The effect of non-linear terms on the process of computing water hammer with regard to friction coefficients for different cast iron pipe

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Abstract: Water Hammer problems are complex and time-consuming even with very simple calculation method and usual boundary conditions. The governing equations of water Hammer are partial differential equation and are expressed based on continuity and momentum equations. One of the important procedures for solving the governing equations of unsteady flows is finite difference method. One procedure for simplifying the governing equations is neglecting the nonlinear terms such as $V\frac{\partial V}{\partial x}, V\frac{\partial H}{\partial x}$ without considering the amount of errors that are created with this process. Therefore in this paper, the phenomenon of water Hammer in the tank, pipe and valve system has been investigated in two manners, one with full equations and other with neglecting the nonlinear terms. For doing this, an FDM code has been written in MATLAB and the amounts of head along the pipe in sequential times and the differences between two manners have been given in diagrams. The obtained results indicate that for iron pipe with different friction coefficient (smooth, perennial and worn) by decreasing Chezy coefficient, wave damping increases and the effect of nonlinear terms decreases.

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1. Introduction

Water Hammer and its destructive potential are in many cases identified by its resultant sound. Water Hammer is caused by an abrupt change of flow velocity in a pipe. When it is created, water Hammer is propagated in the form of a wave through all parts of the system connected to it, and is a function of flow geometry and pipe characteristics. Water Hammer is assumed to be an elastic phenomenon and is expressed by continuity and momentum equations (Koutitas, 1983; Zandi, 2006).

The wide-spread dimensions of water Hammer problem and numerous unknowns and problems caused by this phenomenon, as well as rapid advance in numerical computation in parallel to computer sciences provide an appropriate field for further study. Finite difference method is among techniques commonly used for solving partial differential equations and plenty of its applications exist in. The effect of nonlinear terms on process of solving partial differential equations is an issue which has not been studied much in hydraulics engineering. Therefore in the present research water Hammer phenomenon has been studied for hyperbolic differential equations with similar initial and boundary conditions and different friction coefficients, and the results obtained from finite difference method have been compared in two cases.

Governing equations and formulation

Mathematical model of this phenomenon includes the two principles of continuity and momentum, and the respective nonlinear partial differential equations are as follows:

$$\frac{\partial V}{\partial t}+V\frac{\partial H}{\partial x}+g\frac{\partial H}{\partial x}+\frac{fV|V|}{2D}=0 \quad (1)$$

$$\frac{\partial H}{\partial t}+V\frac{\partial H}{\partial x}+\frac{C^2}{g}\frac{\partial V}{\partial x}=0 \quad (2)$$

where H and V are values of pressure head and velocity as unknown parameters, respectively; g is earth's gravity constant; C is wave velocity; and f is friction coefficient which depends on pipe diameter and pipe smoothness (Daneshfaraz et al, 2009).

An appropriate approach for simplifying the above mentioned nonlinear equations is to neglect the nonlinear terms $\left(V\frac{\partial V}{\partial x},\ V\frac{\partial H}{\partial x}\right)$. If water Hammer phenomenon occurs in a short time period, frictional damping will be small and the equations (1) and (2) are summarized as follows considering small amount of flow velocity compared to wave propagation velocity:

$$\frac{\partial V}{\partial t}+g\frac{\partial H}{\partial x}+\frac{fV|V|}{2D}=0 \quad (3)$$

$$\frac{\partial H}{\partial t}+\frac{C^2}{g}\frac{\partial V}{\partial x}=0 \qquad (4)$$

Equations (1) and (2) are nonlinear differential equations, while equations (3) and (4) are linear differential equations which become complete by defining initial and boundary conditions. The mentioned equations can be solved by numerical computation of finite differences on nodes at time and position intervals. To solve them, the pipe is assumed to be broken down into several discrete segments with defined lengths, and for each segment the values of pressure head (H) on nodes and velocity (V) in middle of the segment are calculated. Fig. 1 schematically demonstrates integration procedure in (x, t) plane (Daneshfaraz et al, 2009; Musavai-Jahromi, 2006).

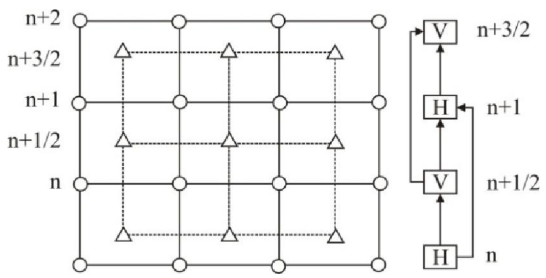

Fig 1: Integration procedure in (x, t) plane using FDM method.

Making use of the algorithm shown in Fig. 1 and finite difference approximation, the equations (1) to (4) are expanded as follows:

$$\frac{V_i^{n+\frac{3}{2}}-V_i^{n+\frac{1}{2}}}{\Delta t}+V^*\frac{V_{i+1}^{n+\frac{3}{2}}-V_i^{n+\frac{3}{2}}}{\Delta x}+g\frac{H_i^n-H_{i-1}^n}{\Delta x}+\frac{fV^*\left|V^*\right|}{2D}=0 \qquad (5)$$

$$\frac{H_i^{n+1}-H_i^n}{\Delta t}+V^*\frac{H_i^n-H_{i-1}^n}{\Delta x}+\frac{C^2}{g}\frac{V_{i+1}^{n+\frac{1}{2}}-V_i^{n+\frac{1}{2}}}{\Delta x}=0 \qquad (6)$$

$$\frac{V_i^{n+\frac{3}{2}}-V_i^{n+\frac{1}{2}}}{\Delta t}++g\frac{H_i^n-H_{i-1}^n}{\Delta x}+\frac{fV^*\left|V^*\right|}{2D}=0 \qquad (7)$$

$$\frac{H_i^{n+1}-H_i^n}{\Delta t}+\frac{C^2}{g}\frac{V_{i+1}^{n+\frac{1}{2}}-V_i^{n+\frac{1}{2}}}{\Delta x}=0 \qquad (8)$$

where indices *i* and *n* denote position and time, respectively. To expand V^* in mentioned equations, Lax formulation has been used as follows:

$$V^*=\frac{V_{i+1}^n-V_{i-1}^n}{2} \qquad (9)$$

Also, for solution's stability, Courant condition has been used as integration criterion as follows:

$$\lambda=\frac{C\Delta t}{\Delta x}<1 \qquad (10)$$

2. Experimental/Applied example

A pipe is assumed to possess following properties: length, 6000 m; diameter, 50 cm; thickness, 4 mm; output head, 5 m; wave velocity, 2980 m/s; output flow velocity, 9.9 m/s. General scheme of the system is illustrated in Fig. 2. Discharge is stopped in 2 seconds by a valve at the end of the pipe.

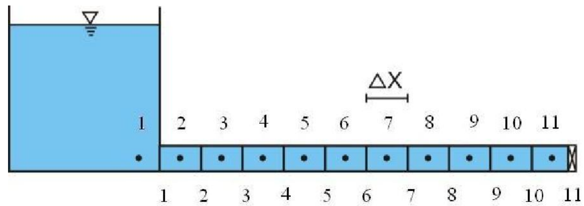

Fig 2: General scheme of tank and pipe system, and its segmentation.

Flow cross-section changes linearly by the valve. At beginning of the pipe, pressure head is equal to water depth in tank and is assumed to be constant. At the end of the pipe, boundary conditions depend on flow velocity and flow cross-section, and are calculated from the following equation. $V=\frac{A_{(t)}}{A_0}\sqrt{2gH}$ (11)

3. Analysis of results and discussion

Numerical solutions by FDM method were yielded from program written in MATLAB environment for smooth iron pipes with Chezy coefficient equal to 130, for perennial iron pipes with Chezy coefficient of 100, and for worn iron pipes with Chezy coefficient of 80 (The website of Southern Tehran Mechanic Society), and the following graphs were obtained.

Figs. 3 and 5 demonstrate changes in water head versus time at end part and middle of the pipe considering its material in two cases, i.e. with and without nonlinear terms, respectively. Furthermore, differences in water head values at the two mentioned cases for end and middle nods are presented in Figs. 5 and 6, respectively. For instance in Figs. 3 and 5 the water head values at the two cases with and without considering nonlinear terms are provided for smooth iron pipes; the difference is not however much obvious due to high level of water head existing in the pipe. Therefore, these difference values are shown in Figs. 4 and 6 and are in the range of 10 and -6 m water depth.

Changes of water head along the pipe at times of 10 and 20 s considering the pipe material in the two mentioned cases, as well as differences between water head values along the pipe at times of 10 and 20 s are illustrated in Figs. 7 to 10, respectively.

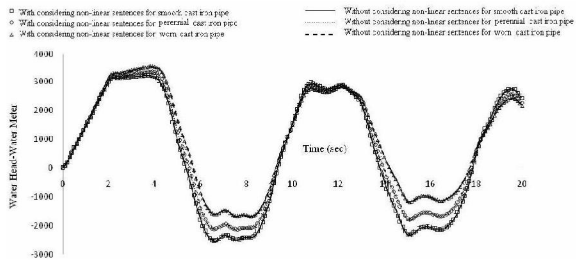

Fig 3: Changes of water head versus time at end of the pipe considering the pipe material in the two cases, i.e. with and without considering nonlinear terms.

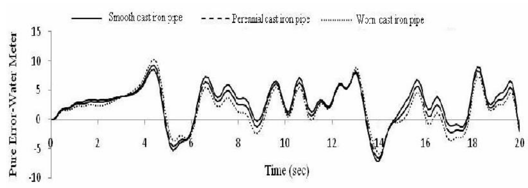

Fig 4: Differences between water head values obtained in the two mentioned cases versus time at end of the pipe considering the pipe material.

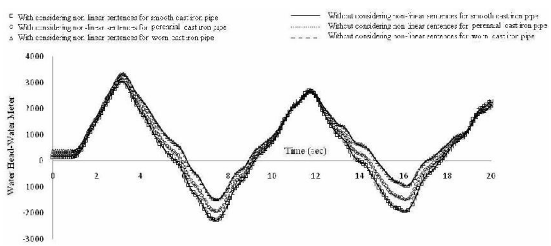

Fig 5: Changes of water head versus time in middle of the pipe considering the pipe material in the two cases, i.e. with and without considering nonlinear terms.

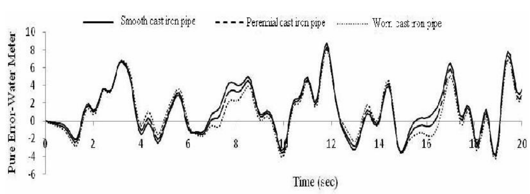

Fig 6: Differences between water head values obtained in the two mentioned cases versus time in middle of the pipe considering the pipe material

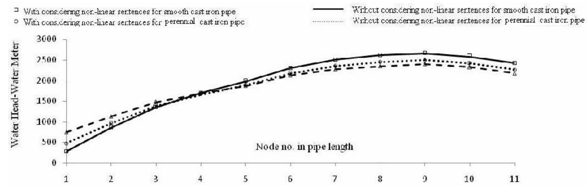

Fig 7: Changes of water head along the pipe at t = 20 s considering the pipe material in the two cases, i.e. with and without considering nonlinear terms.

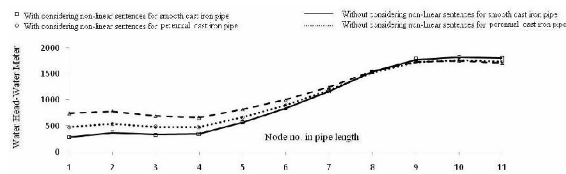

Fig 8: Changes of water head along the pipe at t = 10 s considering the pipe material in the two cases, i.e. with and without considering nonlinear terms.

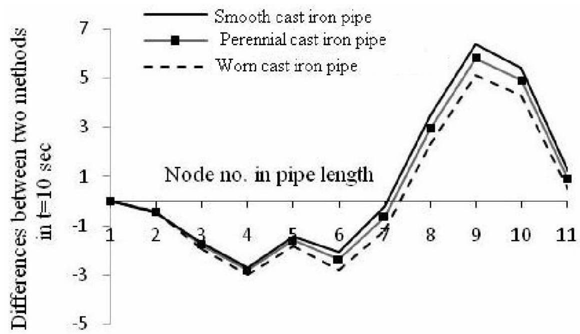

Fig 9: Changes in water head values obtained in the two mentioned cases along the pipe at t = 10 sec.

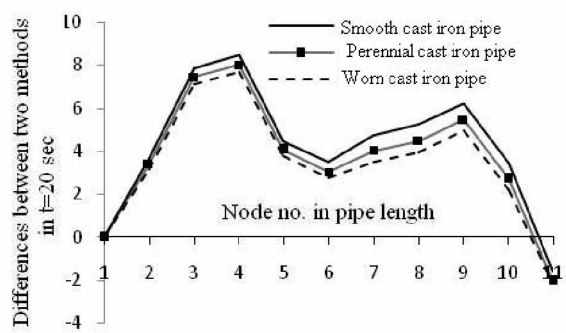

Fig 10: Changes in water head values obtained in the two mentioned cases along the pipe at t = 20 sec.

With numerical considering of mentioned figs, it has concluded that increasing of Chezy coefficient will result in increase of pure mean differences between two methods. Relations between them in pipe length and continued times respectively screened in figs 11, 12.

But the point that is mandatory is that the maximum differences resulted from removing non-linear sentences in pipe length in continued times, were not constant and were in resonance. This pointed in Fig. 13.

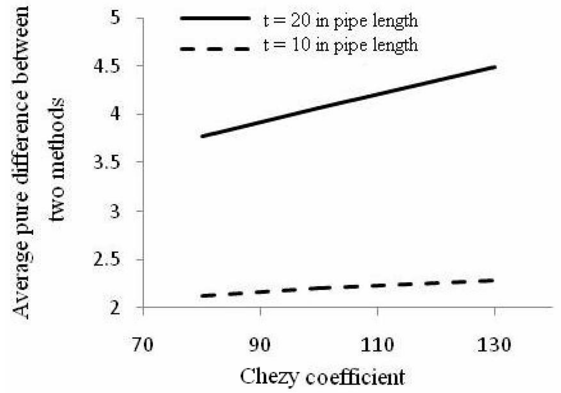

Fig 11: Mean absolute value of the difference between t = 20 s and t = 10 s versus Chezy coefficient.

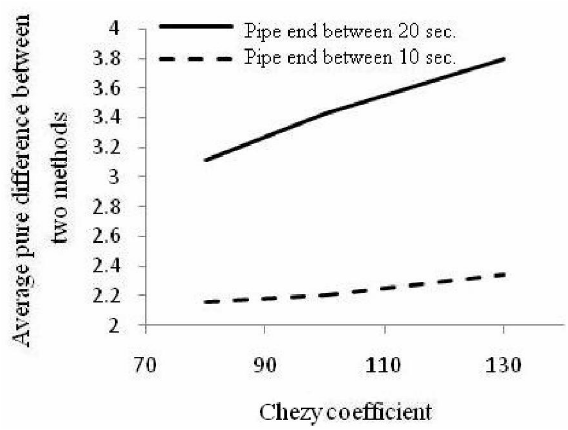

Fig 12: Mean absolute value of the difference at end part and middle of the pipe versus Chezy coefficient.

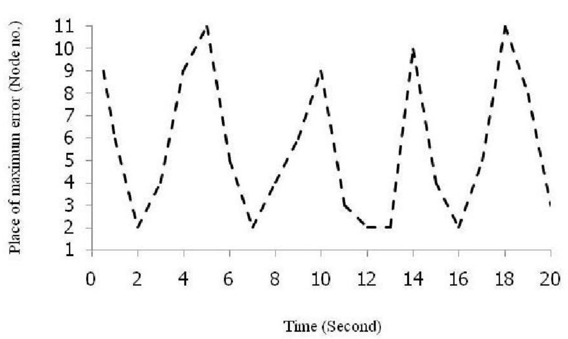

Fig 13: Position of maximum error caused by removal of nonlinear terms.

4. Conclusion

In the present paper, partial differential equations of water Hammer were solved in two cases, i.e. by considering and neglecting nonlinear terms, using FDM method for Chezy coefficients mentioned in the text, and numerous results were yielded. The most significant results are as follows:

1. Increasing Chezy coefficient has led to increase in mean absolute value of the difference between the two mentioned cases (see Figs. 11 and 12).

2. According to Figs. 3 to 6, behavior of wave is in good agreement at nonlinear and linear cases; thus by removing nonlinear terms the solution can be generalized to complete equation with an acceptable error.

3. Difference between the two mentioned cases is not constant along the pipe, and it changes according to Fig. 13.

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References

1. Koutitas, G., 1983. Element of Computational Hydraulics, p. 138.
2. Zandi, Y., 2006. Differences between hydraulic dead flaws of control methods resulted from water hammer, National Conferences of Water Managers, Shahid Chamran University, Ahvaz, Iran.
3. Musavai-Jahromi, H., 2006. Simulation of hydraulic event of water hammer in multi-diameter water transformation pipes with different composition, National Conferences of Water Managers, Shahid Chamran University, Ahvaz, Iran.
4. Daneshfaraz, R., Barzegar, T., Derakhshan, M., 2009. Advanced numerical methods for solving fluids engineering problems, Amidi Press, Tabriz, Iran, p. 244.
5. The website of Southern Tehran Mechanic Society.

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