Effect of fringing filed on the internal stress field of nano cantilever beams in the presence of van der waals attractions

Amir Vosoogh

Young researcher club, Mahshahr Branch, Islamic Azad University, Mahshahr, Iran.

Abstract: In this paper, effect of fringing filed on the internal stress field of nano cantilever beams is studied using homotopy perturbation method. The nano cantilever beam is considered as a distributed parameter model including intermolecular forces, electrostatic forces and fringing filed effects. In the modeling of intermolecular forces the van der Waals attraction and in the modeling of electrostatic forces, the fringing field effect is taken into account. By using the obtained polynomial solution, bending moment and shear force are calculated, for narrow and wide nano beams. Results shows that stress resultants enhances as the fringing field increases.

[Amir Vosoogh. Effect of fringing filed on the internal stress field of nano cantilever beams in the presence of van der waals attractions. Journal of American Science 2011;7(7):436-439]. (ISSN: 1545-1003). http://www.americanscience.org.

Keywords: Nano-cantilever, Fringing field, Homotopy Perturbation, Stress, van der Waals.

1. Introduction

Micro and nanofabrication processes are planar technologies. Therefore, many micro and nano devices consist of beams and plates suspended horizontally over a substrate. On the microscale, suspended beams or plates serve as the active component of accelerometers, rate gyroscopes, pressure sensors, chemical sensors, electrical switches, optical switches, adaptive optical devices, resonators, electrostatic actuators, valves, and pumps (Mastrangelo, 1993). Conductive cantilever nano-actuators are one of the common components in developing nano-electromechanical system (NEMS) switches of nano technology (Ke, 2006). A typical form of NEMS actuator is a nano-beam which is suspended above a conductive flat ground (substrate). Applying voltage difference between the nano-beam and the ground plane causes the nano-beam to deflect downward and be attracted toward to the substrate. The inter-molecular forces significantly influence the deflection and internal stresses of nano-beam, at nano-scale separations (Soroush, 2010).

The van der Waals force results from the interaction between instantaneous dipole moments of atoms. It is significant when separation is less than the retardation length (typically below 20 nm) which corresponds to the transition between the ground and the excited states of the atom (Mastrangelo, 1993). The van der Waals force attraction is proportional to the inverse cube of the separation and is affected by material properties (Israelachvili, 1992) and (Koochi, 2011). Effect of van der Waals attraction on the instability of cantilever NEMS has been investigated by previous researchers (Mastrangelo, 1993), (Soroush, 2010) and (Koochi, 2011).

Design of reliable NEMS requires crucial knowledge about the mechanical stress field in the structure (Jonnalagaddaa, 2008) and (Pugno, 2005). If a nano-beam is not strong enough to bear internal stresses, it might deform or break under influence of electrostatic forces. All lamped models assumed the electrostatic and intermolecular forces uniform along the beam and therefore could not evaluate the internal stress resultants i.e. internal shear force and bending moment along the beam (Israelachvili, 1992).

The fringing field can highly affect electromechnical performance of NEMS (Soroush, 2010). The present paper considers the effect of fringing filed on the internal stress distribution of nano-beams in the presence of van der Waals attractions. The solutions obtained by homotopy perturbation method are compared with numerical results.

2. Mathematical model

Figure 1 shows a nano-cantilever beam, of length L with uniform rectangular cross section of thickness h and width w. the initial gap between the movable beam and the ground plane is g. The constitutive material of the nano-cantilever is assumed linear elastic and only the static deflection of the nano-beam is considered.

In our previous work we obtained the governing equation of nano cantilever beams subject to electrostatic and intermolecular forces in non dimensional form as follow (Soroush, 2010)

$$\frac{d^4u}{dx^4}=\frac{\alpha}{(1-u(x))^3}+\frac{\beta}{(1-u(x))^2}+\frac{\gamma\beta}{(1-u(x))} \quad \text{(1-a)}$$

subject to the following conditions

$$u(0) = u'(0) = 0, \qquad at\ \ x = 0,$$
$$u''(1) = u'''(1) = 0, \qquad at\ \ x = 1, \qquad \text{(1-b)}$$

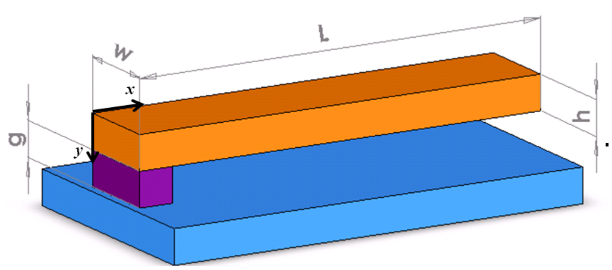

Fig.1. Schematic representation of a cantilever nano-beam

where x is nondimensional position along the beam measured from the clamped end, prime denotes differentiation with respect to x and u is non dimensional deflection. in equation (1) α, β and γ correspond to the values of Casimir force, applied voltage and fringing field respectively. The nondimensional constants are (Soroush, 2010)

$$\alpha = \frac{AwL^4}{6\pi g^4 E_{eff} I}, \beta = \frac{\varepsilon_0 w V^2 L^4}{2g^3 E I},$$
$$\gamma = 0.65\frac{g}{w},\ x = \frac{X}{L},\ u = \frac{y}{g} \qquad (2)$$

where $\varepsilon_0 = 8.854 \times 10^{-12} c^2 N^{-1} m^{-2}$ is the permittivity of vacuum, V is the applied external voltage, y is the deflection of the beam and X is position along the beam. E_{eff} is the effective Young's modulus which is equal to $wh^3/12$, and I is the moment of inertia of the beam cross section (Soroush, 2010).

3. Stress resultants

Design of reliable NEMS requires crucial knowledge about the distribution of internal stress over the length of the beam. The maximum value of shear stress and bending stress at the onset of instability are very important as the most critical state of stress in the engineering applications. Based on Euler beam theory, these parameters can be directly computed from stress resultants (Timoshenko, 1987). In order to determine critical values of stress resultants, we define F and M as the dimensionless maximum value of the shear force and bending moment at the onset of instability, respectively as follows

$$F = \frac{F_0 L^3}{E_{eff} I g} \qquad (3)$$

$$M = \frac{M_0 L^2}{E_{eff} I g} \qquad (4)$$

where F_0 and M_0 are shear force and bending moment at the cross-section of the beam fixed end (x=0). By these definitions, M and F equal to u'' (x=0) and $-u'''$ (x=0), respectively (Timoshenko, 1987).

4. Analytical Solution

To convey the idea of the homotopy perturbation method, we consider a general equation of the type

$$O(u) = 0 \qquad (5)$$

where O is an integral operator. We construct a convex homotopy structure $H(u,p)$ as follows:

$$H(u,p) = (1-p)L(u) + pO(u) \qquad (6)$$

where $L(u)$ is a functional operator with easily determined solutions v_0. It is clear that $H(u,0)=L(u)$ and $H(u,1)=O(u)$. This shows that $H(u, p)$ continuously traces an implicitly defined curve from a starting point $H(v_0,0)$ to a solution $H(f,1)$. The embedding parameter increases monotonically from 0 to 1 as $L(u)$=0 continuously deforms into the $O(u)$=0. The homotopy perturbation method employs the embedding homotopy parameter p as an expanding parameter to obtain

$$u = \sum_{i=0} p^i u_i = u_0 + pu_1 + p^2 u_2 + \ldots \qquad (7)$$

The best approximation for solution is

$$f = \lim_{p \to 1} u = \sum_{i=0}^{\infty} u_i \qquad (8)$$

Series (8) is convergent for most of the cases. A comparison of like powers of p gives appropriate solutions at various orders.

Integrating the Eq. 9(a-d), we get the following system of integral equations:

$$y(x) = 1 + \int_0^x y'(t)dt\,, \tag{9-a}$$

$$y'(x) = 0 + \int_0^x M(t)dt\,, \tag{9-b}$$

$$M(x) = A + \int_0^x F(t)dt\,, \tag{9-c}$$

$$F(x) = B - \int_0^x \left(\alpha\, y(x)^{-3} + \beta\, y(x)^{-2} + \gamma\beta\, y(x)^{-1}\right)dt\,. \tag{9-d}$$

Using relations (7) in Eq. 9(a-d), we have

$$\sum_{k=0}^{\infty} p^k y_k = 1 + p\int_0^x \left(\sum_{k=0}^{\infty} p^k y'_k\right)dt \tag{10-a}$$

$$\sum_{k=0}^{\infty} p^k y'_k = 0 + p\int_0^x \left(\sum_{k=0}^{\infty} p^k M_k\right)dt\,, \tag{10-b}$$

$$\sum_{k=0}^{\infty} p^k M_k = A + p\int_0^x \left(\sum_{k=0}^{\infty} p^k F_k\right)dt\,, \tag{10-c}$$

$$\sum_{k=0}^{\infty} p^k F_k = B - p\int_0^x \left(\alpha\sum_{k=0}^{\infty} p^k \phi_{k,3} + \beta\sum_{k=0}^{\infty} p^k \phi_{k,2} + \gamma\beta\sum_{k=0}^{\infty} p^k \phi_{k,1}\right)dt\,. \tag{10-d}$$

The function $\phi_{k,n}$, which approximates the nonlinear term y_k^{-n}, is determined by Taylor series using Eq. (11) (Koochi, 2011).

$$\phi_{k,n} = \frac{1}{k!}\frac{d^k}{dp^k}\left[\left(\sum_{i=0}^{\infty} p^i y_i\right)^{-n}\right]_{p=0}\,, \tag{11}$$

Therefore, the polynomial solutions for F and M are obtained which can be summarized to

$$\begin{aligned} M(x) = {} & A_1 - A_2 x + (\alpha + \beta + \beta\gamma)\frac{x^2}{2!} \\ & - (3\alpha + 2\beta + \gamma\beta)\frac{A_1 x^4}{4!} - (3\alpha + 2\beta + \gamma\beta)\frac{A_2 x^5}{5!} \\ & + \left(A_1^2(6\alpha + 3\beta + \gamma\beta) + \frac{(3\alpha + 2\beta + \gamma\beta)(\alpha + \beta + \gamma\beta)}{6}\right)\frac{x^6}{5!} \\ & + \ \ldots \end{aligned} \tag{12-a}$$

$$\begin{aligned} F(x) = {} & A_2 - (\alpha + \beta + \beta\gamma)x \\ & + (3\alpha + 2\beta + \gamma\beta)\frac{A_1 x^3}{3!} + (3\alpha + 2\beta + \gamma\beta)\frac{A_2 x^4}{4!} \\ & - \left(6A_1^2(6\alpha + 3\beta + \gamma\beta) + (3\alpha + 2\beta + \gamma\beta)(\alpha + \beta + \gamma\beta)\right)\frac{x^5}{5!} \\ & - \ \ldots \end{aligned} \tag{12-b}$$

In the following text all results are calculated from eight terms of obtained Adomian polynomials.

5. Results

In order to verify the convergence of obtained series, the deflection of a typical nano-actuator is computed analytically and the solutions are compared with the numerical results those from (Soroush, 2010) in table 1. Figure 2, figure 3 and figure 4 are show the effects of fringing field for β=0.2 on *F*, *M* and cantilever tip deflection respectively. As seen, fringing field increases *F* and *M* of the cantilever.

6. Conclusion

Internal stress field of nano-cantilevers were computed using homotopy perturbation method. It is found that the intermolecular forces increase maximum shear force and bending moment at constant voltages. Intermolecular forces also, increase tip deflection of cantilever beams. Results shows that stress resultants enhances as the fringing field increases.

Table 1. The variation of the tip deflection of a typical beam obtained using different selected terms of homotopy perturbation series for α=0.3, β=0.2 , and g/w = 1

Terms	*Tip Deflection*
6 Terms	0.0986
7 Terms	0.0870
8 Terms	0.0940
9 Terms	0.0886
10 Terms	0.0922
11 Terms	0.0898
12 Terms	0.0915
Numerical	0.0908 in (Soroush, 2010)

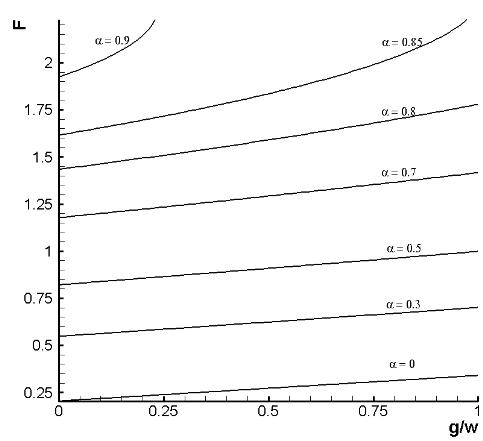

Fig.2. Effect of fringing filed and intermolecular force on the shear force at the fixed end of cantilever nano-beam when β=0.2

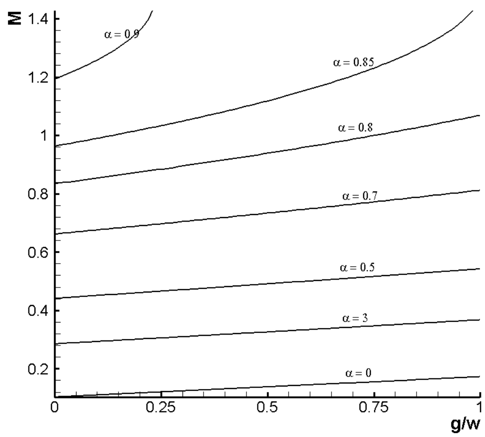

Fig.3. Effect of fringing filed and intermolecular force on the bending moment at the fixed end of cantilever nano-beam when β=0.2

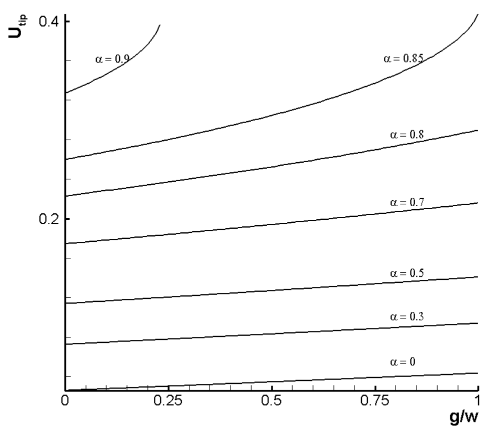

Fig.4. Effect of fringing filed and intermolecular force on the tip deflection of cantilever nano-beam when β=0.2

References

1. Chen E, Lu Z., An algorithm for Adomian decomposition method. Appl. Math. Comput. 2004;159:221–35.
2. Israelachvili JN., Intermolecular and Surface Forces. (London: Academic Press), 1992.
3. Jonnalagaddaa K, Chob SW, Chasiotisa I, Friedmannc T, Sullivanc J., Effect of intrinsic stress gradient on the effective mode-I fracture toughness of amorphous diamondlike carbon films for MEMS. J. Mech. Phys. Solids. 2008; 56 388–401.
4. Ke CH, Espiona HD. Nanoelectromechanical systems (NEMS) and modeling, Handbook of theoretical and computional nanotechnology Ed Rieth M, Schommers W, Gennes PD., (American Scientific Publishers), 2006, Chapter 121.
5. Koochi A, Noghrehabadi A, Abadyan M, Roohi E., Investigation of the effect of van der Waals force on the instability of electrostatic nano-actuators. Int J Mod Phys: B [in press].
6. Mastrangelo CH, CH Hsu, 1993 Mechanical stability and adhesion of microstructures under capillary force-Part I: Basic theory Journal of Microelectromechanical Systems 2 (1): 33-43
7. Koochi A, Kazemi AS, Noghrehabadi A, Yekrangi A., Abayan M., New approach to model the buckling and stable length of multi walled carbon nanotube probes near graphite sheets. Int. J Materials and Design [in press].
8. Momoniat E, Selway T A, Jina K, Analysis of Adomian decomposition applied to a third-order ordinary differential equation from thin film flow Nonlinear Anal. 2007;66:2315– 2324.
9. Pugno N, Peng B, Espinosa HD., Predictions of strength in MEMS components with defects—a novel experimental–theoretical approach. Int. J. Solids and Struct. 2005; 42 647–661.
10. Soroush A, Koochi A, Kazemi AS, Noghrehabadi A, Haddadpour H, Abadyan M. Investigating the effect of Casimir and van der Waals attractions on the electrostatic pull-in instability of nano-actuators. J. Phys. Scr. 2010; 82 045801.
11. Timoshenko S., Theory of Plates and Shells, (New York: McGraw Hill) 1987.

5/22/2011