Wind Turbine Performance – The Betz Limit and Other Factors Nnorom ACHARA

MDPGA, Ministry of Defence, Wethersfield, Braintree, United Kingdom
nnoromeluwa@btinternet.com

Abstract: The power available from the wind has long been recognised and exploited in various ways including powering the sailing ships and wind mills. The Danes are regarded as the pioneers in wind turbine development. Although most studies on wind turbine losses found in the literature deal with tip losses and wake eddies especially in the wind farm environment, other losses relating to mechanical and electrical devices do exist in the system. The maximum energy that can be derived from the wing is given as 59% of the free stream value as established by the Betz limit. It is necessary that wind speed is of the appropriate order before choosing a site. Where this data is not available, the mean annual wind speed together with the Rayleigh Distribution function may be used to calculate the values. Obstacles in the wind route in a site will adversely affect the wind turbine performance. Performance is also affected by tower height. Losses occur in the bearings and the gear trains connecting the turbine and the generator. There are losses that are attributable to the diodes in converting the wild AC to DC for storage in the batteries and in the inverters if the power is used in an AC wired property and some of the energy generated may be used in running the battery bank.

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1. Introduction:

Wind energy is abundantly available and it is one of the few renewable energy forms that are easily exploitable. The power of the wind has long been recognised and this has been exploited in various ways. The technology has therefore evolved and matured over the years. For centuries wing energy has been employed in powering the sailing ships, the wind mills in grain grinding, the water pumps and other mechanical devices. Since the 1890's the Danes (Bhadra, 2004) have channelled their wind energy development efforts to electricity generation and are regarded therefore as pioneers in this sector. Before the oil crisis of the seventies, wind power development was virtually restricted to countries that have no coal, oil or gas resources. The crises and the realisation of the finiteness of fossil fuel, prompted the search for alternative sources of energy. Energy independence has a link with economic independence. The political volatility in the regions from which most of the fossil fuels are sourced does not help matter. This search has had an added urgency as a result of global warming caused by high concentrations of greenhouse gases which are associated with human activities in the form of deforestation and the burning fossil fuels (Achara, 1987, 2011). The main greenhouse gases include carbon dioxide, methane, water vapour, ozone and nitrous oxide. The nuclear sector would have provided a solution but for the possibility of uncontrollable fission reaction, due to human error or natural phenomena. The Chernobyl and Fukushima disasters in Ukraine and Japan are, respectively, glaring examples of what can happen with the nuclear power. Additionally, the ownership of a nuclear generator brings with it handling and storage problems of the spent fuel. This appears therefore to rule out, at least of now, nuclear energy as a credible alternative option. The issue is that global warming has assumed an international dimension; it is no more a cry from the green lobby alone as governments all over the world have woken up to the fact that we all have to reduce our individual carbon footprints. Energy policies have as a result tilted favourably towards renewable non polluting sources such as wind resources. In particular, the wind turbine has benefitted from the wealth of knowledge acquired over the years in the aviation industry especially with regards to blade design

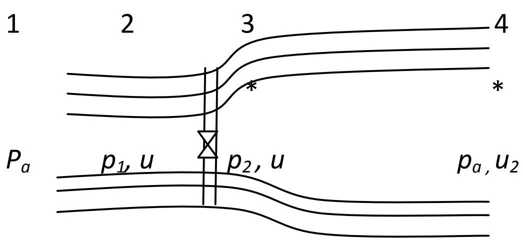
Notwithstanding the wealth of data from the aviation industry, the design of the wind turbine has it peculiarities. The designer has to consider three distinct issues including the wind turbine blade aerodynamic losses, the mechanical drives and the electrical generator. There is abundant material in the literature on blade design but even after getting the best blade configuration, if the mechanical and electrical aspects are not given adequate consideration, operational performance and system efficiency will be severely reduced.

The purpose of this study is therefore, in addition to the wind turbine aerodynamic losses, identify the other performance issues and losses including those relating to the mechanical and electrical components in the wind turbine system.

2. The Maximum Efficiency Attainable

The first task would be to establish the maximum power that can theoretically be extracted from the wind turbine.

Consider the fluid pressure and velocity variation over an ideal rotating disc. The pressure and velocity variations upstream and downstream of the disk are shown in figure 1 below.



Pressure and velocity values are – upstream, free stream: p_a and u_a just before disk: p_1 and u, and downstream of the disk: p_2 and u. At location *4* in the figure, it is assumed that the pressure has recovered sufficiently to approximate its free stream value. Another assumption implicit in this consideration is that the dynamic pressure loss of flow over the disk is negligible.

Difference in pressure across the disk, $p_1 - p_2$

It is assumed that the flow is axial and irrotational. Applying Bernoulli's theorem on two sides of the disk,

Upstream:

$$\tfrac{1}{2}\rho u_a^2 + p_a = \tfrac{1}{2}\rho u^2 + p_1 \quad (1)$$

and downstream

$$\tfrac{1}{2}\rho u_2^2 + p_a = \tfrac{1}{2}\rho u^2 + p_2 \quad (2)$$

Eqn(1) - eqn(2)

$$p_1 - p_2 = \tfrac{1}{2}\rho(u_a^2 - u_2^2) \quad (3)$$

where u_2 is the free stream velocity after the disk.

Thrust, T is given as $\Delta p.A$ where Δp is pressure difference and A = area

$$T = A(p_1 - p_2)$$
$$= \tfrac{1}{2}\rho A(u_a^2 - u_2^2) \quad (4)$$

If mass flow through the blade is m per second then thrust is also given by

$$T = m(u_a - u_2) \quad (5)$$

From eqn(4) and eqn(5)

$$m(u_a - u_2) = \rho Au(u_a - u_2) \quad (6$$

But $m = \rho Au$, therefore

$$\tfrac{1}{2}\rho A(u_a^2 - u_2^2) = \rho Au(u_a - u_2)$$

From which

$$u = \tfrac{1}{2}(u_a + u_2) \quad (6)$$

The axial induction/interference factor a may be defined in terms of:

$$u = u_a(1 - a) \quad (7)$$

From eqn(6) and eqn(7)

$$u_2 = u_a(1 - 2a) \quad (8)$$

Power extracted, P is given by the drop in kinetic energy over the system, thus

$$P = \tfrac{1}{2}\rho Au(u_a^2 - u_2^2) \quad (9)$$

Substituting for u and u_2 from eqn(7) and eqn(8)

$$P = 2\rho A\, u_a^3 a(1 - a)^2 \quad (10)$$

From eqn(10), it can be seen that power is a non linear function of the interference factor, a and that it is zero when $a = 0$ or 1. A point of inflection should therefore occur between $a = 0$ and $a = 1$, differentiating eqn(10) with respect to a

$$dP/da = 2\rho A\, u_a^3(1 - 4a + 3a^2) = 0 \quad (11)$$

From eqn(11)

$a = 1$ and $1/3$

$a = 1$ is spurious as it would give $u = 0$ which is impossible, therefore $a = 1/3$ is the only plausible

solution and this gives the maximum power extractible from eqn(10) as

$$8/27\rho A\, u_a^3 = 16/27 P_a \qquad (12)$$

Let the Power coefficient = C_p, therefore

C_p = power extracted/total power in system

$= (2\rho A\, u_a^3 a(1 - a))^2/(½\rho A\, u_a^3)$

$$\therefore \quad Cp = 4a(1 - a)^2 \qquad (12)$$

Substituting in eqn(12), a = 1/3 to give

$$C_{p,\,max} = 16/27 \qquad ((13)$$

This means that the maximum power extractable from the wind is 16/27 times the power content of the wind free stream and this is the limit established by Betz and it is commonly referred to as the *Betz Limit.* This is the theoretical aerodynamic efficiency. In practice however, efficiency of the turbine is highly influenced by the design of the rotor blade. The maximum efficiency obtainable at optimal rotation speed (the aerodynamic power coefficient) is nearer 35% with efficiency dropping off either side of that speed.

3. Other Performance Issues:

Losses due to eddies at the trailing edge increases as wind speed rises towards the furling speed of the particular design. In wind farm losses can also be sustained depending on the relative location of each of the turbines. The literature abound with studies relating mostly to eddies and wake blade losses and a few of these studies are given below. (Ivanell, 2009) used the actuator disk and line methods to computationally resolve the wake structure of both single turbine and wake interaction in a wind turbine farm and furthermore establish the basic mechanism controlling the length of the wake and tip vortices. (Singer et al, 2010) also used the actuator disk model to study the flow dynamics of both individual turbines and arrays of turbines in a wind farm. . In their own studies, (Madsen et al, 2005) used five pitot holes on an 80m blade to measure the flow dynamics in the wake to find that the upstream rotor creates about 15% additional turbulence intensity. (Sanderse et all, 2010) reviewed the literature on wake interaction whilst (Fletcher and Brown, 2009) used the vorticity transport model to simulate the interaction between rotors in both axial and yawed wind conditions. In their own study, (Mikklesen et al, 2010) studied the wake effects of the wind turbine operating within the turbulent regime.

3.1 Wind Turbine Site:

Before choosing the site, it is necessary to carry out economic assessment to determine its suitability for wind turbine installation. The range of factors that can impact on performance of an installation include availability of high enough wind speed (about 5m/s) minimum and open area with no obstacles within a 150m radius. Obstacles cause turbulence and turbulence leads to losses and consequently reduced turbine efficiency. The direction from which the prevailing wind flows is also very important and obstacles in the path of the prevailing wind should be avoided. Other factors that have to be considered include the availability of public grid. It will make economic sense for a remote site with no public electric grid utility to consider wind turbine installation even where wind availability is moderate. With generous subsidies, it may prove beneficial to build wind energy facility in a site of moderate wind speed where otherwise a wind system will not be competitive as it will produce electricity at a higher rate/kW.

3.2 Annual Wind Hours:

Moderate high wind speed per se is not the determinant but how many hours of it is available in one calendar year. As a result, part of the assessment of the wind site is a record of the annual distribution of wind speeds in hours at the site. Where this record is not available the mean annual wind speed together with Rayleigh distribution function can be used as shown in figure 2 to calculate these values. This function is expressed as:

$$t = 8760 \frac{\pi}{2} \frac{v}{\bar{v}^2} \exp\left(\frac{-\pi v^2}{4\bar{v}^2}\right) \qquad (14)$$

where t is the time in hours in the year, v the wind speed (m/s), and $\vec{v}$ the mean wind speed obtained at the hub height. The Rayleigh distribution is a special form of the more general Weibull distribution function. In the more general form, the Weibull distribution function, correction is made for factors such as vegetation, landscape and obstacles such as nearby houses. For wind speeds below 10m/s, the Rayleigh distribution does not hold. Figure 2 is a plot of the Rayleigh distribution function for a mean speed of 12m/s.

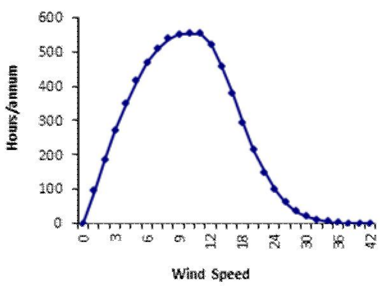


Figure2 Annual Wind Speed Hours

For the Weibull distribution, the effect of obstacles is accounted for by the scaling factor, *A* and form factor *k*. The Weibull distribution is thus expressed as:

$$h_w(v_0) = \frac{k}{A}\left(\frac{v_0}{A}\right)^{k-1} Exp\left(-\frac{v_0}{A}\right)^{k} \quad (15)$$

The values for the parameters A and k may be derived from meteorological data.

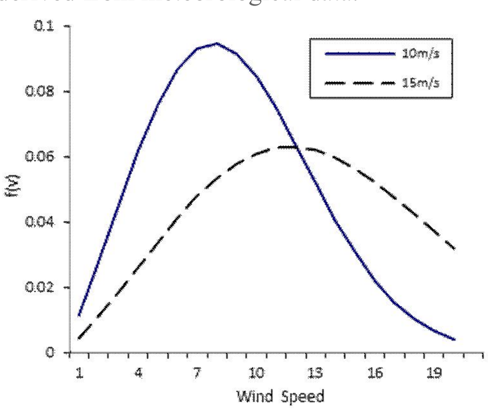


Figure.3 Weibull Wind Speed Distribution

Figure.3 is the Weibull distribution function plotted for mean wind speed values of 10m/s and 15m/s. To calculate the probability that the wind lies between v_i and v_{i+1}

$$f(v_i < v_0 < v_{i+1}) = e^{-\left(\frac{v_i}{A}\right)^k} - e^{-\left(\frac{v_{i+1}}{A}\right)^k} \quad (16)$$

Then the total annual energy production (AEP) can thus be evaluated as:

$$\text{AEP} = \sum_{i=1}^{N-1} \frac{1}{2}\left(P(v_{i+1}) + P(v_i)\right).f(v_i < v_0 < v_{i+1}) \quad (17)$$

where $P(v_i)$ is the power produced at mean wind speed v_i.

3.3 Tower Height

The tower plays important part in the installation of any successful wind turbine system. For the wind turbine to harvest enough energy from the wind, height has to be adequate. The tower design must be such to withstand the violent stresses from the wild wind. The cost of the tower may be as much as that of the turbine itself. Cost is a very important factor when designing the tower but a height of about 10m may be considered the adequate minimum in a site without obstacles (Bartmann and Fink 2009) and (Gipe, 1998).

When designing for an obstacle-rich environment, the height of the obstacles effectively represents ground level for wind speed estimation. Quality of wind improves away from the ground level. Turbulence is intensified near the ground level and this adversely affects the turbine performance. When calculating turbine output the nature of the landscape over which the wind travels is to be considered. This is known as 'Roughness'. It is rated in terms of class with values from 0 to 4. (Klemen, 2010) has rated the surface of the sea as *0* and for a landscape with many trees and buildings the value is given 4. Towers are of three types: fixed guyed, freestanding and tilt-up. The freestanding towers are either monopoles or lattice structures. The freestanding monopole towers are made of high-strength hollow tubular steel. The lattice tower consists of either tubular steel pipes or flat metal bars bolted or welded together.

3.4 Drives and Gear Train:

The mechanical drives and gearings connecting the wind turbine and the generator as well as the bearings supporting the wind turbine shaft will introduce losses that should not be ignored. The generator to be able to produce electricity generally rotates faster than the wind turbine and to achieve this, it is appropriately geared to the wind turbine shaft. (Hansen, 2008) has given the combined gear train and generator losses of the wind turbine system to be approximately 10%

3.5 Bridge Rectifier:

As current passes each diode there is voltage drop of about 0.7V (Piggott, 1977). In charging a battery therefore, since the current has to pass through two diodes about 1.4V voltage drop will be required to charge a 12V battery. This is over 10% of the battery voltage. In practice however, to cater for the period when the battery bank is nearing the full charge and charging performance becomes sluggish, the source voltage is in the range of 14.5V.

3.6 The Inverter:

Many of the wind turbines in use produce wild three-phase AC. The wild AC output is rectified to DC. For a grid-tied wind turbine system, the wild AC is converted in the inverter to AC in sync with the grid and this type is described as asynchronous inverter. The wind turbine system may also be grid – connected with battery backup. In order that the bank of batteries are kept fully charged about 5 to 10% daily power generated is used by the battery bank and this is a loss and is considered a performance issue. There is also the off-grid wind turbine system where the wild AC produced by the wind generator is converted to DC and stored in the battery bank. When electric power is needed the inverter reconverts the DC to AC for used in AC appliances. Where a DC circuit exists, the DC could be used directly thus bypassing further power losses in the inverter. DC circuits and appliances are however more complex and also more expensive than AC circuits.

3.7 Losses due to yaw

Conventional turbines need to turn into the wind to function. In small wind turbines this is achieved through the tail-vain. In the case of big turbines the control may be motorised and activated either by a fan-tail or a small turbine mounted on the main turbine. When there is however, a gust of wind the turbine can move (yaw) erratically around its axis without generating appreciable energy and this can result in power loss. Worse still, this can seriously contribute to mechanical failure.

3.8 The generator:

Generators convert rotational mechanical energy to electrical energy and in doing this part of the rotational energy is lost as heat in bearings and windings.

The generator's resistance to the rotor implies that the rotor will only start spinning at a certain speed known as the 'cut in' speed and is usually much lower than the optimal speed at which maximum power is generated. At the upper end, the cut-out or furling speed, the turbine is turned away from the wind and possibly shut down so as to protect the turbine from being damaged (Chiras D, 2010).

3.9 Cabling losses

Energy is always lost through transmission and as a result the specification and design of the cabling should be considered carefully so as to minimise this losses over distance. There is a difference between cabling designed to deliver power to a battery and cabling designed to link in with a grid (i.e. either directly to a domestic distribution unit or larger grid). Cable sizing for a DC circuit is usually heavier than a corresponding AC circuit.

4. Conclusion:

1. The maximum power extractable from a wind turbine as established by Betz is 16/27 (59.2%)

2. After the Betz limit, other performance issues and losses including mechanical and electrical components have been identified.

3. All the energy content of the wind turbine before the cut-in speed and after the cut-out speed or furling is wasted. It is not unusual to turn the wind turbine away from the wind direction at furling in order to prevent damaging the turbine blades.

4. Performance is enhanced as tower height increases. The minimum tower height in a site without obstacles in the direction of the wind is about 10m for a small wind turbine installation.

5. A site for wind turbine installation should have a minimum radius of 150m clear without obstacles.

6. Designing the cabling for a DC circuit is different from the corresponding AC design. To minimise losses, the DC circuit requires heavier cables and these are quite expensive.

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