem. Section 5, discusses some computational results. Finally, section 6 contains some conclusions and future research development.

2. The Markov Chain Model for forecasting customers' demands

In this section, we propose our new multivariate markov chain model for forecasting customers’ demands. The following multivariate Markov chain model has been proposed in Fung et al. (2003). The model assumes that there are $S$ categorical sequences and each has $m$ possible states in $M = \{1, 2, \ldots, m\}$

Here we adopt the following notations. Let $X^{(k)}_n$ be the state vector of the $k$th sequence at time $n$. If the $k$th sequence is in State $j$ at time $n$ then we write

\[ X^{(k)}_n = j \]
The following relationships among the sequences are assumed:

\[ X_n^{(j)} = \lambda_j p^{(j)} X_n^{(k)} + \sum_{k=1, k \neq j}^s \lambda_{jk} p^{(jk)} X_n^{(k)} \]

for \( j = 1, \ldots, s \),

Where

\[ \lambda_{jk} \geq 0, 1 \leq j, k \leq s \quad \text{and} \quad \sum_{k=1}^s \lambda_{jk} = 1 \]

for \( j = 1, \ldots, s \).

Equation (1) simply means that the state probability distribution of the jth chain (sequence) at time \( (n+1) \) depends only on the weighted average of \( p^{(j)} X_n^{(j)} \) and \( p^{(jk)} X_n^{(k)} \). Here \( p^{(j)} \) is the one-step transition probability matrix of the states from the jth sequence to the states of the ith sequence. In matrix form, one may write

\[
\begin{pmatrix}
X_{n+1}^{(1)} \\
X_{n+1}^{(2)} \\
\vdots \\
X_{n+1}^{(s)}
\end{pmatrix} = 
\begin{pmatrix}
X_{n}^{(1)} \\
X_{n}^{(2)} \\
\vdots \\
X_{n}^{(s)}
\end{pmatrix}
\]

\[
\begin{pmatrix}
\lambda_{11} p^{(1)} & \lambda_{12} p^{(2)} & \cdots & \lambda_{1s} p^{(s)} \\
\lambda_{21} p^{(1)} & \lambda_{22} p^{(2)} & \cdots & \lambda_{2s} p^{(s)} \\
\vdots & \vdots & \ddots & \vdots \\
\lambda_{s1} p^{(1)} & \lambda_{s2} p^{(2)} & \cdots & \lambda_{ss} p^{(s)}
\end{pmatrix}
\begin{pmatrix}
X_{n}^{(1)} \\
X_{n}^{(2)} \\
\vdots \\
X_{n}^{(s)}
\end{pmatrix}
\]

\[ = Q X_n \]

When we forecast the customers’ demands then this forecasting is used as an input to the supply chain model.

### 3. The Supply Chain Model

We assume that there are several capacitated suppliers, one manufacturer, and several retailers. The retailers are confronted with time-varying customer demands, which were predicted in the previous section. In the proposed supply chain model, each supplier can supply one material. The manufacturer produces several products, which consume several materials respectively.

Then the production planning in the manufacturer is a multi-product multi-resource constraints lot sizing problem. The sequence of events in every period is as follows. First, the manufacturer decides on his production quantity for the period, and the materials he needs are transported from suppliers under the resources constraints. Next, the manufacturer consigns his products to several retailers. If the manufacturer cannot satisfy the full order of the retailer, we assume that the retailer acquires the shortage part of the order elsewhere. All happen with no lead time. Then customer demands occur. At last inventory holding or shortage penalty costs are charged.

Now we introduce some indices and parameters, which will be used in the following formulas and models.

#### Index sets

- \( l \): Product index, \( l = 1, \ldots, L \);
- \( n \): Supplier index, \( n = 1, \ldots, N \);
- \( m \): Retailer index, \( m = 1, \ldots, M \);
- \( t \): Index of planning period, \( t = 1, \ldots, T \);

#### Parameters

- \( a_{nl} \): Capacity needed on material \( n \) for one unit product \( l \) in period \( t \);
- \( R_{nl} \): The supplier capacity of available material \( n \) in period \( t \);
- \( b_n \): Unit transportation cost for material \( n \) to the manufacturer;
- \( c_{lt} \): Unit production cost for product \( l \) in period \( t \);
- \( h_l \): Unit inventory holding cost for product \( l \) in the manufacturer in period \( t \);
- \( g_{lm} \): Unit transportation cost for product \( l \) to the retailer \( m \);
- \( h_{lm} \): Unit inventory holding cost for product \( l \) in the retailer \( m \) in period \( t \);
- \( \beta \): The penalty coefficient for the shortage of the manufacturer to the retailer;
- \( D_{ltm} \): The predicted customer demand for product \( l \) in retailer \( m \) in the period \( t \);

#### Decision Variables

- \( I_{lt} \): The inventory of product \( l \) in the manufacturer at the end of period \( t \);
- \( I_{ltm} \): The inventory of product \( l \) in the retailer \( m \) at the end of period \( t \);
- \( O_{ltm} \): The shortage of product \( l \) for the retailer \( m \) from the manufacturer in the period \( t \);
- \( X_{lt} \): The amount of product \( l \) produced in period \( t \);
The quantities of product $l$ distributed to the retailer $m$ in period $t$.

So the problem can be formulated as follows:

$$
\begin{align*}
\text{MIN} & \sum_{n=1}^{N} \sum_{l=1}^{L} \sum_{t=1}^{T} b_{nl} a_{nl} X_{lt} \\
& + \sum_{l=1}^{L} \sum_{t=1}^{T} (c_{lt} X_{lt} + h_{lt} I_{lt}) \\
& + \sum_{m=1}^{M} \sum_{l=1}^{L} \sum_{t=1}^{T} (g_{ltm} f_{ltm} + h_{ltm} l_{ltm} + \beta o_{ltm})
\end{align*}
$$

Subject to:

$$
\sum_{l=1}^{L} a_{nl} X_{lt} \leq R_{nt} \quad \text{for all } n,t
$$

(5)

$$
I_{l,t-1} + X_{lt} - I_{lt} = \sum_{m=1}^{M} f_{ltm} \quad \text{for all } l,t
$$

(6)

$$
I_{l,t-1,m} + f_{ltm} - I_{ltm} + O_{ltm} \geq D_{ltm} \quad \text{for all } l,t,m
$$

(7)

$$
X_{lt} \geq 0 \quad \text{for all } l,t
$$

(8)

$$
f_{ltm} \geq 0 \quad \text{for all } l,t,m
$$

(9)

$$
I_{lt} \geq 0 \quad \text{for all } l,t
$$

(10)

$$
I_{ltm} \geq 0 \quad \text{for all } l,t,m
$$

(11)

$$
o_{ltm} \geq 0 \quad \text{for all } l,t,m
$$

(12)

Constraint (5) is the total capacity for a material $n$ needed to produce all the scheduled products in period $t$. Constraint (6) defines the quantities of product $l$ on consignment by the manufacturer for the retailers in period $t$. Constraint (7) is to satisfy the customer demands. Constraints (8)-(12) enforce the non-negativity restrictions on the corresponding variables.

4. Solution approach

A simulated annealing algorithm is used for solving the problem. The SA methodology draws its analogy from the annealing process of solids. In the annealing process, a solid is heated to a high temperature and gradually cooled to a low temperature to be crystallized. As the heating process allows the atoms to move randomly, if the cooling is done too rapidly, it gives the atoms enough time to align themselves in order to reach a minimum energy state that named stability or equipment. This analogy can be used in combinatorial optimization in which the state of solid corresponds to the feasible solution, the energy at each state corresponds to the improvement in the objective function and the minimum energy state will be the optimal solution. The SA parameters are as follows:

$$
T_{0} : \text{Initial temperature},
$$

$$
C : \text{Rate of the current temperature decreases (cooling schedule)},
$$

$$
ST : \text{Freezing temperature (the temperature at which the desired energy level is reached)},
$$

$$
L : \text{Number of accepted solution at each temperature},
$$

$$
S : \text{Counter for the number of accepted solution at each temperature},
$$

$$
X : \text{A feasible solution}
$$

$$
C(X) : \text{The value of objective function for } X,
$$

In the section 4.1, 4.2, we describe the initial solution construction and generating the candidate move which we use for SA algorithm.

4.1. Representation and Initialization

The procedure for obtaining the initial solution is randomly. The decision variables in our problem are $X_{lt}$ and $f_{ltm}$, they are positive variables of real numbers. We encode variables $X_{lt}$ and $f_{ltm}$ as follows:

$$(X,f) = (X_{11}, X_{12}, ..., X_{1T}, X_{21}, X_{22}, ..., X_{2T}, ..., X_{LT}, f_{111}, f_{112}, ..., f_{1LM}, f_{211}, f_{212}, ..., f_{2TM}, ..., f_{L11}, f_{L12}, ..., f_{LTM})$$

For a capacitated lot-sizing problem, the decision variables $X_{lt}$ are dependent on the available resource capacities $R_{nt}$. In this situation, we can first check it whether the total capacity for a resource needed to produce all the scheduled products in this period exceeds the total available capacity for this resource at this period. If exceeds, we will generate another solution to replace.

4.2. Obtaining the candidate move

For obtaining the candidate move, we randomly select one $X_{lt}$ in $X$ part of the solution and regenerate its value. Then adjust the correlation value of $f$ of the solution according to the replacement and check the feasibility of it.
Table 1. Comparison of optimal solution and SA solution

<table>
<thead>
<tr>
<th>NO.</th>
<th># Retailers</th>
<th># Suppliers</th>
<th>Cost</th>
<th>CPU time</th>
<th>Cost</th>
<th>CPU time</th>
<th>Gap (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>2</td>
<td>19125.6</td>
<td>12</td>
<td>19125.6</td>
<td>1</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
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<td>3</td>
<td>25892.1</td>
<td>28</td>
<td>25892.1</td>
<td>3</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>3</td>
<td>32978.4</td>
<td>54</td>
<td>32978.4</td>
<td>9</td>
<td>0.00</td>
</tr>
<tr>
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<td>9</td>
<td>3</td>
<td>35026.4</td>
<td>132</td>
<td>35026.4</td>
<td>13</td>
<td>0.00</td>
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<tr>
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<td>6</td>
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<td>79226.1</td>
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<td>8</td>
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<td>39</td>
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<tr>
<td>7</td>
<td>40</td>
<td>10</td>
<td>144786.7</td>
<td>847</td>
<td>146157.4</td>
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<td>0.95</td>
</tr>
<tr>
<td>8</td>
<td>50</td>
<td>13</td>
<td>181912.5</td>
<td>1475</td>
<td>183792.7</td>
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<td>1.03</td>
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<td>9</td>
<td>60</td>
<td>16</td>
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<td>213375.1</td>
<td>80</td>
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</tr>
<tr>
<td>10</td>
<td>70</td>
<td>17</td>
<td>254975.9</td>
<td>6748</td>
<td>258138.6</td>
<td>94</td>
<td>1.24</td>
</tr>
<tr>
<td>11</td>
<td>80</td>
<td>18</td>
<td>292381.7</td>
<td>10511</td>
<td>296364.2</td>
<td>110</td>
<td>1.36</td>
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<td>12</td>
<td>90</td>
<td>20</td>
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<td>127</td>
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</tr>
<tr>
<td>13</td>
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<td>383674.1</td>
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<td>369734.2</td>
<td>146</td>
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<tr>
<td>14</td>
<td>120</td>
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<td>469526.4</td>
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<td>435186.4</td>
<td>181</td>
<td>-7.31</td>
</tr>
</tbody>
</table>

Gap(%) = 100*(Heuristic solution value – LINGO best solution value) / LINGO best solution value.

5. Computational results

The computational experiments described in this section were designed to evaluate the performance of our overall solution procedure with respect to a series of test problems. The SA algorithm was coded in Visual Basic 6 and run on a Pentium 4 with 3 GB processor. For simplicity we assume that the number of periods is three and the number of products is two in all of the instances, and also we use the following parameter values.

\[ a_{nh} \text{ is uniformly drawn from } [5, 8] \]
\[ b_n \text{ is uniformly drawn from } [8, 15] \]
\[ c_{lt} \text{ is uniformly drawn from } [8, 15] \]
\[ h_{lt} \text{ is uniformly drawn from } [7, 12] \]
\[ g_{lm} \text{ is uniformly drawn from } [7, 12] \]
\[ h_{lm} \text{ is uniformly drawn from } [7, 12] \]
\[ \beta = 10 \]

And \( R_m \) with regard to the size of the problem is generated randomly.

5.1. Comparison of optimal solution and SA solution

For evaluating the SA algorithm, fourteen problems are solved by LINGO software (Table 1). For each problem, the tuning of the parameters is done by carrying out random experiments.

It can be seen that the SA solutions are optimal (or near optimal) in different problem instances (Table 1). The average CPU time is less than or equal to 181 seconds for the SA method (CPU times are in the seconds). However, the maximal average CPU time for obtaining the optimal solutions is equal to 10511 seconds, and for problem instances 12 to 14 by a reasonable amount of time limit, LINGO can not find the optimal solution, and the SA solutions in these problem instances are better than the best solutions that are obtained by LINGO.

5.2. Validating the markov chain model

For validating the proposed Markov chain model, we comprise it with the Auto Regressive Moving Average (ARMA) model. The order of the ARMA model which we use is (1, 1) (i.e., ARMA (1,1)). After running we see that the absolute error for the Markov chain predictor model is less than the absolute error of the ARMA model, the absolute error for the proposed Markov chain model is 0.24 and the absolute error for the ARMA model is 0.46.

6. Conclusions

In traditional supply chain inventory management, orders are the only information firms exchange, but information technology now allows firms to share demand and inventory data quickly and inexpensively. In order to have an integrated plan, a manufacturer needs to know not only demand information from its customers but also supply information from its suppliers. In this paper, we incorporated information flow in a supply chain model. Also for decreasing the risk of the supply chain system, we first predicted the customers’ demands and then this forecasting was used as an input to the supply chain model. A Markov chain...
model was proposed to forecast the customers’ demands.

A simulated annealing (SA) algorithm was used for solving the distribution network problem. The results of extensive computational tests indicated that the SA method and proposed markov chain model is both effective and efficient for a wide variety of problem sizes.

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