A Method for Defuzzification Based On Central Interval and its Application in Decision Making

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Abstract: In this paper, a new method to rank fuzzy numbers by central interval is proposed. The central interval can be used as a crisp set approximation with respect to a fuzzy quantity. Therefore, a method for ordering the fuzzy numbers is defined accordingly. This method can effectively rank various fuzzy numbers, their images and overcome the shortcomings of the previous techniques. The proposed model is studied for a broad class for fuzzy numbers. The calculation of this method is far simpler than the other approaches. Finally, this study compares the proposed definition with some of the known ones.
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1. Introduction

Since Jain, Dubis and Prade (Dubois and Prade, 1987) introduced the relevant concepts of fuzzy numbers, many researchers have proposed the related methods or applications for ranking fuzzy numbers. For instance, Bortolan and Degani (Bortlan et al., 1985) reviewed some methods to rank fuzzy numbers in 1985, Chen and Hwang (Chen et al., 1992) proposed fuzzy multiple attribute decision making in 1992, Choobineh and Li (Choobineh et al., 1993) proposed an index for ordering fuzzy numbers in 1993, Dias (Dias, 1993) ranked alternatives by ordering fuzzy numbers in 1993, Lee (Lee et al., 1998) ranked fuzzy numbers with a satisfaction function in 1998, Requena utilized artificial neural networks for the automatic ranking of fuzzy numbers in 1994, Fortemps (Fortemps et al., 1996) presented ranking and defuzzification methods based on area compensation in 1996, and Raj (Raj et al., 1999) investigated maximizing and minimizing sets to rank fuzzy alternatives with fuzzy weights in 1999. In recent years, many methods are proposed for ranking different types of fuzzy numbers (Abbasbandy and Asady (Abbasbandy et al., 2006), Abbasbandy and Hajjary (Abbasbandy et al., 2009), Asady and Zendehnam (Asady et al., 2007), Allahviranloo et al. ((Allahviranloo et al., 2011), Saneifard (Saneifard et al., 2007; Saneifard et al., 2010), Wang and Kerre (Wang et al., 2001), and can be classified into four major classes: preference relation, fuzzy mean, and spread fuzzy scoring, and linguistic expression. But each method appears to have advantages as well as disadvantages. Having reviewed the previous methods, this article proposes a novel method to find the order of fuzzy numbers. Representing fuzzy numbers by proper intervals is an interesting and important problem. Besides, an interval representation of a fuzzy number may have many useful applications. By using such a representation, it is possible to apply approaches in fuzzy numbers

from which some results derived in the field of interval number analysis. Many authors (Carlsson et al., 2002; Grzegorzewski, 2002; Chakrabarty et al., 1998;Bodjanova, 2005) have studied the crisp set approximation of fuzzy sets. They proposed a rough theoretic definition of that crisp approximation which is the nearest interval approximation and the nearest ordinary set of a fuzzy set. Based on the reasons mentioned above, this article proposes a conceptual procedure and a method to use the concepts of interval value, median interval and central interval in order to find the order of fuzzy numbers. The advantage of this method is that can distinguish the alternatives clearly. The main purpose of this article is that the central interval can be used as a crisp set approximation of a fuzzy number, in which the researchers obtain a crisp set approximation with respect to a fuzzy quantity, and then define a method for ordering of fuzzy numbers. Therefore, by means of this defuzzification, this article aims to present a novel method for ranking of fuzzy numbers.

The paper is organized as follows: In Section 2, some fundamental results on fuzzy numbers are recalled. In Section 3, a crisp approximation of a fuzzy number is obtained. The proposed method for ranking fuzzy numbers is mentioned in Section 4. In this Section some theorems and remarks are proposed and illustrated. Discussion and comparison of this work and other methods are carried out in Section 5. The paper ends with conclusions in Section 6.

2. Basic Definitions and Notations

The basic definitions of a fuzzy number are given in (Heilpern, 1992;Kauffman et al., 1991;Zimmerman, 1991;Zadeh, 1972) as follows.

Definition 2.1. A fuzzy number A is a mapping $\mu_A(x): \Re \to [0,1]$ with the following properties:

1. μ_A is an upper semi-continuous function on $\Re$.
2. $\mu_A(x) = 0$ outside of some interval $[a_1, b_2] \subset \Re$.
3. There are real numbers a_1, a_2, b_1 and b_2 such that $a_1 \le a_2 \le b_1 \le b_2$ and

3.1 $\mu_A(x)$ is a monotonic increasing function on $[a_1, a_2]$,

3.2 $\mu_A(x)$ is a monotonic decreasing function on $[b_1, b_2]$,

3.3 $\mu_A(x) = 1$ for all x in $[a_2, b_1]$.

The set of all fuzzy numbers is denoted by F.

We assume a fuzzy number A that can be expressed for all $x \in \Re$ in the form

$$A(x) = \begin{cases} g(x) & when \ \ x \in [a,b), \\ w & when \ \ x \in [b,c], \\ h(x) & when \ \ x \in (c,d], \\ 0 & otherwise. \end{cases} \quad (1)$$

Where a, b, c, d are real numbers such that $a < b \le c < d$ and g and h are real valued functions such that g is increasing and right continuous and h is decreasing and left continuous. Based on the basic theories of fuzzy numbers, A is a normal fuzzy number if $w = 1$, whereas A is a non-normal fuzzy number if $0 < w \le 1$. Notice that (1) is an LR fuzzy number. A normal fuzzy number A with shape function g and h defied by

$$g(x) = \left(\frac{x-a}{b-a}\right)^n, \quad (2)$$

and

$$h(x) = \left(\frac{d-x}{d-c}\right)^n, \quad (3)$$

respectively, where $n > 0$, will be denoted by $A = \langle a,b,c,d \rangle_n$. If A be non-normal fuzzy number, it will be denoted by $A = \langle a,b,c,d;w \rangle_n$. If $n = 1$, we simply write $A = \langle a,b,c,d \rangle$, which is known as a normal trapezoidal fuzzy number and if $b = c$, is known as a normal triangular fuzzy number and represented by $A = \langle a,b,d \rangle$. If $n \ne 1$, a fuzzy number $A^* = \langle a,b,c,d \rangle_n$ is a concentration of A. If $0 < n < 1$, then A^* is a dilation of A. Concentration of A by $n = 2$ is often interpreted as the linguistic hedge "very". Dilation of A by $n = 0.5$ is often interpreted as the linguistic hedge "more or less". More about linguistic hedges can be found in (Cheng et al., 2002).

Each fuzzy number A described by (1) has the followinga-level sets (α -cuts) $A_\alpha = [a_\alpha, b_\alpha]$, $a_\alpha, b_\alpha \in \Re, \alpha \in [0,1]$,

1. $A_\alpha = [g^{-1}(\alpha), h^{-1}(\alpha)]$ for all $\alpha \in (0,1)$.
2. $A_1 = [b,c]$.
3. $A_0 = [a,d]$.

If $A = \langle a,b,c,d \rangle_n$ then for all $\alpha \in [0,1]$,

$$A_\alpha = \left[a + \alpha^{\frac{1}{n}}(b-a), d - \alpha^{\frac{1}{n}}(d-c) \right]. \quad (4)$$

Another important notion connected with fuzzy number A is a cardinality of a fuzzy number A. In this paper, we will always refer to fuzzy number A described by (1).

2.1. The measure of interval number

The measure of interval is given first which is different from the measure of traditional interval number, such as the length of interval number.

Generally, interval number is denoted as $A(a_1, a_2) = [a_1, a_2]$, where a_1 and a_2 are respectively called left end point and right end point, $a_1 \le a_2$. Particularly, if $a_1 = a_2$, $A(a_1, a_2)$ denotes real number a1. Let, $A(a_1, a_2)$ and $B(b_1, b_2)$ are arbitrary interval numbers, herein $A(a_1, a_2) = B(b_1, b_2)$ if and only if $a_1 = b_1$ and $a_2 = b_2$.

Definition 2.2. (Yang et al., 2002). Let, $A(a_1, a_2)$ is arbitrary interval number. The measure of interval number A define as follows:

$$M_I(A) = sign(a_1).|a_1 . a_2|. \quad (5)$$

Note that, the geometric meaning of the measure that we defined here is monotone function of a triangle area which is constituted be segment $l(a_1, a_2)$ and two axes (Figure 1). The meaning of symbol function is that we can compare the size between two interval numbers when the end point of interval numbers is negative numbers. Let us introduce some definition which this article need in the following Section.

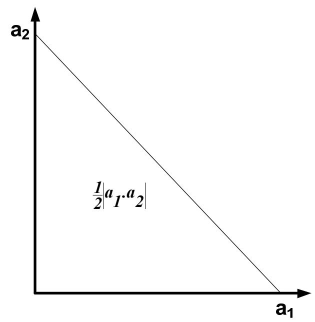

Figure 1. Geometric presentation of Definition 2.2

3. Median Interval and Central value

Various authors have studied the mean interval of a fuzzy number, also called the interval-valued mean (Carlsson et al., 2002;Dubois et al., 1987). According to Dubios and Prade (Dubois et al., 1987), the interval-valued probabilistic mean of a fuzzy number A with α -cuts $A_\alpha = [a_\alpha, b_\alpha]$, $\alpha \in [0,1]$ is the interval $E(A) = [E_*(A), E^*(A)]$, where

$$E_*(A) = \int_0^1 a_\alpha d\alpha \text{ and } E^*(A) = \int_0^1 b_\alpha d\alpha. \quad (6)$$

Carlsson (Carlsson et al., 2002) introduced the interval-valued possibilistic mean of a fuzzy number A as the interval $M(A) = [M_*(A), M^*(A)]$. The lower possibilistic mean $M_*(A)$, is the weighted average of the minima of the α -cuts of A. Similarly, the upper possibilistic mean $M^*(A)$ is the weighted average of the maxima of the α -cuts of A. If A is a fuzzy number characterized by (1), then

$$M_*(A) = \int_0^1 \alpha a_\alpha d\alpha \text{ and } M^*(A) = \int_0^1 \alpha b_\alpha d\alpha. \quad (7)$$

They also proved that if A is a fuzzy number of LR type with strictly monotonous and continuous shape functions, then $M(A) \subset E(A)$. This reflects on the fact that real numbers with small membership grades in A are considered to be less important in the definition of lower and upper possibilistic mean value in the definition of probabilistic ones. We will introduce the median interval (interval-valued median) of A similarly to the definition of $E(A)$ and $M(A)$.

Definition 3.1. (Bodjanova, 2005). Let A be a fuzzy number characterized by (1). Let $m_L \in (a,b)$ and $m_R \in (c,d)$ be such that

$$\int_a^{m_L} A(x)dx = \int_{m_L}^b A(x)dx,$$

and

$$\int_c^{m_R} A(x)dx = \int_{m_R}^d A(x)dx,$$

respectively. Then $M_e(A) = [m_L, m_R]$ is called the median interval (interval-valued median) of A. For a trapezoidal fuzzy number A and for its modifications by selected linguistic hedges we will provide formulas for the location of the median interval.

Proposition 3.1. (Bodjanova, 2005). Let $A = \langle a,b,c,d \rangle_n$. Then $M_e(A) = [m_L, m_R]$, where

$$m_L = a + \frac{b-a}{\sqrt[n+1]{2}}$$

and

$$m_R = d - \frac{d-c}{\sqrt[n+1]{2}}.$$

Corollary 3.1. (Bodjanova, 2005). Let $A = \langle a,b,c,d \rangle_n$. Then $M_e(A) = A_\alpha$, where $\alpha = 2^{\frac{-n}{n+1}}$ and $\alpha \in (0.5,1)$. The median value m_A is always in the median interval $M_e(A)$.

Corollary 3.2. (Bodjanova, 2005). Let $A = \langle a,b,c,d \rangle_n$ be trapezoidal fuzzy number. Then $M_e(A) = A_\alpha$, where $\alpha = 0.707$. In general, if A can not be expressed in the form ha;b;c;din, then $M_e(A)$ is not an α -cut of A.

Proposition 3.2. (Bodjanova, 2005). Let $A = \langle a,b,c,d \rangle_n$. Then $M_e(A) \subset M(A) \subset E(A)$ if $0 < n \le 1.72$, and $M(A) \subset M_e(A) \subset E(A)$ if $n \ge 1.73$.

Corollary 3.3. (Bodjanova, 2005). Let A be a trapezoidal fuzzy number. Then $M_e(A) \subset M(A) \subset E(A)$.

Example 3.1. Let $A = \langle 0,10,11,12 \rangle_n$ be a trapezoidal fuzzy number. Then $E(A) = [5,11.5] = A_\alpha$, where $\alpha = 0.5$. Similarly $M(A) = [6.66,11.33] = A_\alpha$ where $\alpha = 0.6$ and $M_e(A) = [7.07,11.29] = A_\alpha$, where $\alpha = 0.707$. Therefore $M(A) \subset M_e(A) \subset E(A)$.

4. Comparison of fuzzy numbers using a central interval

In this section, the article will propose the ranking of fuzzy numbers associated with the central interval.

Definition 4.1. (Bodjanova, 2005). Let A be a fuzzy number characterized by (1) and $E(A)$, $M(A)$ and $M_e(A)$ be the interval-valued probabilistic mean, interval-valued possibilistic mean and the interval-valued median of A, respectively. Then the interval $C(A)=E(A)\cap M(A)\cap M_e(A)$ is called the central interval of A. The central interval $C(A)=[C_*(A),C^*(A)]$ has the lower bound

$$C_*(A)=\max\{E_*(A),M_*(A),m_L(A)\},$$

and the upper bound

$$C^*(A)=\max\{E^*(A),M^*(A),m_R(A)\}.$$

Also, $core(A)\subset C(A)\subset Supp(A)$. If $E(A)$, $M(A)$ and $M_e(A)$ are associated with α -cuts of A, then $C(A)$ is equal to one of them. If A is a fuzzy number of LR type with strictly monotonous and continuous shape functions, then $C(A)=M_e(A)\cap M(A)$. From Proposition 3.2 it follows that if $A=\langle a,b,c,d\rangle_n$, then for $0<n\le 1.72$, $C(A)=M_e(A)$ and for $n\ge 1.73$, there is $C(A)=M(A)$.

Each central interval can be used as a crisp approximation of a fuzzy number, therefore, the resulting interval is used to rank the fuzzy numbers. Thus, $C(A)$ is used to rank fuzzy numbers.

Definition 4.2. Let A be an arbitrary fuzzy number and $C(A)=[C_*(A),C^*(A)]$ be its central interval. According to definition 2.2, the measure of $C(A)$ which is an interval number is as $M_I(C(A))=sign(C_*(A)).\left|C_*(A).C^*(A)\right|$. We define the measure of fuzzy number A as follows:

$$M_C(A)=\int_0^1 p(\alpha)M_I(C(A))d\alpha. \quad (8)$$

The function $p:[0,1]\to[0,+\infty)$ denotes the distribution density of the importance of the degrees of fuzziness, where $\int_0^1 p(\alpha)d\alpha=1$. In particular cases, it may be assumed that

$$p(\alpha)=(k+1)\alpha^k, k=0,1,\ldots.$$

Throughout this study the researchers assumed that $k=1$, i.e. $p(\alpha)=2\alpha$. Obviously, if a fuzzy number becomes interval numbers, then $M_C(A)$ will be the measure of the interval number which can be denoted as $M_I(A)$. For a certain fuzzy numbers, we can obtain $M_C(A)$ by definite integral. But it is not easy to compute definite integral sometimes. For trapezoid fuzzy numbers and triangular fuzzy numbers, the calculation formulas for the indices are given in the paper.

Proposition 4.1. If $A=\langle a,b,c,d\rangle$ is a trapezoidal fuzzy number, the measure $M_C(A)$ can be denoted as follows:

$$M_C(A)=\frac{B}{2}+\frac{2C}{3}+D, \quad (9)$$

where $B=(b-a)(c-d)$, $C=db-2ad+ac$ and $D=ad$.

Since every measure can be used as a crisp approximation of a fuzzy number, therefore, the resulting value is used to rank the fuzzy numbers. Thus, $M_C(A)$ is used to rank fuzzy numbers.

Let A and B be two arbitrary fuzzy numbers, and $M_C(A)$ and $M_C(B)$ be the measures of A and B, respectively. Define the ranking of A and B by $M_C(.)$ on F, i.e.

(1) $M_C(A)=M_C(B)$ if only if $A\sim B$,

(2) $M_C(A)<M_C(B)$ if only if $A\prec B$,

(3) $M_C(A)>M_C(B)$ if only if $A\succ B$.

Then, this article formulates the order $\succeq$ and $\preceq$ as $A\succeq B$ if and only if $A\succ B$ or $A\sim B$, $A\preceq B$ if and only if $A\prec B$ or $A\sim B$.

Proposition 4.2. Let $A=(a_1,a_2,a_3,a_4)$ and $B=(b_1,b_2,b_3,b_4)$ are two fuzzy numbers

1. If $a_1\le b_1, a_2\le b_2, a_3\le b_3$ and $a_4\le b_4$, then $A\preceq B$,
2. If $a_1=b_1, a_2=b_2, a_3\le b_3$ and $a_4\le b_4$, then $A\preceq B$,
3. If $a_1=b_1, a_2=b_2, a_3=b_3$ and $a_4\le b_4$, then $A\preceq B$.

Proof: (1) Let $A_\alpha=[a_{1\alpha},b_{1\alpha}]$ and $B_\alpha=[a_{2\alpha},b_{2\alpha}]$ are α -cuts of their. If $a_1\le b_1$ and $a_2\le b_2$ then $a_{1\alpha}\le a_{2\alpha}$ and if $a_3\le b_3$ and $a_4\le b_4$ then $b_{1\alpha}\le b_{2\alpha}$. Since, A_α and B_α are two interval numbers, so $M_I(A_\alpha)\le M_I(B_\alpha)$ and $M_I(C(A))\le M_I(C(B))$ thus $M_C(A)\le M_C(B)$. That is $A\preceq B$. Similarly, we can prove (2) and (3).

Remark 4.1. If $A\preceq B$, then $-A\succeq -B$.

Hence, this article can infer ranking order of the images of the fuzzy numbers.

5. Numerical Examples

In this Section, this study compare proposed method with others in (Wang et al., 2001;Chu et al., 2002;Chen, 1985).

Example 5.1. Consider the following sets, see Yao and Wu (Yao et al., 2000).

Set1: $A=(0.4,0.5,1)$, $B=(0.4,0.7,1)$, $C=(0.4,0.9,1)$.

Set2: $A=(0.3,0.4,0.7,0.9)$ (trapezoidal fuzzy number), $B=(0.3,0.7,0.9)$, $C=(0.5,0.7,0.9)$.

Set3: $A=(0.3,0.5,0.7)$, $B=(0.3,0.5,0.8,0.9)$ (trapezoidal fuzzy number), $C=(0.3,0.5,0.9)$.

Set 4: $A=(0.3,0.4,0.8,0.9)$ (trapezoidal fuzzy number), $B=(0.2,0.5,0.9)$, $C=(0.1,0.6,0.8)$.

To compare with other methods, researchers refer the reader to Table (1). Note that, in Table (1) and in set 4, for Sign Distance(p=1), Distance Minimization, Chu-Tsao and Yao-Wu methods, the ranking order for fuzzy numbers B and C is $B \sim C$, which seems unreasonable regarding the figures.

Table 1. Comparative results of Example 5.1.

Authors	Fuzzy numbers	Set1	Set2	Set3	Set4
Proposed method	A	0.3	0.28	0.24	0.36
	B	0.47	0.43	0.36	0.24
	C	0.68	0.48	0.27	0.28
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \prec C$
Sing Distance method with p=1	A	1.20	1.15	1.0	0.09
	B	1.40	1.30	1.25	1.05
	C	1.60	1.40	1.10	1.05
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \sim C$
Sing Distance method with p=2	A	0.88	0.87	0.72	0.78
	B	1.01	0.95	0.94	0.79
	C	1.16	1.00	0.81	0.83
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \prec C$
Distance Minimization	A	0.6	0.57	0.5	0.47
	B	0.7	0.65	0.62	0.52
	C	0.9	0.7	0.55	0.52
Result		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \sim C$
Abbasbandy and Hajjari	A	0.53	0.55	0.50	0.52
	B	0.70	0.63	0.64	0.50
	C	0.86	0.70	0.51	0.57
Result		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$B \prec A \prec C$
Choobineh and Li	A	0.33	0.54	0.33	0.50
	B	0.50	0.58	0.41	0.58
	C	0.66	0.66	0.54	0.61
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Chu and Tsao	A	0.29	0.28	0.25	0.24
	B	0.35	0.32	0.31	0.262
	C	0.39	0.35	0.27	0.261
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec C \prec B$
Yao and Wu	A	0.60	0.57	0.50	0.47
	B	0.70	0.65	0.62	0.52
	C	0.80	0.70	0.55	0.52
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \sim C$
Cheng CV uniform distribution	A	0.027	0.03	0.01	0.06
	B	0.021	0.02	0.03	0.03
	C	0.022	0.00	0.02	0.04
Results		$A \prec C \prec B$	$A \prec B \prec C$	$B \prec C \prec A$	$A \prec C \prec B$
Cheng CV proportional distribution	A	0.018	0.02	0.00	0.04
	B	0.012	0.01	0.02	0.023
	C	0.013	0.00	0.01	0.025
Results		$A \prec C \prec B$	$A \prec B \prec C$	$B \prec C \prec A$	$A \prec C \prec B$

Example 5.2. Consider the three fuzzy numbers $A=(1,2,5)$, $B=(0,3,4)$ and $C=(2,2.5,3)$. By using this new approach $M_C(A)=6.2$, $M_C(B)=6.5$ and $M_C(C)=4.83$. Hence, the ranking order is $C \prec A \prec B$ too. To compare with some of the other

methods in (Abbasbandy et al., 2006;Asady et al., 2007;Chu et al., 2002), the reader can refer to Table 2.

Table (2). Comparative results of Example 5.2.

Fuzzy number	New approach	Sign Distance with p=1	Sign Distance with p=2	Distance Minimization	Chu and Tsao
A	6.2	3	2.16	2.50	0.74
B	6.5	3	2.70	2.50	0.74
C	4.8	3	2.70	2.50	0.75
Results	$C \prec A \prec B$	$C \sim A \sim B$	$C \prec A \sim B$	$C \sim A \sim B$	$A \sim B \prec C$

All the above examples show that this method is more consistent with institution than the previous ranking methods. This method can overcome the shortcomings of other methods.

5.1. Using The Proposed Ranking Method In Selecting Army Equip System

From experimental results, the proposed method with some advantages: (a) without normalizing process, (b) fit all kind of ranking fuzzy number, (c) correct Kerre's concept. Therefore we can apply measure of fuzzy ranking method in practical examples. In the following, the algorithm of selecting equip systems is proposed, and then adopted to ranking a army example.

5.1.1. An algorithm for selecting equip system

We summarize the algorithm for evaluating equip system as below:

Step 1: Construct a hierarchical structure model for equip system.

Step 2: Build a fuzzy performance matrix $\widetilde{A}$. We compute the performance score of the subfactor, which is represented by triangular fuzzy numbers based on expert's ratings, average all the scores corresponding to its criteria. Then, build a fuzzy performance matrix $\widetilde{A}$.

Step 3: Build a fuzzy weighting matrix $\widetilde{W}$. According to the attributes of the equip systems, experts give the weight for each criterion by fuzzy numbers, and then form a fuzzy weighting matrix $\widetilde{W}$.

Step 4: Aggregate evaluation. To multiple fuzzy performance matrix and fuzzy weighting matrix $\widetilde{W}$, then get fuzzy aggregative evaluation matrix $\widetilde{R}$. (i.e. $\widetilde{R} = \widetilde{A} \otimes \widetilde{W}$).

Step 5: Determinate the best alternative. After step 4, we can get the fuzzy aggregative performance for each alternative, and then rank fuzzy numbers by measure of fuzzy numbers.

5.1.2. The selecting of best main battle tank

In (Cheng et al., 2002), the authors have constructed a practical example for evaluating the best main battle tank, and they selected $x_1 = M_1A_1$ (USA), $x_2 = Challenger2$ (UK), $x_3 = Leopard2$ (Germany) as alternatives. In (Cheng et al., 2002), the experts opinion were described by linguistic terms, which can be repressed in triangular fuzzy numbers. The fuzzy Delphi method is adopted to adjust the fuzzy rating of each expert to achieve the consensus condition. The evaluating criteria of main battle tank are $a_1 : attackcapability$, $a_2 : mobilitycapability$, $a_3 : self\text{-}defencecapability$ and $a_4 : communication\,and\,controlcapability$. In this example, we adopted the hierarchical structure constructed in (Cheng et al., 2002) for selection of five main battle Tanks as shown in Fig. 2, and the step-by-step illustrations based on Sec. 5.1.1s algorithm are described bellow:

Step 1: Construct a hierarchical structure model for equip system (the result is shown in Fig. 2).

Step 2: Build a fuzzy performance matrix $\widetilde{A}$. The basic performance data for five types of main battle tanks are summarized in Table 4. Then based on the linguistic values in Table 3, the fuzzy preference of five tanks toward four criteria are collected and shown in Table 7.

Step 3: Build a fuzzy weighting matrix $\widetilde{W}$. The aggregative fuzzy weights of four criteria, according to the linguistic values of importance in Table 5, are shown in Table 6.

Step 4: Aggregate evaluation. To multiple fuzzy performance matrix $\widetilde{A}$ and fuzzy weighting matrix $\widetilde{W}$, then get fuzzy aggregative evaluation matrix $\widetilde{R} = \widetilde{A} \otimes \widetilde{W}^t$. Therefore, from Table 7 and 6, we have

$$\widetilde{R} = \begin{bmatrix} (0.72.0.89,1.0) & (0.67,0.83,1.00) & (0.50,0.75,0.83) & (0.58,0.75,0.92) \\ (0.39,0.56,0.7) & (0.42,0.58,0.75) & (0.50,0.67,0.83) & (0.58,0.75,0.92) \\ (0.61,0.78,0.8) & (0.71,0.88,0.96) & (0.42,0.58,0.75) & (0.58,0.75,0.92) \\ (0.22,0.33,0.5) & (0.29,0.46,0.63) & (0.33,0.50,0.67) & (0.42,0.58,0.75) \\ (0.67,0.83,0.9) & (0.75,0.92,1.00) & (0.58,0.75,0.93) & (0.67,0.83,1.00) \end{bmatrix}$$

$$\otimes \begin{bmatrix} (0.72,0.89,1.0) \\ (0.78,0.94,1.0) \\ (0.56,0.72,0.8) \\ (0.33,0.5,0.67) \end{bmatrix} = \begin{bmatrix} (0.37,0.60,0.82) \\ (0.27,0.47,0.69) \\ (0.35,0.57,0.77) \\ (0.17,0.34,0.54) \\ (0.40,0.64,0.84) \end{bmatrix}$$

Step 5: Determinate the best alternative. According to Eq. 8, we can get the measure value of fuzzy numbers of Tanks A-E, which are equal to 0.558, 0.420, 0.522, 0.211 and 0.590, respectively. Therefore, we find that the ordering of measure is Tank D < Tank B < Tank C < Tank A < Tank F. So, the best type of main battle Tank is Tank F.

Table 3. Linguistic values for the ratings

Linguistic values	TFNs
Very Poor (VP)	(0,0,0.16)
Poor	(0,0.16,0.33)
Slightly(SP)	(0.16,0.33,0.5)
Fair(F)	(0.33,0.5,0.66)
Slightly good(SG)	(0.5,0.66,0.83)
Good	(0.66,0.83,1)
Very good	(0.83,1,1)

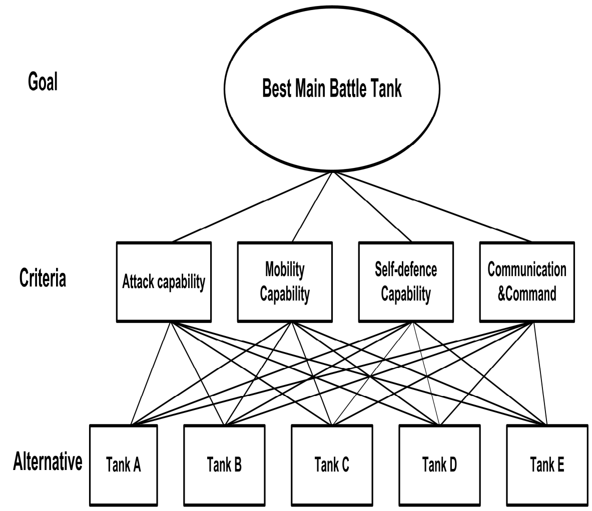

Figure 2. Hierarchical structure model for evaluating the best main battle Tank

Table 4. Basic Performance data for five types of main battle tanks

Item	Type				
	Tank A	Tank B	Tank C	Tank D	Tank E
Armament	120 mm gun	120 mm gun	120 mm gun	105 mm gun	120 mm gun
	15.2 mm MG	15.2 mm MG	15.2 mm MG	15.2 mm MG	7.62 mm MG
	12.7 mm MG				12.7 mm MG
Ammunition	40	Up to 50	42	40	44
	1000	4000	4750	4	1500
	11400				10000
Smoke grenade Discharges	2×6	2×5	2×8	None	2×9
Power to weigh Ratio(hp/t)	26.2	19.2	27.2	19.0	27.5
Max.road Speed	67	56	72	60	71
Max.range(Km)	480	450	550	300	550
Fording(m)	1.21	1.07	1.0	1.2	1.23
Gradient	60	60	60	55	60
Trench	2.74	2.43	3.00	2.51	2.92
Armor protection	Good	Excellent	Good	Fair	Excellent
Acclimatization	Good	Fair	Good	Fair	Good
Communication	Fair	Fair	Fair	Poor	Fair
Scout	Medium	Medium	Medium	Medium	Good

Table 5. Linguistic values of the importance weights

Linguistic values	TFNs
Very Low (VL)	(0.0,0.00,0.167)
Low	(0.0,0.167,0.33)
Slightly(SP)	(0.167,0.33,0.5)
Medium(M)	(0.33,0.5,0.667)
Slightly High(SH)	(0.5,0.667,0.83)

High(H)	(0.667,0.833,1.)
Very High(VH)	(0.833,1.00,1.0)

Table 6. The importance weights of linguistic criteria and its mean

Criteria	Expert			Mean of TFNs
	D_1	D_2	D_3	
Attack($\tilde{W}_1$)	VH	H	H	(0.72,0.89,1.00)
Mobility($\tilde{W}_2$)	VH	H	VH	(0.78,0.94,1.00)
Self-defence($\tilde{W}_3$)	M	VH	SH	(0.56,0.72,0.83)
Communication	M	M	M	(0.33,0.50,0.67)

Table 7. Basic performance data for five types of main battle tanks

Criteria	Type				
	Tank A	Tank B	Tank C	Tank D	Tank E
Attack					
Armament	G	SG	SG	F	SG
Ammunition	VG	SG	SG	F	G
Smoke grenade	G	SP	VG	VP	VG
Mean	(0.7,0.8,1)	(0.3,0.5,0.7)	(0.6,0.7,0.8)	(0.2,0.3,0.5)	(0.6,0.8,0.9)
Mobility					
Power to weight	G	F	G	F	G
Max. road speed	G	F	VG	SG	VG
Max. range	G	SG	VG	P	VG
Fording/Gradient	G	SG	SG	F	G
Mean	(0.67,0.83,1)	(0.42,0.58,0.75)	(0.71,0.88,0.96)	(0.29,0.46,0.63)	(0.75,0.92,1)
Self-defence					
Armor Prot.	SG	G	F	F	G
Acclimatization	SG	F	SG	F	G
Mean	(0.5,0.67,0.83)	(0.5,0.67,0.83)	(0.42,0.58,0.75)	(0.33,0.5,0.67)	(0.58,0.75,0.92)
Communication					
Communication	G	G	G	F	G
Scout	SG	SG	SG	SG	G
Mean	(0.5,0.7,0.92)	(0.58,0.75,0.92)	(0.58,0.75,0.92)	(0.42,0.58,0.75)	(0.67,0.83,1)

6. Conclusion

The existing ranking fuzzy numbers methods all have their advantages and some shortcomings. They may valuable in solving some types of fuzzy numbers (i.e. normal, non-normal, positive, and negative fuzzy numbers). For above reasons, this study proposes a new results of ranking reversal. Roughly, there is not much difference in our method and theirs. In this paper, the researchers proposed a central interval method to rank fuzzy numbers. The method can effectively rank various fuzzy numbers and their images (normal/non-normal/trapezoidal/general). The calculations of this method are simpler than the other approaches.

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