Temperature distribution in porous fins in natural convection condition

Seyfolah Saedodin [1], Mehdi Olank [2]

[1.] Assistant professor, Department of Mechanical Engineering, Science and Research Branch, Islamic Azad University, Semnan, Iran, s_sadodin@iust.ac.ir
[2.] MS.C student, Department of Mechanical Engineering, Science and Research Branch, Islamic Azad University, Semnan, Iran, mehdi.olank@gmail.com

Abstract: This paper investigates the temperature distribution in porous fins in natural convection condition and compares it with temperature distribution in conventional fins. To formulate the heat transfer equation, the energy balance and Darcy's model used. This study is based on finite-length fin with insulated tip. The porous fin allows the flow infiltrate through it. The theory section addressed the derived governing equation. The effect on porosity parameters "S" and convection parameter in porous fin "n", and convection parameter in conventional fin are discussed. The result suggests that by increasing "S" the heat transfer increase but in some cases it has exemption.
[Seyfolah saedodin, Mehdi olank. Temperature distribution in porous fins in natural convection condition. Journal of American Science 2011;7(6):476-481]. (ISSN: 1545-1003). http://www.americanscience.org.

Key words: porous fin, Darcy's model, temperature distribution, heat transfer, natural convection

Nomenclature

c_p : Specific heat
Da: Darcy number, $\frac{k}{t^2}$
G: gravity constant
Gr :Grashoff $\frac{g\beta(\theta_b - \theta_\infty)}{\upsilon^2}$
k_r : Thermal conductivity ratio, k_{eff}/k_f
K: permeability of the porous fin
L: fin length
Nu: Nusselt number, hL/k_f
Pr: Prandtl number, $\frac{\upsilon}{\alpha}$
q :Heat transfer rate
Ra: Rayleigh number, Gr Pr
θ (x) :Temperature at any point
θ_b : Temperature at the fin base
$v_w(x)$:velocity of the fluid passing through the fin at any point
R: Radius of the fin
X: Axial coordinate
P: perimeter of the fin

Symbols

α: Thermal diffusivity
β: Coefficient of volumetric thermal expansion
Δ: Temperature difference
ε :Porosity or void ratio
T: Dimensionless temperature, $T = \frac{\theta(x) - \theta_\infty}{\theta_b - \theta_\infty}$
θ_b : Base temperature difference, $(\theta_b - \theta_\infty)$
μ :Dynamic viscosity
υ :Kinematics viscosity
ρ :Density

Subscripts

s :Solid properties
f :Fluid properties
eff :Porous properties

1. Introduction

In the design of heat exchanger, finned surfaces are often employed to improve performance. In the other hand, for many years reduction of the size and cost of fins are the main targets of fin industries. Some engineering applications also require lighter fin with higher rate of heat transfer where they use high thermal conductivity metals in applications such as airplane and motorcycle applications. However, cost of high thermal conductivity metals is also high. Thus, the enhancement of heat transfer can be achieved by increasing the heat transfer rate and decreasing the size and cost of fin. The major heat transfer from surface to surrounding fluid takes place by convection process. Increasing the heat transfer mainly depend on heat transfer coefficient (h), surface area available and the temperature difference between surface and surrounding fluid.

Fins are frequently used in many heat transfer applications. Meyer [1] in his famous book with a simple manner describes the temperature distribution in conventional fins. But in the recent years, porous fins consider as a potential field for increasing heat transfer. The basic philosophy behind using porous fins is to increase the effective area through which heat converted to ambient fluid.

Extensive research has been done in this area and many references are available especially for heat transfer in porous fins.

1.1Review of the literature

Kiwan [2] conducted thermal analysis of natural Convection porous fins. He grouped all the geometric and flow parameters that influence the temperature distribution in to one parameter called "s_h". Korla [3] investigate thermal analysis of natural convection and radiation in porous fins. Abu-Hijleh [4] investigates numerically the effect of using porous fins on the forced convection heat transfer from a horizontal cylinder. Yo and Chen [5] performed a study on optimization of circular fin with variable thermal parameter. Nguyen and Aziz [6] compare the heat transfer rates from convecting-radiating fins for different profile shapes. An Analysis for Y-shaped fins for determining fin efficiency by a new approach has been demonstrated by Lorenzini and Moretti[7]. Kundu and Bhanja [8] have determined analytically the performance and optimum dimensions of T-shape fin with variable thermal conductivity and convective heat transfer coefficient. Also Kundu and Bhanja [9] investigate an analytical prediction for performance and optimum design analysis of porous fin. Mobedi and Sunden [10] study the Natural convection heat transfer from a thermal heat source located in a vertical plate fin.

In this study it is intended to use the simple approach for finding the temperature distribution in porous fin and compare it with the temperature distribution in conventional fins. The cylindrical fins attached to a vertical surface, have been used.

2. Governing equations

As shown in Fig.1 a cylindrical fin profile is considered. The dimensional of the fin are length L and the radius R. the cross-sectional area of the fin is constant.

For the conventional fin, the defining equation obtain by making an energy balance on element of the fin of thickness Δx as shown in the Fig.1.

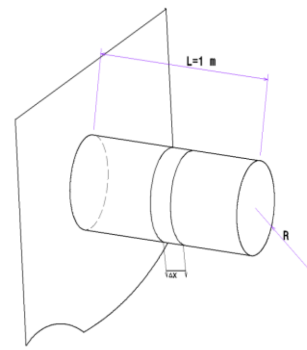

figure 1. Schematic diagram for the problem under consideration

Figure 1. The defining equation obtain by making an energy balance on element

Energy in left face=energy out right face+ energy lost by convection (1)

The defining equation for the energy lost by convection heat transfer is

$$q = hp\Delta x\left(\theta(x) - \theta_\infty\right) \quad (2)$$

$$Energy\ \textbf{in}\ left\ face = q_x = -kA\,\frac{d\theta}{dx} \quad (3)$$

$$Energy\ out\ right\ face = q_{x+\Delta x} = -kA\left(\frac{d\theta}{dx} + \frac{d^2\theta}{dx^2}\right) \quad (4)$$

By substitution the equations 2, 3, 4 in the equation 1, the energy balance yields

$$\frac{d^2\theta}{dx^2} - \frac{hp}{kA}\left(\theta(x) - \theta_\infty\right) = 0 \quad (5)$$

Let $\theta = \left(\theta(x) - \theta_\infty\right)$ the equation (5) becomes

$$\frac{d^2\theta}{dx^2} - \frac{hp\,\theta}{kA} = 0 \quad (6)$$

One boundary condition is

$\theta = \theta_0 = \theta_b - \theta_\infty$ at x=0 (7)

The other boundary condition is the end of fin is insulated so that

$\frac{d\theta}{dx} = 0$ at x=L (8)

If $m^2 = \frac{hp}{kA}$ the solution can be written (9)

$$\theta - \theta_\infty = \left(\theta_b - \theta_\infty\right) \frac{\cosh[m(L-x)]}{coshmL} \qquad (10)$$

If $\dfrac{\theta(x) - \theta_\infty}{\theta_b - \theta_\infty} = T$ the solution may be written (11)

$$T = \frac{\cosh[m(L-x)]}{coshmL} \qquad (12)$$

For the porous fin, Due to this fact that the fin being porous, It allow for the flow to infiltrate through it. In This study to simple the governing equations the following assumptions are considered: (1) length=1(m), the porous medium is homogeneous, isotropic, (2) both the fluid and the solid matrix have constant physical properties. (3) the surface radiant exchanges are ignored,(4) the temperature inside the fin is only function of x, (5) no temperature variation across the fin thickness, and (6) the interaction between the porous medium and the clear fluid can be simulated by the Darcy formulation(7).

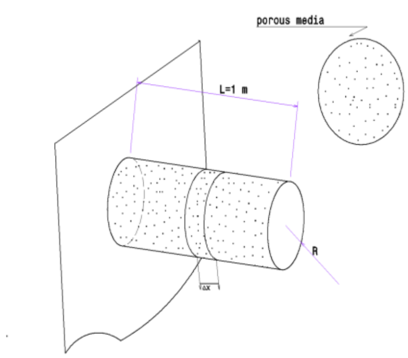

figure 2. Schematic diagram for the problem under consideration

Figure 2. The energy balance to the slice segment

The energy balance to the slice segment of the fin of thickness Δx, shown in Fig.2, requires that

$$q(x) - q(x+\Delta x) = \dot{m}c_p(\theta(x) - \theta_\infty) + h(p.\Delta x)\left(\theta(x) - \theta_\infty\right) \qquad (13)$$

In the right hand of equation, the second terms represent the natural convection around the fin and the first term represents the heat transfer lost to the fluid passing through the porous media. This fluid is induced by the buoyancy force created due to the temperature difference between the fin and the surroundings. It should be noted that Equation (13) assumes that the fluid enters the fin at θ_∞ and leaves at θ (x).

The mass flow rate of the fluid passing through the porous material can be written as,

$$\dot{m} = \rho v_w 2\pi R\Delta x \qquad (14)$$

The flow in the porous medium shall be considered next to account for the value of v_w . Referring to assumption (7) above, Darcy's model gives,

$$v_w = \frac{gk\beta}{\upsilon}\left(\theta(x) - \theta_\infty\right) \qquad (15)$$

By substitution of equations (14) and (15) into equation (13), the result is:

$$\frac{\left(q(x) + q(x+\Delta x)\right)}{\Delta x} = \frac{\rho c_p gk\beta 2\pi R}{\upsilon}\left(\theta(x) - \theta_\infty\right)^2 + hp\left(\theta(x) - \theta_\infty\right) \qquad (16)$$

If Δx $\longrightarrow$0, equation (16) transfer to

$$\frac{dq}{dx} = \frac{\rho c_p gk\beta 2\pi R}{\upsilon}\left(\theta(x) - \theta_\infty\right)^2 + hp\left(\theta(x) - \theta_\infty\right) \qquad (17)$$

According to Fourier's law of conduction,

$$q = -k_{eff} A \frac{d\theta}{dx} \qquad (18)$$

In this equation, A is the cross-sectional area of the fin ($A = \pi R^2$) and k_{eff} is the effective thermal conductivity of the porous fin.

Substitution Equation (18) in to Equation (17) yields,

$$\frac{d^2\theta}{dx^2} - \frac{2\rho c_p gk\beta}{R\upsilon c_p k_{eff}}\left(\theta(x) - \theta_\infty\right)^2 - \frac{hp}{k_{eff}A}\left(\theta(x) - \theta_\infty\right) = 0 \qquad (19)$$

By introducing the non-dimensional temperature function, $\dfrac{\theta(x) - \theta_\infty}{\theta_b - \theta_\infty} = T$ into equation (19) becomes

$$\frac{d^2T}{dx^2} - ST^2 - nT = 0 \qquad (20)$$

The constant, $S = \dfrac{2\rho c_p gk\beta}{R\upsilon c_p k_{eff}}$ and $n = \dfrac{hp}{k_{eff}A}$ input all the geometric and flow parameters that influence the solution of the problem into definite parameter. To solve this equation we need two boundary condition. One boundary condition here is that the temperature at the base of the fin is θ_b . Then

$$T(0) = 1 \qquad (21)$$

The second boundary condition depends on the condition of the fin at the tip. The case that considered in this study is finite-length fin with insulated tip.

2. Results

The governing equation is solved numerically with curve fitting method. This method is a well-known method and has been widely used in literature.

In the finite-length fin with insulated tip, the second boundary condition at x=L will be $\frac{dT}{dx}=0$.

In this study to simplify the governing equation, assumed that L=1 (m). For solving the governing equation, It has been used "Maple 14" and this is the codes of solving this equation in MAPLE 14.

```
> with(linalg) : with(CurveFitting) :
>

BVPsol := proc(S, n, k)  # k shows number of subintervals
  local bvp, sol, d, X, T, P;
  X := [seq(i/k, i = 0 ..k)];
  bvp := {diff(T(x), x, x) - S·(T(x))^2 - n·T(x) = 0, D(T)(1)
     T(0) = 1};
  sol := dsolve(bvp, numeric, output = array(X));
  d := eval(sol[2, 1]);
  T := col(d, 2);
  T := convert(T, list);
  P := PolynomialInterpolation(X, T, x);
  return X, T, P
end proc:
```

Some result of this solving is:

$a := BVPsol(1, 0.5, 5)$

$$\left[0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right], [1.00000000000000, 0.844829011352\ldots, 0.735552970947914, 0.662924310953779, 0.6213431362\ldots, 0.607799313129204], -0.0275497731897726\,x^5 + 0.150091023196619\,x^4 - 0.345217117806639\,x^3 + 0.742097610001791\,x^2 - 0.911622429072794\,x + 1.00000000000000$$

$a := BVPsol(10, 0.5, 5)$

$$\left[0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right], [1.00000000000000, 0.625843381743979, 0.433444784724519, 0.328888341361934, 0.275775922301351, 0.259412586136504], -0.932670580109573\,x^5 + 3.36319892649033\,x^4 - 5.05974870625561\,x^3 + 4.47804925940624\,x^2 - 2.58941631339488\,x + 1.00000000000000$$

$a := BVPsol(50, 0.5, 5)$

$$\left[0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right], [1.00000000000000, 0.399012751529879, 0.213794957240962, 0.136724326781819, 0.102698050806002, 0.0928305228070700], -5.11203849387947\,x^5 + 16.5396884722636\,x^4 - 21.1443853884917\,x^3 + 13.8660812573918\,x^2 - 5.05651532447720\,x + 1.00000000000000$$

$a := BVPsol(100, 0.5, 5)$

$$\left[0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right], [1.00000000000000, 0.301273595675838, 0.143179108974865, 0.0854082244018191, 0.0614332936463886, 0.0546543261016980], -8.43367212191453\,x^5 + 26.6012157851039\,x^4 - 32.6608767263614\,x^3 + 19.9181252409074\,x^2 - 6.37013785163360\,x + 1.00000000000000$$

$a := BVPsol(150, 0.5, 5)$

$$\left[0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right], [1.00000000000000, 0.248703251824705, 0.110313731783634, 0.0633994874838542, 0.0445807929007307, 0.0393314388690428], -10.6562163934608\,x^5 + 33.2408761819294\,x^4 - 40.0960006997073\,x^3 + 23.6902414077773\,x^2 - 7.13956905766954\,x + 1.00000000000000$$

The variation of the dimensionless temperature is shown in figures 3, 4, 5 and 6.

For drawing the result in figure, the "MATLAB FIGURE" has been used.

It is clear that, in the Equation (20) when the s=0, the Equation transfer to the conventional fin equation and the governing equation becomes as same as the equation (12). In this particular condition the constant $n = m^2$ .In the figures 3,4,5,6 for every figure the "n" is constant and "S" is variable.

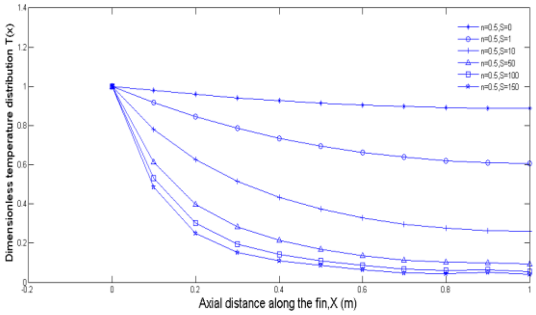

Figure 3. The distribution of the axial nondimensional temperature along the insulated fin with n=0.5 and for different value of S.

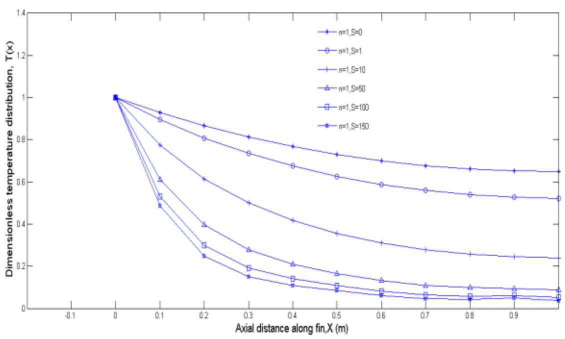

Figure 4. The distribution of the axial nondimensional temperature along the insulated fin with n=1 and for different value of S.

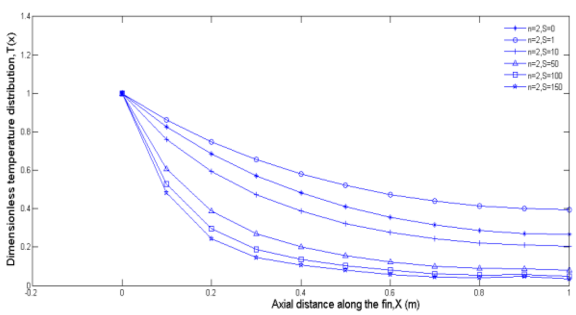

Figure 5. The distribution of the axial nondimensional temperature along the insulated fin with n=2 and for different value of S.

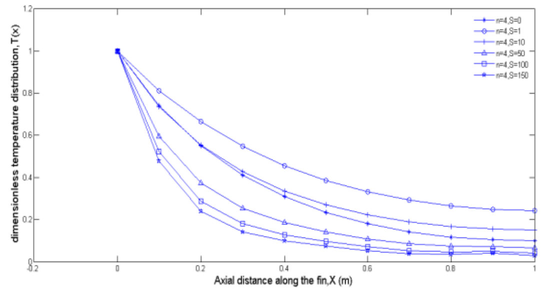

Figure 6. The distribution of the axial nondimensional temperature along the insulated fin with n=4 and for different value of S.

The figures 3, 4, 5 and 6 indicate that increasing the value of "S" will increase the heat transfer rate from the fin base. And it is clear from this figures that as the value of S increase the fin will attain the temperature of the surrounding fluid faster. Of course the figure 5 illustrate that the heat transferring in condition n=2, S=0 more than n=2, S=1. In fact in this condition, the conventional fin has more heat transfer than the porous media fin with S=1. Also the situation in figure 6 is same and the heat transferring in condition n=4, S=0 more than the both conditions n=4, S=1 and n=4, S=10.

3. Conclusion

Temperature distribution of porous fin for natural convection has been performed here. A second order non-linear ordinary differential equation has been derived as the governing equation for this problem. It has been solved using the curve fitting method. It is also found that all geometric and flow parameter influencing the temperature distribution has been grouped in three parameters called "m", "S" and "n". this temperature distribution analysis was performed for finite porous fin with insulated tip. It was found that increasing S, increase the heat transfer from fin. Of course in some condition the conventional fin has more heat transfer than porous fin.

Corresponding Author:
Mehdi Olank
MS.C student, Department of Mechanical Engineering, Science and Research Branch, Islamic Azad University, Semnan, Iran
mehdi.olank@gmail.com

References

1. E. Meyer, 1971. Analytical Methods in conduction Heat transfer, McGraw-Hill Book Company, New York

2. S. Kiwan,. 2006. Thermal analysis of natural convection in porous fins, Transport in porous media, 10.1007/s11242-006-0010-3.
3. R.S.R. Gorla, A.Y. Bakier. 2011. Thermal analysis of natural convection and radiation in porous fins, Int. Commun. Heat Mass Transf.doi:10.1016/j.icheatmasstransfer.2010.12.024.
4. Abu-Hijleh. 2003. Enhanced forced convection heat transfer from a cylinder using permeable fins. ASME J. Heat transfer 125,804-811.
5. L.T. Yu, C.K. Chen. 1999. Optimization of Circular Fins with Variable Thermal Parameters, J. Franklin Institute 336, 77-95.
6. A. Nguyen, A. Aziz. 1992. The heat transfer rate from convecting –radiating fins for different profile shapes, Heat and Mass -Transfer 27(2), 67-72.
7. G. Lorenzini, S. Moretti, 2007. Numerical analysis on heat removal from Y-shaped fins: efficiency and volume occupied for a new approach to performance optimization. Int.j. Thermal Science 46, 573-579.
8. B. Kundu, D. Bhanja, 2010. Performance and optimization analysis of a constructal T-shaped fin subject to variable thermal conductivity and convective heat transfer coefficient. Int.J. Heat Mass Transfer 53,254-267.
9. B. Kundu, D. Bhanja, 2010. An analytical prediction for performance and optimum design analysis of porous fins. Int.J. of refrigeration 34,337-352.
10. M. Mobedi, B. Sunden, 2006. Natural convection heat transfer from a thermal heat source located in a vertical plate fin. Int.J. Heat Mass Transfer 33, 943-950.

5/23/2011