Applying fuzzy AHP and fuzzy TOPSIS to Machine Selection

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Abstract: Machine selection is a multi-criteria decision problem and has a strategic importance for many companies. The conventional methods for Machine selection are inadequate for dealing with the imprecise or vague nature of linguistic assessment. To overcome this difficulty, fuzzy multi-criteria decision-making methods are proposed. The aim of this study is to use fuzzy analytic hierarchy process (AHP) and the fuzzy technique for order preference by similarity to ideal solution (TOPSIS) methods for the selection of Machine. The proposed methods have been applied to Machine selection problem of an electerofan company in Iran. After determining the criteria that affect the decisions, fuzzy AHP and fuzzy TOPSIS methods are applied to the problem and results are presented.
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1. Introduction

The selection of appropriate machines is one of the most crucial decisions for a manufacturing company to develop an efficient production environment. Improperly selected machines can negatively affect the overall performance of a production system. Since the selection of machines is a time consuming and difficult process, requiring advanced knowledge and experience, that it may cause several problems for the engineers and managers (Yang et al. 1997). The evaluation data of machine selection problem for various subjective criteria and the weights of the criteria are usually expressed in linguistic terms. Thus in this paper, fuzzy AHP and fuzzy TOPSIS methods are proposed for Machine selection, where the ratings of various alternative under various subjective criteria and the weights of all criteria are represented by fuzzy numbers. In this paper, in spite of other researches, fuzzy AHP and fuzzy TOPSIS methods are proposed for Machine selection. The remainder of this paper is organized as follows. Fuzzy sets, linguistic variables and fuzzy numbers are briefly explained in Sect. 2. Then in Sect. 3, fuzzy AHP method is introduced. In Sect. 4, fuzzy TOPSIS method is explained and the steps of proposed method are summarized. In Sect. 5, the application of proposed methods in Electerofan Company is illustrated and finally Sect. 6 concludes the paper.

2. Fuzzy sets

In order to deal with vagueness of human thought, Zadeh (1965) first introduced the fuzzy set theory. A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function which assigns to each object a grade of membership ranging between zero and one (Zadeh 1965). A fuzzy set is an extension of a crisp set. Crisp sets only allow full membership or non membership at all, whereas fuzzy sets allow partial membership. In other words, an element may partially belong to a fuzzy set (Ertuğrul et al. 2006). Fuzzy sets and fuzzy logic are powerful mathematical tools for modeling: uncertain systems in industry, nature and humanity; and facilitators for commonsense reasoning in decision-making in the absence of complete and precise information. Their role is significant when applied to complex phenomena not easily described by traditional mathematical methods, especially when the goal is to find a good approximate solution (Bojadziev et al. 1998). Fuzzy sets theory providing a more widely frame than classic sets theory, has been contributing to capability of reflecting real world (Ertuğrul et al. 2007). Modeling using fuzzy sets has proven to be an effective way for formulating decision problems where the information available is subjective and imprecise (Zimmermann 1992).

2.1. Linguistic variable

A linguistic variable is a variable whose values are words or sentences in a natural or artificial language (Zadeh 1975). As an illustration, *age* is a linguistic variable if its values are assumed to be the fuzzy variables labeled *young, not young, young, not very young*, etc. rather than the numbers 0, 1,2,3… (Bellman et al. 1977) .The concept of a linguistic variable provides a means of approximate characterization of phenomena which are too complex or too ill-defined to be amenable to description in conventional quantitative terms. The main applications of the linguistic approach lie in the realm of humanistic systems-especially in the fields of artificial intelligence, linguistics, human decision processes, pattern recognition, psychology, law, medical diagnosis, information retrieval, economics and related areas (Zadeh 1975).

2.2. Fuzzy numbers

A fuzzy number $\tilde{M}$ is a convex normalized fuzzy set $\tilde{M}$ of the real line R such that (Zimmermann 1992):

– It exists such that one $x_0 \in R$ with $\mu_{\tilde{M}}(x_0) = 1$(x_0 is called mean value of ($\tilde{M}$)

– $\mu_{\tilde{M}}(x)$ is piecewise continuous.

It is possible to use different fuzzy numbers according to the situation. In applications it is often convenient to work with triangular fuzzy numbers (TFNs) because of their computational simplicity, and they are useful in promoting representation and information processing in a fuzzy environment. In this study TFNs are adopted in the fuzzy AHP and fuzzy TOPSIS methods. Triangular fuzzy numbers can be defined as a triplet (l, m, u). The parameters l, m, and u. respectively, indicate the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event. A triangular fuzzy number $\tilde{M}$ is shown in Fig. 1 (Deng 1999).

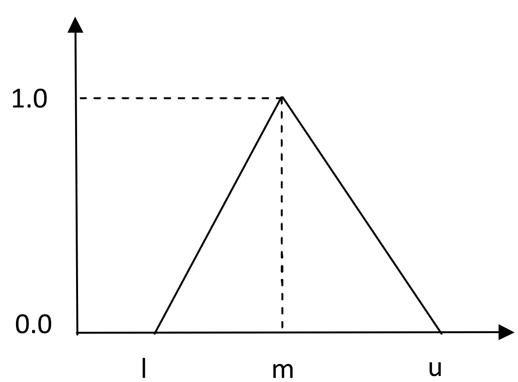

Fig. 1. Triangular fuzzy number

There are various operations on triangular fuzzy numbers. But here, only important operations used in this study are illustrated. If we define, two positive triangular fuzzy numbers (l_1, m_1, u_1) and (l_2, m_2, u_2) then:

$$(l_1, m_1, u_1) + (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2) \quad (1)$$

$$(l_1, m_1, u_1) \,.\, (l_2, m_2, u_2) = (l_1 . l_2, m_1 . m_2, u_1 . u_2) \quad (2)$$

$$(l_1, m_1, u_1)^{-1} \approx (1/u_1, 1/m_1, 1/l_1) \quad (3)$$

$$(l_1, m_1, u_1) \,.\, k = (l_1 k, m_1 k, u_1 k) \quad (4)$$

(k is a positive real number)

The distance between two triangular fuzzy numbers can be calculated by vertex method (Chen 2000):

$$d_v(\tilde{m}, \tilde{n}) = \sqrt{\frac{1}{3}[(l_1 - l_2)^2 + (m_1 - m_2)^2 + (u_1 - u_2)^2]} \quad (5)$$

3. Fuzzy analytic hierarchy process

First proposed by Thomas L. Saaty (1980), the analytic hierarchy process (AHP) is a widely used multiple criteria decision-making tool. The analytic hierarchy process, since its invention, has been a tool at the hands of decisionmakers and researchers, becoming one of the most widely used multiple criteria decision-making tools (Vaidya et al. 2006). Although the purpose of AHP is to capture the expert's knowledge, the traditional AHP still cannot really reflect the human thinking style (Kahraman et al. 2003). The traditional AHP method is problematic in that it uses an exact value to express the decision maker's opinion in a comparison of alternatives (Wang et al. 2007). And AHP method is often criticized, due to its use of unbalanced scale of judgments and its inability to adequately handle the inherent uncertainty and imprecision in the pairwise comparison process (Deng 1999). To overcome all these shortcomings, fuzzy analytical hierarchy process was developed for solving the hierarchical problems. Decision-makers usually find that it is more accurate to give interval judgments than fixed value judgments. This is because usually he/she is unable to make his/her preference explicitly about the fuzzy nature of the comparison process (Kahraman et al. 2003). The first study of fuzzy AHP is proposed by Van Laarhoven and Pedrycz (1983), which compared fuzzy ratios described by triangular fuzzy numbers. Buckley (1985) initiated trapezoidal fuzzy numbers to express the decision maker's evaluation on alternatives with respect to each criterion Chang (1996) introduced a new approach for handling fuzzy AHP, with the use of triangular fuzzy numbers for pair-wise comparison scale of fuzzy AHP, and the use of the extent analysis method for the synthetic extent values of the pair-wise comparisons. Fuzzy AHP method is a popular approach for multiple criteria decision-making. In this study the extent fuzzy AHP is utilized, which was originally introduced by Chang (1996). Let X = {x_1, x_2, x_3,....., x_n} an object set, and G = {g_1, g_2, g_3,....., g_n} be a goal set. Then, each object is taken and extent analysis for each goal is performed, respectively. Therefore, m extent analysis values for each object can be obtained, with the following signs:

$\tilde{M}_{g_i}^1, \tilde{M}_{g_i}^2, \ldots, \tilde{M}_{g_i}^m$, i =1, 2,…,n

Where $\tilde{M}_{g_i}^j$(j=1,2,3,…, m) are all triangular fuzzy numbers. The steps of the Chang's (1996) extent analysis can be summarized as follows:

Step **1**: The value of fuzzy synthetic extent with respect to the ith object is defined as:

$$S_i = \sum_{j=1}^{m} \tilde{M}_{g_i}^j \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}_{g_i}^j\right]^{-1} \quad (6)$$

Where $\otimes$ denotes the extended multiplication of two fuzzy numbers. In order to obtain $\sum_{j=1}^{m} \tilde{M}_{g_i}^j$,we perform the addition of m extent analysis values for a particular matrix such that,

$$\sum_{j=1}^{m} \tilde{M}_{g_i}^j = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right) \quad (7)$$

And to obtain $\left[\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}_{g_i}^j\right]^{-1}$, we perform the fuzzy addition operation of $\tilde{M}_{g_i}^j$ (j =1,2,…,m) values such that,

$$\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}_{g_i}^j = \left(\sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i\right) \quad (8)$$

Then, the inverse of the vector is computed as,

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}_{g_i}^j\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i}\right) \quad (9)$$

Where u_i , m_i , l_i >0

Finally, to obtain the S_j, we perform the following multiplication:

$$S_i = \sum_{j=1}^{m} \tilde{M}_{g_i}^j \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}_{g_i}^j\right]^{-1} = \left(\sum_{j=1}^{m} l_j \otimes \sum_{i=1}^{n} l_i, \sum_{j=1}^{m} m_j \otimes \sum_{i=1}^{n} m_i, \sum_{j=1}^{m} u_j \otimes \sum_{i=1}^{n} u_i\right) \quad (10)$$

Step 2: The degree of possibility of $\tilde{M}_2 = (l_2, m_2, u_2) \geq \tilde{M}_1 = (l_1, m_1, u_1)$ is defined as

$$V(\tilde{M}_2 \geq \tilde{M}_1) = \sup[\min(\tilde{M}_1(x), \tilde{M}_2(y))] \quad (11)$$

This can be equivalently expressed as,

$$V(\tilde{M}_2 \geq \tilde{M}_1) = \operatorname{hgt}(\tilde{M}_1 \cap \tilde{M}_2) = \tilde{M}_2(d) = \begin{cases} 1 & \text{if } m_2 \geq m_1 \\ 0 & \text{if } l_1 > u_2 \\ \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & \text{otherwise} \end{cases} \quad (12)$$

Fig. 2 illustrates $V(\tilde{M}_2 \geq \tilde{M}_1)$ for the case d for the case $m_1 < l_1 < u_2 < m_1$, where d is the abscissa value corresponding to the highest crossover point D between $\tilde{M}_1$ and $\tilde{M}_2$,To compare $\tilde{M}_1$ and $\tilde{M}_2$, we need both of the values $V(\tilde{M}_1 \geq \tilde{M}_2)$ and $V(\tilde{M}_2 \geq \tilde{M}_1)$.

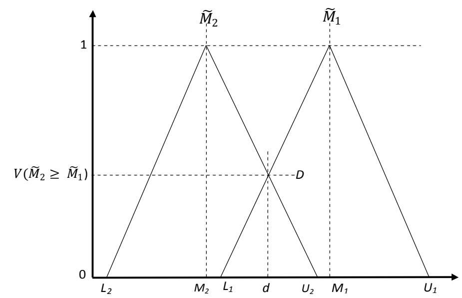

Fig. 2. The intersection between M_1 and M_2 (Chang 1996)

Step 3: The degree of possibility for a convex fuzzy number to be greater than k convex fuzzy numbers M_i (i=1, 2… K) is defined as

$V(\tilde{M} \geq \tilde{M}_1, \tilde{M}_2, \ldots, \tilde{M}_k) = \min V(\tilde{M} \geq \tilde{M}_i)$,
i =1,2,…,k

Step 4: Finally, W=(min V($s_1 \geq s_k$) , min V($s_2 \geq s_k$),….,min V($s_n \geq s_k$))^T, is the weight vector for k = 1,. . .,n.

4. Fuzzy TOPSIS method

The TOPSIS method was firstly proposed by Hwang and Yoon (1981). The basic concept of this method is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from negative ideal solution. Positive ideal solution is a solution that maximizes the benefit criteria and minimizes cost criteria, whereas the negative ideal solution maximizes the cost criteria and minimizes the benefit criteria (Wang et al.2006). In the classical TOPSIS method, the weights of the criteria and the ratings of alternatives are known precisely and crisp values are used in the evaluation process. However, under many conditions crisp data are inadequate to model real-life decision problems. Therefore, the fuzzy TOPSIS method is proposed where the weights of criteria and ratings of alternatives are evaluated by linguistic variables represented by fuzzy numbers to deal with the deficiency in the traditional TOPSIS. There are many applications of fuzzy TOPSIS in the literature. For instance, Triantaphyllou and Lin (1996) developed a fuzzy version of the TOPSIS method based on fuzzy arithmetic operations, which leads to a fuzzy relative closeness for each alternative. Chen (2000) extended the TOPSIS to the fuzzy environment and gave a numerical example of system analysis engineer selection for a software company. Tsaur et al. (2002) applied fuzzy set theory to evaluate the service quality of airline. Chu (2002) presented a fuzzy TOPSIS model under group decisions for solving the facility location selection problem. Chu and Lin (2003) proposed the fuzzy TOPSIS method for robot selection. Abo-Sinna and Amer (2005) extended the TOPSIS approach to solve the multi-objective large-scale nonlinear programming problems with block angular structure. Saghafian and Hejazi (2005) proposed a modified fuzzy TOPSIS method for the multi-criteria selection problem when there is a group of decision-makers. And they proposed a new distance measure for fuzzy TOPSIS. Wang and Elhag (2007) proposed a fuzzy TOPSIS method based on alpha level sets and presented a nonlinear programming solution procedure for bridge risk assessment. Jahanshahloo et al. (2006) extended the TOPSIS method to decision-making problems with fuzzy data and they used the concept of α-cuts to normalize fuzzy numbers. Chen et al. (2006) presented a fuzzy TOPSIS approach to deal with the supplier selection problem in supply chain system. Bottani and Rizzi (2006) proposed a multattribute approach based on TOPSIS technique and fuzzy set theory for the selection and ranking of the most suitable service provider. Wang and Chang (2007) developed an evaluation approach based on the fuzzy TOPSIS to help the Air Force Academy in Taiwan to choose initial training aircraft. Li (2007) gave a comparative analysis of compromise ratio method and the extended fuzzy TOPSIS method and illustrated a numerical example by showing their similarity and differences. Benitez et al. (2007) presented a fuzzy TOPSIS method for measuring quality of service in the hotel industry. Yang and Hung (2007) proposed to use TOPSIS and fuzzy TOPSIS methods for plant layout design problem. Wang and Lee (2007) generalized TOPSIS to fuzzy multiple-criteria group decision-making in a fuzzy environment. They proposed two operators Up and Lo that are employed to find ideal and negative ideal solutions. In this paper, the extension of TOPSIS method is considered which was proposed by Chen (2000) and Chen et al. (2006) the algorithm of this method can be described as follows:

Step 1: First of all a committee of decision-makers is formed. In a decision committee that has K decision-makers; fuzzy rating of each decision maker D_k = (k=1, 2,...K) can be represented as triangular fuzzy number $\tilde{R}_k$ = (k=1, 2,...K) with membership function $\mu_{\tilde{R}_k}(x)$.

Step 2: Then evaluation criteria are determined.

Step 3: After that, appropriate linguistic variables are chosen for evaluating criteria and alternatives.

Step 4: Then the weight of criteria are aggregated (Chen et al. 2006). If the fuzzy ratings of all decision-makers are described as triangular fuzzy numbers $\tilde{R}_k$ = (ak ,bk ,ck) ,k = 1,2,…,k, then the aggregated fuzzy rating can be determined as

$\tilde{R}$ = (a,b,c) , k =1,2,…,k. Here;

$$a = \min_k[a_k] \ , \ b = \frac{1}{k}\sum_{k=1}^{k} b_k \ , \quad c = \max_k[c_k] \qquad (13)$$

If the fuzzy rating and importance weight of the kth decision-maker are $\tilde{x}_{ijk}$ = (a_{ijk}, b_{ijk}, c_{ijk}) and $\tilde{w}_{ijk}$ = (w_{jk1} ,w_{jk2} ,w_{jk3}), i= 1,2,…,m , j= 1,2,…,n respectively, then the aggregated fuzzy ratings ($\tilde{x}_{ij}$) of alternatives with respect to each criterion can be found as ($\tilde{x}_{ij}$) = (a_{ij} ,b_{ij}, c_{ij})

Here,

$$a_{ij} = \min_k[a_{ijk}] \ , \ b_{ij} = \frac{1}{k}\sum_{k=1}^{k} b_{ijk} \ , \quad c_{ij} = \max_k[c_{ijk}] \qquad (14)$$

Then the aggregated fuzzy weights ($\tilde{w}_{ij}$) of each criterion are calculated as:

$$(\tilde{w}_j) = (w_{j1}, w_{j2}, w_{j3}) \qquad (15)$$

Here,

$$W_{j1} = \min_k[w_{jk1}] \ , \ w_{j2} = \frac{1}{k}\sum_{k=1}^{k} w_{jk2} \ , \quad w_{j3} = \max_k[w_{jk3}]$$

Step 5: Then the fuzzy decision matrix is constructed as:

$$\tilde{D} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix}$$

$$\tilde{w} = [\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n]$$

Here $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ and $(\tilde{w}_j) = (w_{j1}, w_{j2}, w_{j3})$; $i=1,2,\ldots,m$, $j=1,2,\ldots,n$ can be approximated by positive triangular fuzzy numbers.
Step 6: After constructing the fuzzy decision matrix, it is normalized. Instead of using complicated normalization formula of classical TOPSIS, the linear scale transformation can be used to transform the various criteria scales into a comparable scale. Therefore, we can obtain the normalized fuzzy decision matrix $\tilde{R}$ (Chen 2000).

$$\tilde{R} = [\tilde{r}_{ij}]_{m\times n} \quad i = 1,2,\ldots,m \quad j=1,2,\ldots,n \tag{16}$$

Where:

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}\right),$$

$$c_j^* = \max_i c_{ij}$$

Step 7: Considering the different weight of each criterion, the weighted normalized decision matrix is computed by multiplying the importance weights of evaluation criteria and the values in the normalized fuzzy decision matrix. The weighted normalized decision matrix $\tilde{V}$ is defined as:

$$\tilde{V} = [\tilde{v}_{ij}]_{m\times n} \quad i = 1,2,\ldots,m \quad j=1,2,\ldots,n \tag{17}$$

$$\tilde{V}_{ij} = \tilde{r}_{ij} \,(.)\, \tilde{w}_j$$

Here $\tilde{w}_j$ represents the importance weight of criterion C_j.
According to the weighted normalized fuzzy decision matrix, normalized positive triangular fuzzy numbers can also approximate the elements $\tilde{V}_{ij}$, $\forall i, j$.

Step 8: Then, the fuzzy positive ideal solution (FPIS, A^*) and fuzzy negative ideal solution (FNIS,A^-) are determined as (Chen et al. 2006):

$$A^* = (\tilde{V}_1^*, \tilde{V}_2^*, \ldots, \tilde{V}_n^*) \tag{18}$$

$$A^- = (\tilde{V}_1^-, \tilde{V}_2^-, \ldots, \tilde{V}_n^-), \tag{19}$$

Where

$\tilde{V}_j^* = \max_i [w_{ij3}]$ and $\tilde{V}_j^- = \min_i [w_{ij1}]$,$i=1,2,\ldots,m$, $j=1,2,\ldots,n$.

Step 9: Then the distance of each alternative from FPIS and FNIS are calculated as:

$$d_i^* = \sum_{j=1}^{n} d_v(\tilde{v}_{ij}, \tilde{v}_j^*), \; i=1,2,\ldots,m \tag{20}$$

$$d_i^- = \sum_{j=1}^{n} d_v(\tilde{v}_{ij}, \tilde{v}_j^-), \; i=1,2,\ldots,m \tag{21}$$

where $d_v(.,.)$ is the distance measurement between two fuzzy numbers.

Step 10: A closeness coefficient (CC_i) is defined to rank all possible alternatives. The closeness coefficient represents the distances to the fuzzy positive ideal solution (A^*) and fuzzy negative ideal solution (A^-) simultaneously. The closeness coefficient of each alternative is calculated as (Chen 2000):

$$CC_i = \frac{d_i^-}{d_i^- + d_i^*}, \; i=1,2,\ldots,m \tag{22}$$

Step 11: According to the closeness coefficient, the ranking of the alternatives can be determined. Obviously, according to Eq. (22) an alternative A_i would be closer to FPIS and farther from FNIS as CC_i approaches to 1. The general steps of fuzzy TOPSIS method (Chen 2000) can be summarized as in the Fig. 3.

5. Application in Electerofan Company

Electerofan Company desires to select best machine from three alternatives (A_1, A_2, A_3). First of all, a committee of decision-makers is formed. There are three decision-makers (D_1, D_2, D_3) in the

committee. Then evaluation criteria are determined as Quality (C_1), Payment Terms (C_2), After-Sale Service (C_3), Capacity (C_4), and Technology (C_5). The hierarchical structure for the selection of the best machine is seen as in Fig. 4.

5.1. Application with TOPSIS method

In this section fuzzy TOPSIS method is proposed for machine selection problem of this company. Firstly, three decision-makers evaluated the importance of criteria by using the linguistic variables in Table 1. The importance weights of the criteria determined by these three decision-makers are shown in Table 2.

Table 1: Linguistic variables for importance weight of each criterion

Linguistic variables	Triangular fuzzy numbers
Very low (VL)	(0,0,0.2)
Low (L)	(0.1,0.2,0.3)
Medium low (ML)	(0.2,0.35,0.5)
Medium (M)	(0.4,0.5,0.6)
Medium high (MH)	(0.5,0.65,0.8)
High (H)	(0.7,0.8,0.9)
Very high (VH)	(0.8,1,1)

Table 2: Importance weight of criteria from three decision-makers

	Decision-makers		
Criteria	D_1	D_2	D_3
C_1	VH	H	VH
C_2	H	MH	M
C_3	VH	VH	H
C_4	MH	H	H
C_5	H	MH	VH

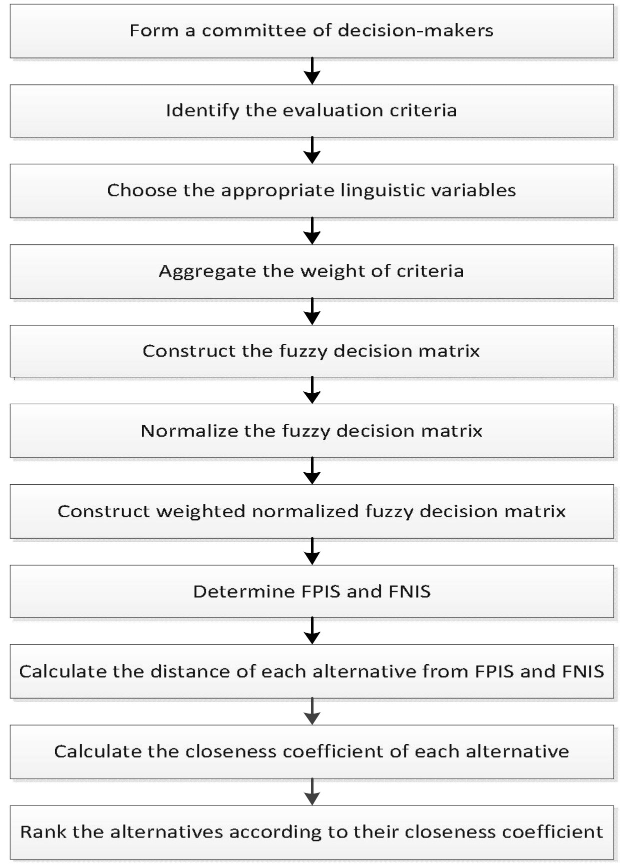

Fig. 3. The steps of the proposed method

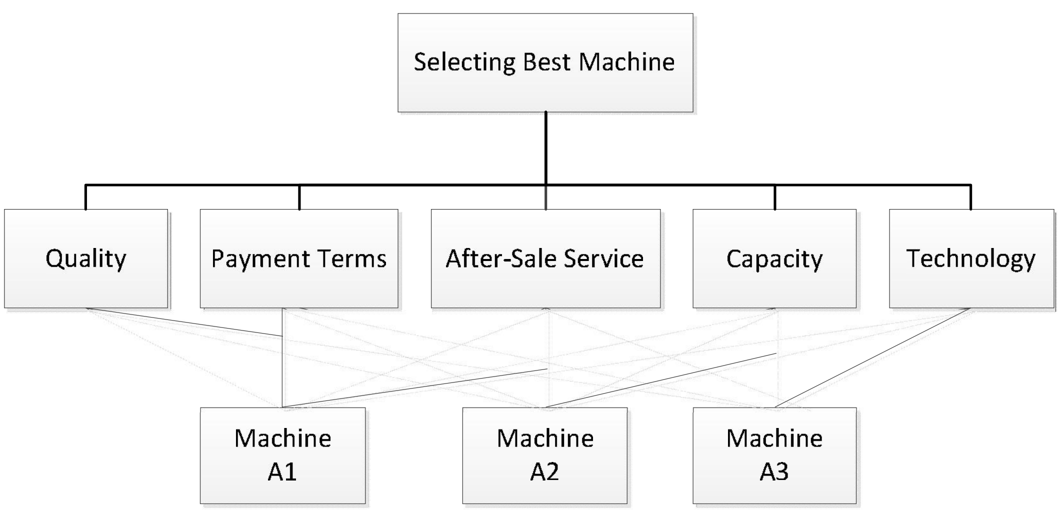

Fig. 4. Hierarchical structure of facility location selection process

Three decision-makers use the linguistic variables shown in Table 3 to evaluate the ratings of alternatives with respect to each criterion. The ratings of three alternatives under five criteria are shown in Table 4.

Then linguistic variables shown in Tables 2 and 4 are converted into triangular fuzzy numbers to form fuzzy decision matrix as shown in Table 5. The normalized fuzzy decision matrix is formed as in Table 6.

Then weighted normalized fuzzy decision matrix is formed as in Table 7. After a weighted normalized fuzzy decision matrix is formed, fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS) are determined as in the following:

$A^* = [\ (.49,.87,1)\ ,\ (.56,.93,1)\ ,(.35,.76,1)]$

$A^- = [(.2,.49,.81)\ ,\ (.05,.29,.54)\ ,\ (.14,.47,.8)]$

Then the distance of each alternative from FPIS and FNIS with respect to each criterion is calculated .Here only the calculation of the distance of the first alternative to FPIS and FNIS for the first criterion is shown, as the calculations are similar in all steps. The results of all alternatives' distances from FPIS and FNIS are shown in Tables 8 and 9. d_i^* and d_i^- of three alternatives are shown in Table 10.

According to the closeness coefficient of three alternatives, the ranking order of three alternatives is determined as $A_3>A_1>A_2$. The third alternative is determined as the most appropriate machine for this company. In other words, the third alternative is closer to the FPIS and farther from the FNIS.

Table 3: Linguistic variables for ratings

Linguistic ariables	Triangular fuzzy numbers
Very poor (VP)	(0,0,2)
Poor (P)	(1,2,3)
Medium poor (MP)	(2,3.5,5)
Fair (F)	(4,5,6)
Medium good (MG)	(5,6.5,8)
Good (G)	(7,8,9)
Very good (VG)	(8,10,10)

Table 4: Ratings of the three alternatives by decision-makers under five criteria

		Decision-makers		
Criteria	Alternatives	D_1	D_2	D_3
C_1	A_1	VG	G	G
	A_2	VG	VG	VG
	A_3	MG	VG	G
C_2	A_1	G	G	MG
	A_2	MG	F	G
	A_3	G	MG	VG
C_3	A_1	VG	G	VG
	A_2	MG	G	VG
	A_3	MP	F	MG
C_4	A_1	VG	G	MG
	A_2	F	F	MP
	A_3	VG	VG	G
C_5	A_1	MG	G	MG
	A_2	P	F	MP
	A_3	G	G	G

Table 5: Fuzzy decision matrix and fuzzy weights of three alternatives

	A_1	A_2	A_3	Weight
C_1	(7,8.67,10)	(8,10,10)	(5,8.17,10)	(.7,.93,1)
C_2	(5,7.5,9)	(5,7,9)	(4,7.67,10)	(.4,.65,.9)
C_3	(7,9.33,10)	(5,8.17,10)	(2,5,8)	(.7,.93,1)
C_4	(5,8.17,10)	(2,4.5,6)	(7,9.33,10)	(.5,.75,.9)
C_5	(5,7,9)	(1,3.5,6)	(7,8,9)	(.5,.82,1)

Table 6: Normalized fuzzy decision matrix

	A_1	A_2	A_3
C_1	(.7,.87,1)	(.8,1,1)	(.5,.82,1)
C_2	(.5,.75,.9)	(.5,.7,.9)	(.4,.77,1)
C_3	(.7,.93,1)	(.5,.82,1)	(.2,.5,.8)
C_4	(.5,.82,1)	(.2,.45,.6)	(.7,.93,1)
C_5	(.5,.7,.9)	(.1,.35,.6)	(.7,.8,.9)

Table7: Weighted normalized fuzzy decision matrix

	A_1	A_2	A_3
C_1	(.49,.81,1)	(.56,.93,1)	(.35,.76,1)
C_2	(.2,.49,.81)	(.2,.46,.81)	(.16,.5,.9)
C_3	(.49,.87,1)	(.35,.76,1)	(.14,.47,.8)
C_4	(.25,.61,.9)	(.1,.34,.54)	(.35,.7,.9)
C_5	(.25,.57,.9)	(.05,.29,.6)	(.35,.65,.9)

Table 8: Distances between A_i (i =1, 2, 3) and A* with respect to each criterion

	C_1	C_2	C_3	C_4	C_5
$d(A_1,A^*)$	0.04	0.30	0.00	0.21	0.23
$d(A_2,A^*)$	0.00	0.36	0.16	0.51	0.53
$d(A_3,A^*)$	0.00	0.20	0.24	0.07	0.09

Table 9: Distances between A_i (i =1 ,2, 3) and A^- with respect to each criterion

	C_1	C_2	C_3	C_4	C_5
$d(A_1,A^-)$	0.27	0.00	0.30	0.09	0.08
$d(A_2,A^-)$	0.54	0.20	0.42	0.04	0.03
$d(A_3,A^-)$	0.24	0.06	0.00	0.19	0.17

Table 10: Computations of d_i^* , d_i^- and CC_i

	A_1	A_2	A_3	Ranking order
d_i^*	0.78	1.56	0.59	$A_3 > A_1 > A_2$
d_i^-	0.74	1.24	0.66	
CC_i	0.49	0.44	0.53	

5.2. Application with fuzzy AHP method

In this section, fuzzy AHP method is proposed for the same problem of Electerofan

Company. We proposed a group decision based on fuzzy AHP. Firstly each decision-maker (D_p), individually carry out pair-wise comparison by using Saaty's 1–9 scale as in Eq. (23). (Chen 2004):

$$D_p = \begin{bmatrix} b_{11p} & b_{12p} & \cdots & b_{1mp} \\ b_{21p} & b_{22p} & \cdots & b_{2mp} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1p} & b_{m2p} & \cdots & b_{mmp} \end{bmatrix}, p = 1,2,\ldots,t \quad (23)$$

Three decision-makers' pair-wise comparisons of for the five criteria are as follows:

$$D_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 1 & 3 & 3 & 7 & 7 \\ 1/3 & 1 & 1 & 5 & 3 \\ 1/3 & 1 & 1 & 3 & 1/3 \\ 1/7 & 1/5 & 1/3 & 1 & 1/3 \\ 1/7 & 1/3 & 3 & 3 & 1 \end{bmatrix} \end{array}$$

$$D_2 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 1 & 1 & 7 & 1/5 & 3 \\ 1 & 1 & 5 & 1/3 & 3 \\ 1/7 & 1/5 & 1 & 1/3 & 1/5 \\ 5 & 3 & 3 & 1 & 3 \\ 1/3 & 1/3 & 5 & 1/3 & 1 \end{bmatrix} \end{array}$$

$$D_3 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 1 & 3 & 5 & 1/5 & 1/3 \\ 1/3 & 1 & 3 & 1/7 & 1/5 \\ 1/5 & 1/3 & 1 & 1/5 & 1/7 \\ 5 & 7 & 5 & 1 & 3 \\ 3 & 5 & 7 & 1/3 & 1 \end{bmatrix} \end{array}$$

Then, a comprehensive pair-wise comparison matrix is built as in Table 11 by integrating three decision-makers' grades through Eq. (24) (Chen 2004). By this way, decision-makers' pair-wise comparison values are transformed into triangular fuzzy numbers.

$$L_{je} = \min (b_{jep}), \quad m_{je} = \frac{\sum_{p=1}^{t} b_{jep}}{p}$$

$$U_{je} = \max (b_{jep}), \; p = 1,2,\ldots,t \quad j = 1,2,\ldots,m \quad e = 1,2,\ldots,m \quad (24)$$

Table 11: The fuzzy evaluation matrix with respect to goal

D	C_1	C_2	C_3	C_4	C_5
C_1	(1,1,1)	(1,2.33,3)	(3,5,7)	(.2,2.47,7)	(.33,3.44,7)
C_2	(.33,.56,1)	(1,1,1)	(1,3.25,5)	(.14,1.83,5)	(.2,2.07,3)
C_3	(.14,.23,.33)	(.2,.51,1)	(1,1,1)	(.2,1.18,3)	(.14,.23,.33)
C_4	(.14,3.38,5)	(.2,3.4,7)	(.33,.85,5)	(1,1,1)	(.33,2.11,3)
C_5	(.14,1.16,3)	(.33,1.89,5)	(3,1,7)	(.33,1.22,3)	(1,1,1)

From Table 11, according to extent analysis synthesis values respect to main goal are calculated like in Eq. (6):

$Sc_1 = (5.53, 14.24, 25.00) \otimes (0.0116, 0.0232, 0.0598) = (0.064591, 0.330553, 1.495726)$
$Sc_2 = (2.68, 8.70, 15.00) \otimes (0.0116, 0.0232, 0.0598) = (0.03124, 0.201835, 0.897436)$
$Sc_3 = (1.69, 3.14, 5.67) \otimes (0.0116, 0.0232, 0.0598) = (0.019678, 0.072859, 0.339031)$
$Sc_4 = (2.01, 10.74, 21.00) \otimes (0.0116, 0.0232, 0.0598) = (0.023457, 0.249256, 1.25641)$
$Sc_5 = (4.81, 6.27, 19.00) \otimes (0.0116, 0.0232, 0.0598) = (0.056142, 0.145497, 1.136752)$

These fuzzy values are compared by using Eq. (12), and these values are obtained:

$V(Sc_1 \geq Sc_2) = 1$ $V(Sc_1 \geq Sc_3) = 1$
$V(Sc_1 \geq Sc_4) = 1$ $V(Sc_1 \geq Sc_5) = 1$
$V(Sc_2 \geq Sc_1) = 0.623495$ $V(Sc_2 \geq Sc_3) = 1$

$V(Sc_2 \geq Sc_4) = 0.742075$ $V(Sc_2 \geq Sc_5) = 1$
$V(Sc_3 \geq Sc_1) = 0.225255$
$V(Sc_3 \geq Sc_2) = 0.411057$
$V(Sc_3 \geq Sc_4) = 0.297617$
$V(Sc_3 \geq Sc_5) = 0.270853$
$V(Sc_4 \geq Sc_1) = 0.856825$ $V(Sc_4 \geq Sc_2) = 1$
$V(Sc_4 \geq Sc_3) = 1$ $V(Sc_4 \geq Sc_5) = 1$
$V(Sc_5 \geq Sc_1) = 0.854633$ $V(Sc_5 \geq Sc_2) = 1$
$V(Sc_5 \geq Sc_3) = 1$
$V(Sc_5 \geq Sc_4) = 1.015309$

Then priority weights are calculated as follow:

$d'(C_1) = \min(1,1,1,1) = 1$
$d'(C_2) = \min(0.623495, 1, 0.742075, 1) = 0.623495$

d' (C_3) = min (0.225255, 0.411057, 0.297617, 0.270853) = 0.225255
d' (C_4) = min (0.856825,1,1,1) = 0.856825
d' (C_5) = min (0.854633,1,1, 1.015309) = 0.854633

Priority weights form w' = (1, 0.623495, 0.225255, 0.856825, 0.854633) vector. After the normalization of these values priority weight respect to main goal is calculated as (0.280883, 0.175129, 0.06327, 0.240667, 0.240052). After the priority weights of the criteria are determined, the priority of the alternatives will be determined for each criterion. From the pair-wise comparisons of the decision makers for three alternatives, evaluation matrixes are formed as in Tables 12, 13, 14, 15 and 16. Then, priority weights of alternatives for each criterion are determined by making the same calculation like in Table 17.

Table 12: The fuzzy evaluation matrix with respect to C_1

C_1	A_1	A_2	A_3
A_1	(1,1,1)	(1.75,3.25,4)	(.9,1,1.2)
A_2	(.25,.31,.57)	(1,1,1)	(2,3.5,5)
A_3	(.83,1,1.11)	(.2,.29,.5)	(1,1,1)

Table 13: The fuzzy evaluation matrix with respect to C_2

C_2	A_1	A_2	A_3
A_1	(1,1,1)	(3,4.26,5.6)	(1,1.7,2.9)
A_2	(.18,.23,.33)	(1,1,1)	(1.8,3.6,4.9)
A_3	(.34,.59,1)	(.2,.28,.56)	(1,1,1)

Table 14: The fuzzy evaluation matrix with respect to C_3

C_3	A_1	A_2	A_3
A_1	(1,1,1)	(3.3,4.2,6.1)	(1,3,4.3)
A_2	(.16,.24,.3)	(1,1,1)	(.2,.3,.9)
A_3	(.23,.33,1)	(1.11,3.33,5)	(1,1,1)

Table 15: The fuzzy evaluation matrix with respect to C_4

C_4	A_1	A_2	A_3
A_1	(1,1,1)	(2,3.5,5)	(.14,1.95,2.64)
A_2	(.2,.29,.5)	(1,1,1)	(1,1,1)
A_3	(1,1,1)	(1,1,1)	(1,1,1)

Table 16: The fuzzy evaluation matrix with respect to C_5

C_5	A_1	A_2	A_3
A_1	(1,1,1)	(2.5,3.2,5.1)	(1,3,4.3)
A_2	(.2,.31,.4)	(1,1,1)	(1.8,3,4)
A_3	(.23,.33,1)	(.25,.33,.56)	(1,1,1)

Table 17: Summary of priority weights of the main-attributes of the goal

	C_1	C_2	C_3	C_4	C_5	Alternative priority weight
A_1	0.465674	0.640543	0.63554	0.710878	0.784262	0.642536
A_2	0.465674	0.359457	0	0.104753	0.215738	0.27075
A_3	0.068651	0	0.36446	0.184369	0	0.086714
Weight	0.280883	0.175129	0.06327	0.240667	0.240052	

The weight vector from Table 12 is calculated as (0.465674, 0.465674, 0.068651).
The weight vector from Table 13 is calculated as (0.640543, 0.359457, 0).
The weight vector from Table 14 is calculated as (0.63554, 0, 0.36446).
The weight vector from Table 15 is calculated as (0.710878, 0.104753, 0.184369).
The weight vector from Table 16 is calculated as (0.784262, 0.215738, 0).

Alternative A_1 which has the highest priority weight is selected as a best Machine for this company. The ranking order of the alternatives with fuzzy AHP method is $A_1>A_2>A_3$. We have reached the same result with fuzzy TOPSIS. The company management found the application results satisfactory and decided to select the first alternative. Fuzzy AHP and fuzzy TOPSIS methods are both appropriate for the selection of Machine or other multi-criteria decision-making problems of the company.

6. Conclusions

Decision-makers face up to the uncertainty and vagueness from subjective perceptions and experiences in the decision-making process (Ertuğrul et al. 2006). By using fuzzy AHP and fuzzy TOPSIS, uncertainty and vagueness from subjective perception and the experiences of decision-maker can be effectively represented and reached to a more effective decision. In this study machine selection with fuzzy AHP and fuzzy TOPSIS method has been proposed. The decision criteria were Quality, Payment Terms, After-Sale Service, Capacity and Technology. These criteria were evaluated to determine the order of alternatives for selecting the most appropriate one. Although two methods have the same objective of selecting the best Machine for the company, they have differences. In fuzzy TOPSIS decision makers used the linguistic variables to asses the importance of the criteria and to evaluate the each alternative with respect to each criterion. These linguistic variables converted into triangular fuzzy numbers and fuzzy decision matrix was formed. Then normalized fuzzy decision matrix and weighted normalized fuzzy decision matrix were formed. After FPIS and FNIS were defined, distance of each alternative to FPIS and FNIS were calculated. And then the closeness coefficient of each alternative was calculated separately. According to the closeness coefficient of three alternatives, the ranking order of three alternatives has been determined as $A_3>A_1>A_2$. In fuzzy AHP, decision-makers made pair-wise comparisons for the criteria and alternatives under each criterion. Then these comparisons integrated and decision-makers' pair-wise comparison values are transformed into triangular fuzzy numbers. The priority weights of criteria and alternatives are determined by Chang's (1996) extent analysis. According to the combination of the priority weights of criteria and alternatives, the best alternative is determined. According to the fuzzy AHP, the best alternative is A_1 and according to the fuzzy TOPSIS, the best alternative is A_3. Companies should choose the appropriate method for their problem according to the situation and the structure of the problem they have.

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