# Deformation of Circular Holes at Flanging Process of Thin Sheets

Najmeddin Arab

Department of Engineering, Islamic Azad University, Saveh Branch, Iran
najmarab@iau/saveh.ac.ir

**Abstract:** The theoretical analysis of flanging process of circular holes in thin plates taking into account the interconnected changes of a thickness and deformation hardening is carried out. Possibility of definition of current values of strains in non-stationary processes of forming is shown. In process of flanging the co-ordinate of the material element differentiating area of compression and a stretching in meridinal direction is defined.
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## 1. Introduction

Numerous problems of forming sheet metal demand solutions of problems on definition of the stress-strain state at big plastic deformations. These problems primarily include the prediction of the parameters of strength and precision products manufactured by forming stamping operations. A distinctive feature of such problems is that the same source material elements of the sheet blank, strain in the field of plastic strain, acquire the characteristic features caused by these strains. The peculiarity of these elements is characterized by indicators such as change in thickness, the accumulated strain, yield stress, etc. In the process of forming a certain strain accumulates in this element and, as a result of strain hardening, the yield stress of this element changes. Consequently, the prediction parameters of strength and accuracy is possible with the known distribution of strain intensity, since, on the one hand, it is uniquely determined by the components of strain, on the other - indicates the degree of strain hardening. The given article serves as a continuation of a series of works [Nazaryan, Konstantinov, and Nazaryan, Konstantinov, Arakelyan and Nazaryan, Arakelyan] and is devoted the theoretical analysis of flanging process circular apertures in thin plates taking into account the interconnected change of thickness and strain hardening (while the known aspects of flanging process are ignored).

## 2. New Theory

Let us consider the process of forming when flanging circular holes in thin plates carried out with a flat end punch [Priests, Kovalev]. In papers [Nazaryan, Konstantinov, and Nazaryan, Konstantinov, Arakelyan and Nazaryan, Arakelyan] it is shown that the original equations describing the plastic plane stress condition, which occurs when flanging circular holes in thin plates, namely:

- equation of balance taking into account variability of thickness,
- equation of stress and strain increments,
- constant volume condition,
- Mises' plasticity condition

are reduced on the deviatoric plane of the plasticity cylinder to a fairly simple differential equation:

$$d\sigma_\rho = \sigma_s d\varepsilon_i. \qquad \text{(Equ. 1)}$$

In deriving (1) meridional $\sigma_\rho$ and $\sigma_\theta$ circumferential stresses are expressed through the parameter deformed state, used as:

$$\sigma_\rho = \frac{2}{\sqrt{3}}\sigma_s \cos(\varphi + \pi/6); \quad \sigma_\theta = -\frac{2}{\sqrt{3}}\sigma_s \sin\varphi, \qquad \text{(Equ. 2)}$$

whereas components of strain increments are presented by known dependences

$$d\varepsilon_\rho = d\varepsilon_i \cos\varphi; \quad d\varepsilon_\theta = d\varepsilon_i \cos(\varphi + 2\pi/3)$$
$$d\varepsilon_z = d\varepsilon_i \cos(\varphi + 4\pi/3) \qquad \text{(Equ.3)}$$

which unequivocally satisfy intensity of strain increment defined taking into account condition of volume constancy with the following parity

$$d\varepsilon_i = \frac{2}{\sqrt{3}}\sqrt{d\varepsilon_\rho^2 + d\varepsilon_\rho d\varepsilon_\theta + d\varepsilon_\theta^2}. \qquad \text{(Equ. 4)}$$

The isotropic hardening is represented on the deviatoric plane of the cylinder plasticity in the form of circles with radiuce $\sigma_s/\sqrt{3}$, which increase in the radial direction by an amount determined by the value of the accumulated strain [3]

$$\sigma_s = A\varepsilon_i^n \qquad \text{(Equ. 5)}$$

where A and n are the constants characterising strain hardening. Current value of the accumulated strain, taking into account a condition of volume constancy is defined by dependence

$$\varepsilon_i = \frac{2}{\sqrt{3}}\sqrt{\varepsilon_\rho^2 + \varepsilon_\rho\varepsilon_\theta + \varepsilon_\theta^2}.$$
(Equ. 6)

From comparison (4) and (6) follows that generally intensity of strain increments $d\varepsilon_i$ is not equal to an increment of intensity $d\bar{\varepsilon}_i$ defined by differentiation of dependence (6). Therefore integration of the differential equation (1) taking into account (5) is possible only for radial directions of strains at which for the given material element the relation of strain increment components remains constant. Limits of parameter change $\varphi$ in the considered problem are established proceeding from flanging process definition at which for all center current values of district deformations are positive, increasing from zero on border of non-deformable part of blank to the greatest value on aperture edge. From the analysis of dependences (2) follows that on border of non-deformable part of blank meridional stretching stresses cannot exceed value $2\sigma_s/\sqrt{3}$ corresponding to direction $\varphi = 11\pi/6$ at which $\varepsilon_\theta = 0$, and from the absence condition of meridional stretching stresses on aperture edge it follows that the boundary element is deformed under conditions of radial stretching in a positive direction of axis $\varepsilon_\theta$ that corresponds to direction $\varphi = 4\pi/3$.

From the above arguments it follows that theoretically possible range of realisation of flanging process at which for all material elements current value of circumferential strain is $\varepsilon_\theta \geq 0$, it is located on deviatorial planes of plasticity cylinder within the change of parameter $\varphi$ in range $4\pi/3 \leq \varphi \leq 11\pi/6$. Hence, all types of deformations in flanging process are completely characterised by the radial beams located in sector with central angle $\varphi = \pi/2$ (fig. 1).

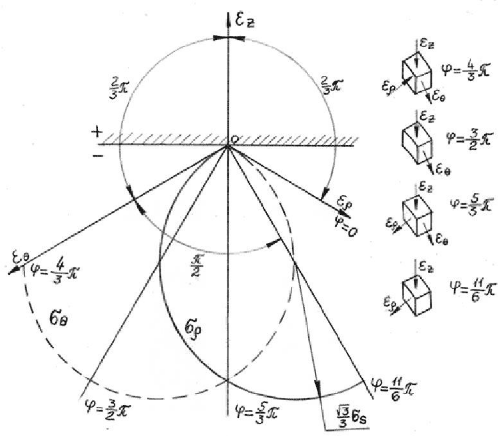

Fig.1. Stress state and strain types in flanging

If the strain direction coincides with axies $\varepsilon_\theta$, then $\varepsilon_\theta$ is stretch strain in circular direction, and $\varepsilon_\rho$ and $\varepsilon_z$ are compression strains numerically equal to $\varepsilon_\theta/2$. If the strain direction coincides with the negative direction of axis $\varepsilon_z$, then $\varepsilon_z$ is strain compression in thickness, and $\varepsilon_\rho$ and $\varepsilon_\theta$ are strech strains numerically equal to $\varepsilon_z/2$. If the strain direction coincides with direction $\varepsilon_\rho = 0$, and $\varepsilon_\theta$ and $\varepsilon_z$ are equal in value and are opposite in sign then pure shear or plane strain in plane ($\theta,\ z$) takes place. If the strain direction coincides with direction $\varepsilon_\theta = 0$, and $\varepsilon_\rho$ and $\varepsilon_z$ are equal in value and are opposite in sign pure shift or plane strain in plane ($\rho,\ z$) takes place. Thus, strain direction with angles $\varphi = 3\pi/2$ and $11\pi/6$ correspond to pure shift, accordingly, in planes ($\theta,\ z$) and ($\rho,\ z$), and with angles $\varphi = 4\pi/3$ and $5\pi/3$ - pure tension and compression, respectively, in the circumferential direction and in the direction of the thickness.

To establish a correspondence between the coordinates of the elements and strain state parameters let's integrate equation (1) i naccord with previously adopted assumption about the radial nature of the current strain accumulation. Equating the result of integrating the value of the meridional stresses, determined by the dependence (2), we obtain

$$\varepsilon_i^{1+n} - \varepsilon_{\kappa p}^{1+n} = (1+n)\varepsilon_i^n \frac{2}{\sqrt{3}}\cos(\varphi + \pi/6)$$
(Equ. 7)

When we integrate (1) the boundary condition is used according to which the boundary element (aperture edge is deformed in the circumferential direction in a linear stretch throughout the entire process of forming).The known distribution of the strain intensity at the angle $\varphi$ allows to establish the relationship between the coordinate $\rho$ of the element and the parameter $\varphi$ on the deviatoric plane of the cylinder plasticity. For the initial stage of the process of forming ($\varepsilon_{\kappa p} = 0$), differentiating the expression (7) and equating the result (3)

($d\bar{\varepsilon}_i = d\varepsilon_i,\ d\varepsilon_\theta = d\rho/\rho$) [2], we obtain:

$$d\varepsilon_\theta = \frac{d\rho}{\rho} = (1+n)(\sin\varphi\cdot\cos\varphi + \frac{\sqrt{3}}{2}\sin^2\varphi + \frac{\sqrt{3}}{6}\cos^2\varphi)d\varphi$$

(Equ. 8)

Expression (8) after integration will become:

$$\ln\rho = (1+n)\left(\frac{\sqrt{3}}{3}\varphi - \frac{1}{4}\cos 2\varphi - \frac{\sqrt{3}}{12}\sin 2\varphi\right) + c$$

(Equ. 9)

where the constant of integration $c$ is from the boundary condition, according to which at $\varphi = 4\pi/3$ $\rho = r_0$ (r0 is the initial radius of aperture). Substituting integration constant in (9), after simple transformationswe get:

$$\frac{\rho}{r_0} = \exp(1+n)\left(\frac{\sqrt{3}}{3}(\varphi - 4\pi/3) - \frac{1}{4}\cos 2\varphi - \frac{\sqrt{3}}{12}\sin 2\varphi\right)$$

(Equ. 10)

From (10) follows, that strain area has the greatest values at $\varphi = 11\pi/6$ ($\varepsilon_\theta = 0$), $\rho = \rho_0$ ($\rho_0$ - the radius of matrix die) and equals

$$\frac{\rho_0}{r_0} = \exp(1+n)\frac{\sqrt{3}}{3}\frac{\pi}{2} \approx 2{,}475\cdot(1+n)$$

(Equ.11)

At $n = 0$ the limit value of relation $\rho_0/r_0$ coincides with the result of work [1] which was received when solving the problem taking into account thickness change for ideally rigid plastic model of deformable material.

Dependence (10), establishing unequivocal connection between initial co-ordinate of considered element and parameter $\varphi$ allows to newly consider the flanging process dividing it into two stages.

At the initial stage of strain, as the lowering of the punch takes place, the area of plastic strains is formed from the external border (at the edge of the matrix die) to te internal, with blank elements moving in the meridional direction, and their thickness decreases. Consequently, there is an element deformable in the direction $\varphi = 3\pi/2$, for which meridional strain is $\varepsilon_\rho = 0$ throughout the forming process. To define co-ordinate $\rho$ for this element, substituting in (10) $\varphi = 3\pi/2$, we obtain

$$\frac{\rho}{r_0} = \exp(1+n)\left(\frac{\sqrt{3}}{3}\frac{\pi}{6} + \frac{1}{4}\right) \approx 1{,}737\cdot(1+n)$$

(Equ.12)

Comparing the received result for the relative co-ordinate differentiating the area of shortening from the area of lengthening in meridional direction at $n = 0$ with the result of work [4] ($\rho/r_0 \approx 2$) which is obtained without considering the change in thickness and influence of strain hardening, we see, that in [4] it is a little overestimated. Thus, all material elements with a relative co-ordinate less in value defined by (12), in are shortened in meridional direction, being exposed to radial compression, whereas elements with relative co-ordinate more in value in meridional direction are extended, being exposed to radial stretching. Hence, estimation of the flanging process parameters should be based on the flanging coefficient. At small values of flanging coefficient of deformations $\varepsilon_\rho \le 0$, the total height of the bead is less than the width of flanging part of the blank. At great values of flanging coefficient when in the strain area $\varepsilon_\rho$ changes its sign, compressive strains in meridial direction are somewhat compensated by stretching strains, and the estimation application, which is based on equality of length sweep length of bead in the midline on an average line and the width of the flanging part of the bead. Knowing the character of strain intensity distribution , it is possible to define the strain components:

$$\varepsilon_\rho = \frac{1+n}{2}\left(1 + \cos 2\varphi - \frac{\sqrt{3}}{3}\sin 2\varphi\right),$$

$$\varepsilon_\theta = -\frac{1+n}{2}\left(\cos 2\varphi + \frac{\sqrt{3}}{3}\sin 2\varphi\right),$$

$$\varepsilon_z = -\frac{1+n}{2}\left(1 - \frac{2\sqrt{3}}{3}\sin 2\varphi\right);$$

(Equ.13)

Fig. 2 shows the changes of strain intensity and components upon the change of parameter $\varphi$ in range $4\pi/3 \le \varphi \le 11\pi/6$ at the initial stage of strain ($\varepsilon_{kp} = 0$) at $n = 0.2$.

### 3. Result and discussion

From (13) and fig. 2 it follows, that in the strain area some element with parameter of strained state $\varphi = 19\pi/12$ during the initial moment of deformation receives the greatest circumferential strain, which equal to $\frac{\sqrt{3}}{3}(1+n)$. Relative co-ordinate of the material element which has received the greatest circumferential strain is found from (10).

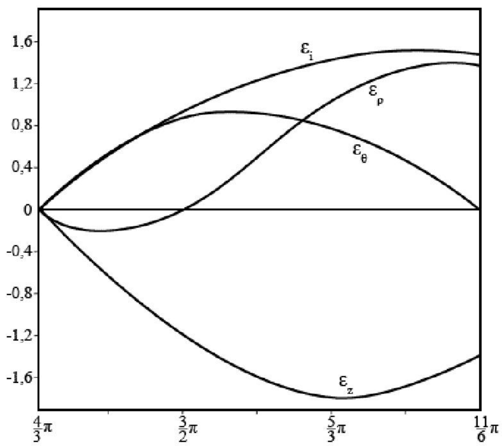


Fig. 2. Distribution of strain intensity and components at the strain initial stage of strain.

$$\rho / r_0 \approx 2{,}1 \cdot (1 + n)$$

At the second stage of strain together with the increase in current radius of aperture edge formation of a vertical wall of bead begins, and at the final moment of strain all material elements in circumferential direction receive the set size of strain defined by the relation of the sizes of an aperture and the deforming tool. On fig. 3 ways and distributions of deformations are presented at different strain values of aperture edge for parameter of strain hardening $n = 0.2$.

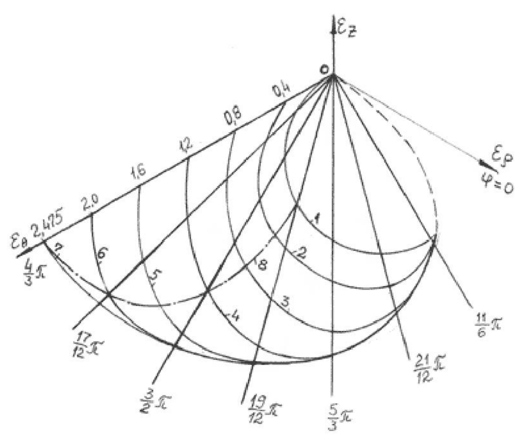


Fig. 3. Ways and distribution of deformations at the second stage of strain (curves 1 and 2-7 of strain distributions on initial and the subsequent stages of strain, directions $4\pi/3 \ldots\ldots 11\pi/6$ - ways of deformations, line 8 - line of the greatest circumferential strains.)

Given that the strain could not take zero values in leaps at the boundary of the un deformed blank ($\varphi = 11\pi/6$), on fig. by 3 dashed lines the area of smooth reduction of strain intensity from current sizes to zero is shown. At the second stage of strain forming process has strongly pronounced non-stationary character at which there is a continuous change of the values of the plastic strain area. The impossibility to define the current values of strain components from (7) in analytical form is connected with the impossibility of its representation in the form of explicit function from parametre $\varphi$ and strain value of aperture edge. Therefore definition of current values and character of change strain components change is carried out by graphic imaging of vector function $\bar{\varepsilon}_i(\varphi, \varepsilon_{кр.})$ on oblique axes $\varepsilon_\rho, \varepsilon_\theta, \varepsilon_z$. Fig. 4 presents the charts of current value changes of strain components at the variable strain area for n=0.2.

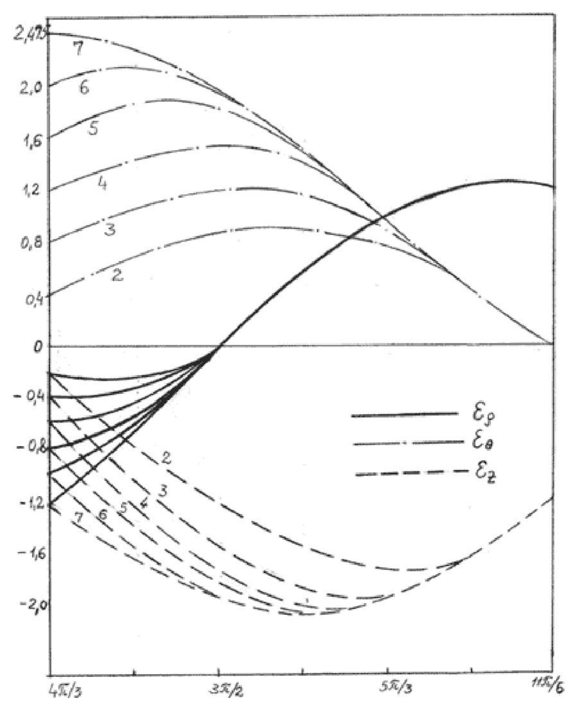


Fig. 4. Charts of Current value chamges of strain components at variable area (curves 2 - 7).

From the presented figures it follows, that for the entire area strain in thickness is negativ, whereas in circumferential it is positive. Radial strains in direction $\varphi = 3\pi/2$ change their sign as a result of which part of the area finds itself in the state of radial compression during the forming process, and part of it is in the state of biaxial stretching.

**Corresponding Author:** Najmeddin Arab
Department of Engineering, Saveh Branch,
Islamic Azad University, Iran
najmarab@iau/saveh.ac.ir

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