# Application and Assessment of Kriging and Cokriging Methods on Groundwater Level Estimation

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**Abstract:** Due to increasing demand for water using and shortage of surface water resources, managed use of groundwater has been so important in recent decades. Understanding spatial and temporal changes in groundwater has very important role in planning the use of groundwater as a one of most valuable water resources in the world. Kriging and cokriging methods are from those statistical categories methods which use magnitude, distance and vectorial information of nearby stations for estimation. In this study, kriging and cokriging methods compared with common arithmetic averaging methods for calculating the monthly average level of ground water in "Mian ab" basin and its changes has been set over the years. Error criteria RMSE and MEE are used for comparing methods. Results indicate that cokriging method's accuracy is higher than kriging in calculating groundwater level and also the arithmetic averaging method (which has lower accuracy) has been led to higher level estimation of groundwater. **.** Journal of American Science 2011;7(7):34-39]. (ISSN: 1545-1003). http://www.americanscience.org.

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## 1. Introduction

Groundwater is one of the main water resources. Due to increasing demand for water in different purposes these resources management is very important. Determine the average groundwater level over time in a basin led to groundwater balance estimation and also estimate the amount of extractable groundwater. So the good estimation of the average groundwater level is important. Groundwater level estimation is done using the observation wells data which is the most important information sources for groundwater resources studying.

In the arithmetic averaging method which is a common method, the average of water level in observation wells is considered as average groundwater level in the basin. This way considers a constant value for the whole basin which not matches reality in practice. The arithmetic averaging method is a method from classical statistics methods class.

The difference between classical statistics and geostatistics is that in classical statistics the method assumes that samples (groundwater level here) are independent of each other, so any sample can present no information about the next sample. But in geostatistics the samples are not considered independent from each other, rather according to this theory adjacent samples have dependence in a certain distance of the space. Overall, the geostatistical estimation is a process which on its during the amount of a variable in known coordinates can be achieved by using the same variable values in other points with known coordinates. To do this networking is done. Networking means creating a continues numeral model of an area using limited data points that are placed on that page which in this case data includes longitude(X), latitude(Y) and the depth of groundwater level(Z).

Many researchers have studied geostatistics for keeping and sustainable development of the groundwater resources. Kumar and Emaderi(2006) in the interpolation of groundwater level for a region evaluated simple kriging method in West India. In this study different models of Spherical, Gaussian and Exponential curve fitted to the experimental variogram and the best model was identified ultimately by the variance of estimation. Ahmadi and Sedghamiz (2007) have used kriging for evaluating spatial and temporal changes in water level of 39 observation wells. They have shown that changes in groundwater level have a strong spatial and temporal structure. Olea and Davis (1999) in a conceptual study have used kriging method for groundwater level estimation in observation wells by cross validation. They also suggested few new observation wells based on the kriging standard deviation. Christakos (2000) has done a statistical analysis of groundwater level for 70 wells in Kansas. The water level measurement has been observed regularly during the period. Due to some uncertainty and error in the water level reading, he used geostatistics method to simulate those unreliable values. He also drew annual simulated water level and geographic direction of water level reduction mapped.

Nowadays, the kriging methods are widely used for interpolation of groundwater depth or elevation (Desbarats et al. 2002; Sharda et al. 2006 ;). Desbarats et al. (2002) defined the depth of water table as a linear function of a deterministic trend, given by TOPMODEL topographic index (Beven and Kirkby 1979) and the residual random error. Rivest et al. (2008) estimated the hydraulic head field by utilizing kriging with external drift method. He used the conceptual model results to approximate the external drift in hydraulic heads. Abedini et al. (2008) predicted the piezometric head in west Texas/New Mexico by utilizing the ordinary kriging on clustered piezometric head data. Hoeksema et al. (1989) used cokriging method to estimate water level in unknown points. Water level and ground level as primary and secondary variables are used by them. Deutsch and Journel (1992) and Goovaerts (1997) presented a method in which the ground elevation is used as the secondary variable for interpolation of groundwater level by cokriging.

This Study targets include:

1- Estimate the accuracy of kriging and cokriging methods in estimating average groundwater level on the watershed in different month of the year.

2- The comparison between kriging , cokriging and arithmetic averaging method results.

## 2. Material and Methods

### A. Theoretical Foundation

Basic theory of geostatistics is described by several authors (Webster and Oliver (2001), etc.). Semivarigram is the main tool in geostatistics which shows the dependence between neighboring observations. Semivariogram can be defined by half the variance of the difference between attribute values at all points with (h) intervals as follows:

$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [Z(x_i) - Z(x_i + h)]^2$$

Which $Z(x_i)$ is the total amount of variable and N (h) is the number of attribute pairs from points that are separated by distance (h). Before geostatistical estimation one model is needed to calculate variogram values for each sampling interval which may be used. The most important models are Spherical, Exponential, Gaussian, Circular and Nugget net effect (Isaaks and Sirvastava(1989)) . Adequacy and appropriateness and accuracy of variogram developed model must be tested by cross validation technique satisfactorily. Cross validation idea includes the removal of a data and estimate the amount of it by different variogram models at the same time. Actual and estimated values are compared and the most accurate model that can have the best prediction remains (Goovaerts (1997)). Estimated values in the graph cross the actual values show the correlation coefficient. The most appropriate variogram is obtained based on the correlation coefficient and on the tried and error. Leuangthong et al. (2004) reported that the variograms obtained through cross-validation satisfy the minimum acceptance criteria for geostatistical analysis. Kriging is an accurate interpolation method and also the best unbiased linear estimator. The best unbiased linear estimator must have the minimum variance estimation error. Kriging general equation is as follows:

$$Z^*(x_p) = \sum_{i=1}^{n} \lambda_i Z(x_i)$$

In order to achieve the unbiased kriging estimates total following equations must be solved simultaneously.

$$\begin{cases} \sum_{i=1}^{n} \lambda_i \gamma(x_i, x_j) - \mu = \gamma(x_i, x) \\ \sum_{i=1}^{n} \lambda_i = 1 \end{cases}$$

Where $Z^*(x_p)$ is the calculated amount in the place ( $x_p$ ), $Z(x_i)$ the known amount in the location ( $x_i$ ), $\lambda_i$ the weight related to i-th data, $\mu$ the Lagrange coefficient and $\gamma(x_i, x_j)$ the variogram value corresponding to a vector starting at $x_i$ and $x_j$ the ultimate limit of the vector.

General form of cokriging equations is as follows:

$$\begin{cases} \sum_{l=1}^{v} \sum_{i=1}^{n_l} \lambda_{il} \gamma_{lv}(x_i, x_j) - \mu_v = \gamma_{uv}(x_j, x) \\ \sum_{i=1}^{n_l} \lambda_{il} = \begin{cases} 1, & 1 = \mathrm{u} \\ 0, & 1 \neq \mathrm{u} \end{cases} \end{cases}$$

Where u and v are primary and secondary variables. In cokriging method the variation of u and v has cross correlation so that secondary variable participates in determination of primary variable variation. For cokriging analysis cross semivariogram must be determined first. At any point u and v should be measured and cross semivariogram is estimated with the following equation:

$$\gamma_{uv} = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [Z_u(x_i) - Z_u(x_i + h)][Z_v(x_i) - Z_v(x_i + h)]$$

**B. Study area Location**

"Mian-ab" basin is located between two cities: "Shushtar" and "Bandghir" and is on the 40 kilometer north of "Ahwaz" approximately, between longitude 29° 66″ to 29° 92″ east and latitude 35° 20″ to 35° 41″ north. Study area is an island between the river "Gargar" and "Shatit" and that is enclosed by "Gargar "river in the east and "Shatit" river in the west. In figure 1 position of study area in the "Khuzestan" province taken by Landsat satellite is marked.

**3. Results**

According to recorded data for observation wells in the twelve months, data belong to the minimum and maximum months and also from annual average was used for finding the best variogram model. Chosen model is used for calculation in all month of the year. Data used in the kriging should be normalized which according to calculated skewness and kurtosis criteria logarithmic conversion for normalizing the data was used. Also, trend testing is done to find out any possible existence trend in the process which the result shows no trend in variables. For cokriging analysis at least two variables is required. In this way, these two variables are covariates in which the primary variable is estimated by the second variable assistance. In this study the cokriging method was used to affect ground level (topography) in groundwater level estimating. To do this, the correlation between these two variables must be evaluated. Amounts of groundwater level in the months of maximum and minimum (May and October) and annual mean in addition of corresponding ground level values for observation wells is used to check this correlation. High obtained values for $R^2$ (The correlation coefficient between primary and secondary variables in cokriging method) confirms correlation between mentioned variables. Figure 2 shows correlation between ground level and groundwater level in case of different modes. The results are shown in table 1. . Ultimately in both methods and for maximum and minimum months and also for annual mean according to (RMSE) criterion best model was selected. The selected model and its profile are expressed in table 1. Mean estimation error (MEE) is defined as: $MBE = \frac{1}{n}\sum_{i=1}^{n}(Z^*(x_i) - Z(x_i))$, and also root mean square error (RMSE) is defined as: $RMSE = \sqrt{\frac{(Z^*(x_i) - Z(x_i))^2}{n}}$.

Monthly groundwater levels for basin's wells (46 wells in water year 2006-2007) are considered for this study. The wells in the basin have been scattered to control groundwater level fluctuation. Also ground level for cokriging calculations is used as secondary variable. According to Goovaerts (1997) in environmental applications large values may indicate potentially critical points so they should be removed only if they are clearly wrong.

Table 1. Selected model profiles in kriging and cokriging methods

| METHODS | VARIABLE | MODEL | RANGE | Nugget | Sill | $\frac{Nugget}{SILL}$ | $R^2$ |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| KRIGING | OCTOBER | EXPONENTIAL | 32000 | 0.00002 | 0.02461 | 0.00082 | |
| | MAY | EXPONENTIAL | 32000 | 0 | 0.02010 | 0 | |
| | ANNUAL MEAN | EXPONENTIAL | 32000 | 0.00019 | 0.02228 | 0.00852 | |
| COKRIGING | OCTOBER | EXPONENTIAL | 32000 | 0.00002 | 0.02461 | 0.00082 | 0.76 |
| | MAY | EXPONENTIAL | 32000 | 0 | 0.02010 | 0 | 0.75 |
| | ANNUAL MEAN | EXPONENTIAL | 32000 | 0.00019 | 0.02228 | 0.00852 | 0.76 |

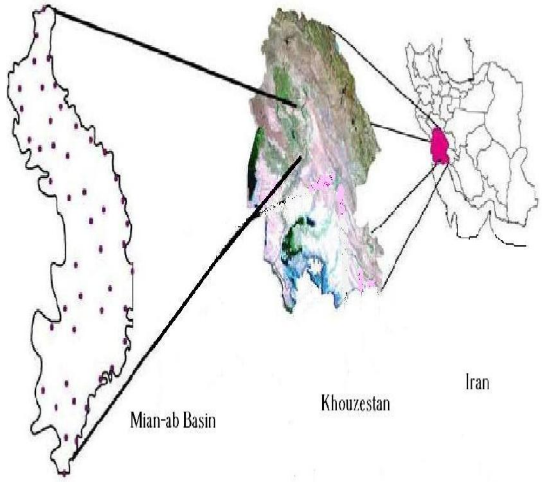


Figure 1. Study area position

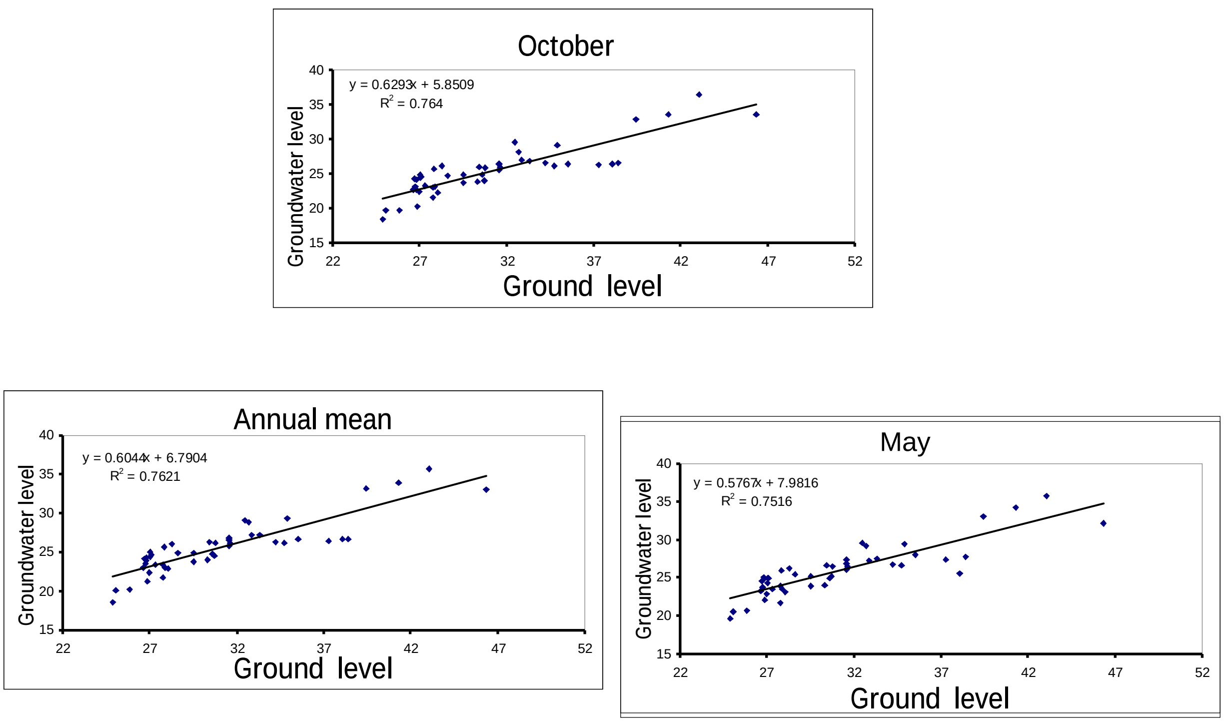


Figure 2. Correlation between ground level depth and groundwater

Values for MEE and RMSE criteria according to chosen model for each month in kriging and cokriging methods are shown in table2. Also values of skewness and kurtosis parameters for data before and after logarithmic conversion are expressed.

Table 2. Evaluation results from kriging and cokriging methods and parameters of statistical data

| MONTH | KRIGING | | COKRIGING | | DATA PARAMETERS | | DATA PARAMETERS AFTER LOGARITMIC TRANSFORMATION | |
|---|---|---|---|---|---|---|---|---|
| | RMSE | MEE | RMSE | MEE | S | K | S | K |
| October | 2.49 | 0.075 | 1.65 | 0.030 | 1.05 | 4.87 | 0.46 | 3.09 |
| November | 2.43 | 0.076 | 1.63 | 0.032 | 0.95 | 4.45 | 0.41 | 2.80 |
| December | 2.45 | 0.073 | 1.61 | 0.031 | 0.92 | 4.52 | 0.38 | 2.8 |
| January | 2.38 | 0.081 | 1.61 | 0.032 | 0.79 | 4.35 | 0.24 | 2.68 |
| February | 2.25 | 0.083 | 1.46 | 0.033 | 0.82 | 4.40 | 0.27 | 2.70 |
| March | 2.15 | 0.068 | 1.39 | 0.034 | 0.84 | 4.37 | 0.28 | 2.72 |
| April | 2.18 | 0.071 | 1.37 | 0.034 | 0.81 | 4.22 | 0.26 | 2.85 |
| May | 2.14 | 0.077 | 1.35 | 0.035 | 0.89 | 4.62 | 0.41 | 2.63 |
| June | 2.28 | 0.083 | 1.46 | 0.034 | 1.00 | 4.62 | 0.46 | 2.87 |
| July | 2.31 | 0.078 | 1.53 | 0.033 | 0.88 | 4.78 | 0.42 | 2.91 |
| August | 2.33 | 0.078 | 1.58 | 0.032 | 0.97 | 4.55 | 0.32 | 3.11 |
| September | 2.40 | 0.075 | 1.56 | 0.033 | 0.92 | 4.24 | 0.30 | 3.03 |
| Annual Mean | 2.34 | 0.078 | 1.51 | 0.033 | 0.93 | 4.36 | 0.38 | 2.86 |

Ratio $\frac{Nugget}{Sill}$ is used for classification of spatial correlation. If the ratio for a variable be less than 0.25 ,then this variable would have high spatial correlation and if the ratio is between 0.25-0.75 it has a restricted correlation and if it is upper than 0.75 ,it has weak correlation(Liu et al.(2006)). Figure 3 shows average groundwater level on different months in “Mian-ab” basin. In this diagram arithmetic average method is compared with kriging and cokriging results (which are considered as interpolation methods).

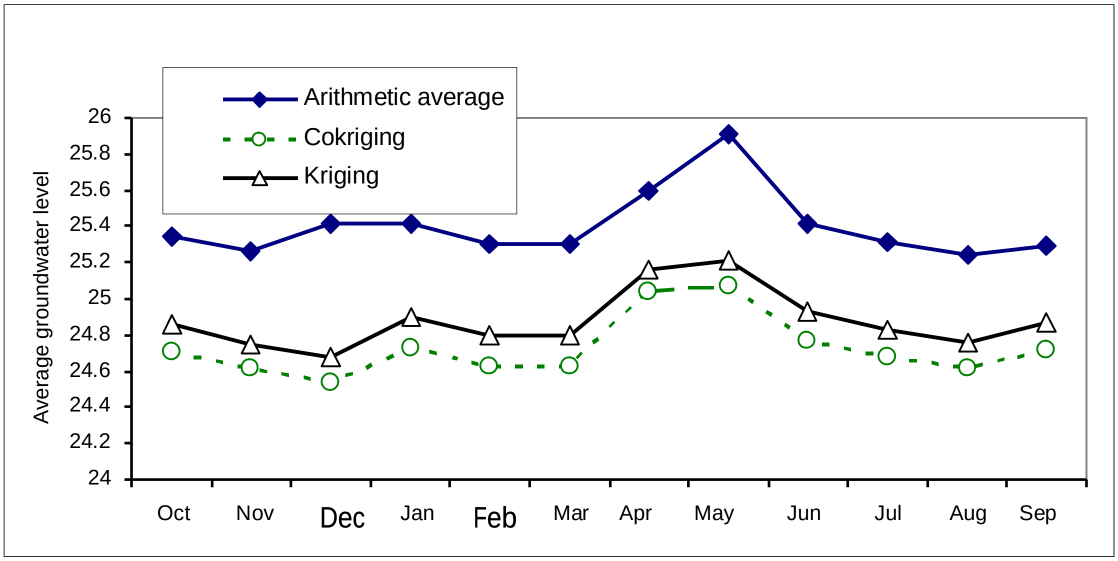


Figure 3: Average groundwater level in different month using kriging, cokriging and arithmetic average method

**4. Conclusion**

Complete and sustainable water resources management requires adequate knowledge from amount and changes (spatial and temporal) of groundwater. Adequate and accurate information plays a key role in management decisions. In order to check the anisotropy, the conventional approach is to compare variograms in several directions. In this study major angles of 0°, 45°, 90°, and 135° with an angle tolerance of ±22.5 were used for detecting anisotropy possibility. However, there were no distinct differences among the structures of the calculated variograms in the four main directions. Results indicate that monthly average data obtained from the arithmetic average method has higher estimates of groundwater potential on the mentioned plain. Both kriging and cokriging have low values of error criteria (MEE and RMSE) which these results confirm unbiased assumption (estimation error close to zero) in these methods. Among the kriging and cokriging methods (interpolation methods) kriging estimation has a higher groundwater level. But according to RMSE criteria, cokriging method has higher accuracy although this increased accuracy is not so significant. The results also reflect the fact that groundwater level changes in the plain during December and January does not follow by data of the arithmetic average values.

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