A fuzzy VIKOR approach for plant location selection

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Abstract: Plant location selection is a multi-criteria decision problem and has a strategic importance for many companies. The aim of this study is to propose a fuzzy approach for Plant location selection. This paper is based on a fuzzy VIKOR (Serbian: Vlsekriterijumsko Kompromisno Rangiranje) method. In this method, the ratings of various alternatives versus various subjective criteria and the weights of all criteria are assessed in linguistic variables represented by fuzzy numbers. Fuzzy numbers try to resolve the ambiguity of concepts that are associated with human being's judgments. By using fuzzy VIKOR, uncertainty and vagueness from subjective perception and the experiences of decision maker can be effectively represented and reached to a more effective decision.
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1. Introduction

In order to minimize cost and maximize the use of resources, selecting a suitable plant location has become one of the most important issues for manufacturing companies. Many potential criteria, such as investment cost, human resources, availability of acquirement material, climate etc., must be considered in selecting a particular plant location (Liang et al.1991). Therefore, plant selection can be viewed as multiple-criteria decision-making (MCDM) problem. The VIKOR method, one of the well known classical MCDM methods, initiated by Opricovic (1998), of which the compromise solution should have a maximum group utility (majority rule) and minimum individual regret of the opponent, is proposed to deal with multicriteria decision-making problems. The lowest performance rating with respect to specified criteria was frequently ignored among them. The primary goal of MCDM is finding an optimal solution with maximum effectiveness via minimum cost. In the human physical world, crisp data are inadequate to present the real situation since human intuition, judgment; perception and preference are always vague and difficult to measure. Dubois et al (1985) pointed out that statistical decision methods do not measure the imprecision of human behavior; rather they model insufficient knowledge about the external environment. Fuzzy set theory approaches toward decision-making take human subjectivity into consideration, rather than applying merely objective probability means. Therefore, that the information in decision-making is indistinct, uncertain, and vague or represented in linguistic terms leads to the study of a new decision analysis field, Fuzzy Multi-Criteria Decision Making (FMCDM). In this paper, we use Fuzzy VIKOR, as described by Wang et al (2005), to solve fuzzy multi-criteria decision-making problems with a best solution and compromise solution in reality confirmed situation.

2. Fuzzy sets and Fuzzy numbers

The concepts of fuzzy set and fuzzy logic were introduced by Zadeh (1965). Zadeh's intention in introducing fuzzy set theory was to deal with problems involving knowledge expressed in vague, linguistic terms. Classically, a set is defined by its members. An object may be either a member or a non-member: the defining characteristics of traditional (crisp) set. The connected logical proposition may also be true or false. This concept of crisp set may be extended to fuzzy set with the introduction of the idea of partial truth. Any object may be a member of a set "to some degree"; and a logical proposition may hold true "to some degree". Often, we communicate with other people by making qualitative statements, some of which are vague because we simply do not have the precise datum at our disposal e.g., a person is tall (we have no exact numerical value at that moment) or because the datum is not measurable in any scale e.g. a beautiful girl (for beautiful, no metric exists). Here, tall and beautiful are fuzzy sets. So, fuzzy concepts are one of the important channels by which we mediate and exchange information, ideas and understanding among ourselves. Fuzzy set theory offers a precise mathematical form to describe such fuzzy terms in the form of fuzzy sets of a linguistic variable. To represent the shades of meaning of such linguistic terms, the concept of grades of membership or the concept of possibility values of membership has been introduced. In Fuzzy set theory the membership grade can be taken as a value intermediate between 0 and 1 although in the normal case of set theory membership the grade can be taken only as 0 or 1. The function of the membership grade is called its "membership function" in Fuzzy theory. The membership function will be defined by the user in consideration of the fuzziness. Let X be the universe of discourse, $\{X = x_1, x_2, ..., x_n\}$. A fuzzy set $\tilde{A}$ of X is a set of order pairs $\{(X_1, f_{\tilde{A}}(x_1)), (x_2, f_{\tilde{A}}(x_2)) (x_n, f_{\tilde{A}}(x_n))\}$,

$f_{\tilde{A}} : X \rightarrow [0,1]$, is the membership function of $\tilde{A}$, and $f_{\tilde{A}}(x_i)$ stands for the membership degree of x_i in $\tilde{A}$. The value $f_{\tilde{A}}$ is closer to 0, the degree is low. The value $f_{\tilde{A}}$ is closer to 1, the degree is high. Dubois et al (1985) show that the membership functions has more types. The paper adopts the type of a triangular fuzzy number. A triangular fuzzy number can be denoted as a triplet (a_1,a_2,a_3) and $a_1<a_2<a_3$,the membership function $f_{\tilde{A}}(x)$ of the fuzzy number $\tilde{A}$ is taken as:

$$f_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1 \\ (x \quad a_1) / a_2 \quad a_1, & a_1 \le x \le a_2 \\ (a_3 - x) / a_3 - a_2, & a_2 \le x \le a_3 \\ 0, & x > a_3 \end{cases}$$

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers parameterized by the triplet (a_1, a_2, a_3) and (b_1, b_2, b_3), then the operations of triangular fuzzy numbers (Cheng et al. 2002) are performed as:

$\tilde{A} (+) \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
$r (\times) \tilde{A} = (ra_1, ra_2, ra_3)$

When X is continuum rather than a countable or finite set, the fuzzy set $\tilde{A}$ is denoted as:

$\tilde{A} \int_X f_{\tilde{A}}(X)/(X)$, where $x \in X$.

When X is a countable or finite set, the fuzzy set $\tilde{A}$ is represented as:

$\tilde{A} = \sum_i f_{\tilde{A}}(X_i)/(X_i)$, where $x_i \in X$.

A fuzzy set $\tilde{A}$ of the universe of discourse X is convex if and only if for all x_1, x_2 in X,

$f_{\tilde{A}}(\lambda x_1 + (1-\lambda) x_2) \ge \text{Min}\,[f_{\tilde{A}}(x_1), f_{\tilde{A}}(x_2)]$,where $\lambda \in [0,1]$, $x_1,x_2 \in X$

The α-cut $\tilde{A}_\alpha$ and strong α-cut $\tilde{A}_{\alpha+}$ of the fuzzy set $\tilde{A}$ in the universe of discourse X is defined by:

$\tilde{A}_\alpha = \{x_i | f_{\tilde{A}}(x_i) \ge \alpha, x_1 \in X\}$, where $\alpha \in [0,1]$.

$\tilde{A}_{\alpha+} = \{x_i | f_{\tilde{A}}(x_i) \ge \alpha, x_1 \in X\}$, where $\alpha \in [0,1]$.

The operations of $\vee$(max) and $\wedge$ (min) are defined as follow:

$\tilde{A} (\vee) \tilde{B} = (a_1 \vee b_1, a_2 \vee b_2, a_3 \vee b_3)$
$\tilde{A} (\wedge) \tilde{B} = (a_1 \wedge b_1, a_2 \wedge b_2, a_3 \wedge b_3)$

3. The Fuzzy VIKOR Method

The optimum in multi-criteria decision-making is the process to decide the compromise ranking in the ensured rules. In reality, there is no avoidance of the coexistence of qualitative and quantitative data, and they are often full of fuzziness and uncertainty. So, the optimum is often the noninferior solutions or compromise solutions depend on the decision-maker. The concepts of compromise solutions were first initiated by Yu et al (1973). The compromise solutions will be presented by comparing the degree of closeness to the ideal alternative. The method of VIKOR initiated by Opricovic (1998), works on the principle that each alternative can be evaluated by each criterion function; the compromise ranking will be presented by comparing the degree of closeness to the ideal alternative. To solve fuzzy multi-criteria decision-making problems with a best solution and compromise solution in reality confirmed situation, Fuzzy VIKOR was described by Wang et al (2005). The following was the stages in Fuzzy VIKOR.

Step1: Form a group of decision-makers (denoted in n), then determine the evaluation criteria (denoted in k) and feasible alternatives (denoted in m).

Step2: Identify the appropriate linguistic variables for the importance weight of criteria, and the rating for alternatives with regard to each criterion (as shown in Table 1 and Table 2). The membership degree of fuzzy numbers in the weight of criteria and the rating of alternatives will be presented in Figure 1 and Figure 2.

Table l: Linguistic Variables for the Weight of Criteria

Linguistic Variables	Fuzzy Numbers
Very Low (VL)	(0.00,0.00,0.25)
Low (L)	(0.00,0.25,0.50)
Medium (M)	(0.25,0.50,0.75)
High (H)	(0.50,0.75,1.00)
Very High (VH)	(0.75,1.00,1.00)

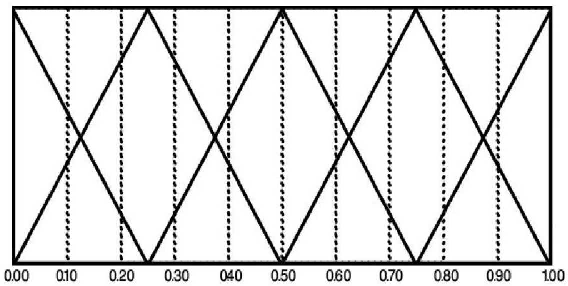

Figure 1. The Membership Degree of Fuzzy Numbers in the Weight of Criteria

Table 2: Linguistic Variables for the Rating of Alternative

Linguistic Variables	Fuzzy Number
Worst (W)	(0.0,0.0,2.5)
Poor (P)	(0.0,2.5,5.0)
Fair (F)	(2.5,5.0,7.5)
Good(G)	(5.0,7.5,10)
Best (B)	(7.5, 10, 10)

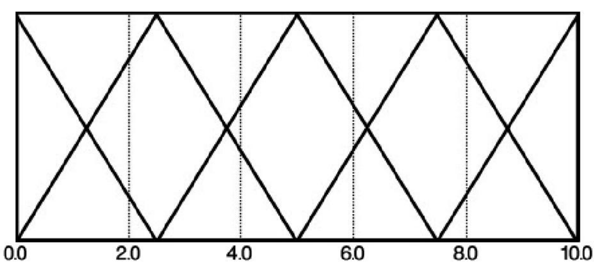

Figure 2. The Membership Degree of Fuzzy Numbers in the Rating of Alternative

Step3: Pull the decision makers' opinions to get the aggregated fuzzy weight of criteria, and aggregated fuzzy rating of alternatives. If there are *n* persons in a decision committee, the importance weight of each criterion and rating of each alternative can be measured by:

$$\tilde{W}_j - {}^{1}/_{k}[\tilde{W}_j^{1} \oplus \tilde{W}_j^{2} \oplus \dots \oplus \tilde{W}_j^{k}] \qquad (1)$$

$$\tilde{x}_{ij} = {}^{1}/_{k}[\tilde{x}_{ij}^{1} \oplus \tilde{x}_{ij}^{2} \oplus \dots \oplus \tilde{x}_{ij}^{k}] \qquad (2)$$

Step 4: Construct a fuzzy decision matrix. Formally, a typical fuzzy multicriteria decision making problem can be expressed in matrix format as:

$$\tilde{D} = \begin{pmatrix} \tilde{x}_{11} & \Lambda & \tilde{x}_{1n} \\ M & M & M \\ \tilde{x}_{m1} & \Lambda & \tilde{x}_{mn} \end{pmatrix}, \text{ i=1,2,…,m ; j=1,2,…,n} \qquad (3)$$

$$\tilde{W} = [\tilde{W}_1, \tilde{W}_2, \dots, \tilde{W}_n], j = 1,2,\dots,n \qquad (4)$$

where $\tilde{x}_{ij}$ the rating of alternative A_i with respect to C_j, $\tilde{w}_j$ the importance weight of the *j* th criterion holds, $\tilde{x}_{ij}$ and $\tilde{w}_j$ are linguistic variables denoted by triangular fuzzy numbers.

Step 5: Determine the fuzzy best value (FBV, $\tilde{f}_j^*$) and fuzzy worst value (FWV, $\tilde{f}_j^-$) of all criterion functions.

$$\tilde{f}_j^* = \underbrace{Max}_{i} \tilde{x}_{ij}, \quad j \in B; \ \tilde{f}_j^- = \underbrace{Min}_{i} \tilde{x}_{ij}, \quad j \in C \qquad (5)$$

Step 6: Compute the values $\tilde{S}_i$ and $\tilde{R}_i$:

$$\tilde{S}_i = \sum_{j=1}^{n} \tilde{W}_j [{(\tilde{f}_j^* - \tilde{x}_{ij})}/{(\tilde{f}_j^* - \tilde{f}_j^-)}] \qquad (6)$$

$$\tilde{R}_i = \underbrace{Max}_{j} [\frac{\tilde{W}_j(\tilde{f}_j^* - \tilde{x}_{ij})}{\tilde{f}_j^* - \tilde{f}_j^-}] \qquad (7)$$

where $\tilde{S}_i$ refers to the separation measure of A_i from the fuzzy best value, similarly, $\tilde{R}_i$ is the separation measure of A_i from the fuzzy worst value.

Step 7: Calculate the value $\tilde{S}^*$, $\tilde{S}^-$, $\tilde{R}^*$, $\tilde{R}^-$ and $\tilde{Q}_i$:

$$\tilde{S}^* = \underbrace{Min}_{i} \tilde{S}_i, \quad \tilde{S}^- = \underbrace{Max}_{i} \tilde{S}_i, \quad \tilde{R}^* = \underbrace{Min}_{i} \tilde{R}_i, \quad \tilde{R}^- = \underbrace{Max}_{i} \tilde{R}_i \qquad (8)$$

$$\tilde{Q}_i = \frac{v(\tilde{S}_i - \tilde{S}^*)}{\tilde{S}^- - \tilde{S}^*} + \frac{(1-v)(\tilde{R}_j - \tilde{R}^*)}{\tilde{R}^- - \tilde{R}^*} \qquad (9)$$

The index $\min_i \tilde{S}_i$ is with a maximum majority rule, and $\min_i \tilde{R}_i$ is with a minimum individual regret of an opponent strategy. As well, v is introduced as weight of the strategy of the maximum group utility, usually v = 0.5.

Step 8: Defuzzify triangular fuzzy number $\tilde{Q}_i$ and rank the alternatives by the index Q_i.

The process converting a fuzzy number into a crisp value is called defuzzify. Various defuzzification strategies were suggested, in this paper, Chen's (1985) method of maximizing set and minimizing set is applied. The maximizing set is defined as:

R = {(x, f_R (x)) x ϵ R}, with the membership function

$$f_R(x) = \begin{cases} (x - x_1)/(x_2 - x_1), & x_1 \le x \le x \\ 0, & Otherwise \end{cases} \qquad (10)$$

Similarly, the minimizing set is defined as:
$L = \{(x, f_l(x)) | x \in R\}$, with membership function:

$$f_L(x) = \begin{cases} (x - x_2)/(x_1 - x_2), & x_1 \le x \le x_2 \\ 0, & Otherwise \end{cases} \quad (11)$$

Then the right utility $U_R(\tilde{Q}_i)$ and left utility $U_L(\tilde{Q}_i)$ can be denoted as:

$$U_R(F_i) = \underset{x}{Sup}(f_{F_i}(x) \wedge f_M(x)) \quad (12)$$

$$U_L(F_i) = \underset{x}{Sup}(f_{F_i}(x) \wedge f_G(x)) \quad (13)$$

As a result, the crisp value can be obtained by combining the right and left utilities.

$$U_T(F_i) = [U_R(F_i) + 1 - U_L(F_i)]/2 \quad (14)$$

The index Q_i implies the separation measure of A_i from the best alternative. That is, the smaller the value, the better the alternative.

Step 9: To determine a compromise solution (a') by the index Q in double conditions. To fit in with below double conditions, we should point out a' is our compromise solution.
[Condition1] Acceptable advantage:

$Q(a) - Q(a) \ge DQ$
$DQ = (1/m-1)$ $(DQ=0.33 \text{ if } m=3)$ (15)

[Condition 2] Acceptable stability in decision making: $Q(a)$ must in $S(a)$ or/and $R(a)$, a'' is the alternative with second position ranked by index Q. If condition1 is not satisfied, and $Q(a^m) - Q(a') < DQ$, a', a'', K, a^m are compromise solutions in the same. If condition 2 is not satisfied, a' and a'' are compromise solutions in the same.

Step 10: Determine the best alternative.
The best alternative is $Q(a')$, which is one with the minimum of Q_i.

4. Numerical example

Assume that a company is looking to select a location to build a new plant. Three locations (A_1, A_2, A_3) are to be evaluated by three decision-making experts (D_1, D_2, D_3) based on four criteria, skilled workers (C_1), expansion possibility (C_2), availability of acquirement material (C_3) and investment cost (C_4). We will use Fuzzy VIKOR methods to solve fuzzy multi-criteria decision-making problems. Fuzzy VIKOR aims to find the decision-maker's preferable compromise solution that suits the human objective cognition. In this paper, we will sort out the systematic solution process, and eventually come up with a best solution and compromise solution that can be an important reference for decision-making. The computational illustrations are inducted as follows:

Step 1: Form a group of decision-makers (denoted in *n*), then determine the evaluation criteria (denoted in *k*) and feasible alternatives (denoted in *m*). In this case, we have three decision makers, four evaluation criteria and three alternatives.

Step 2: 1dentify the appropriate linguistic variables for the importance weight of criteria, and the rating for alternatives with regard to each criterion (as shown in Table 1 and Table 2). The membership degree of fuzzy numbers in the weight of criteria and the rating of alternative are presented in Figure 1 and Figure 2.

Step 3: Pull the decision makers' opinions. The importance weights of the criteria are shown in Table 3 and, the rating of alternatives under four criteria is expressed as Table 4.

Table 3: The Importance Weight of the Criteria

Criteria	(D_1,D_2,D_3)
C_1	(VH,VH,H)
C_2	(H,H,M)
C_3	(L,M,L)
C_4	(H,VH,M)

Table 4: The Rating of Candidates under four Criteria

	A_1	A_2	A_3
C_1	(G,F,G)	(F,F,G)	(B,B,P)
C_2	(F,F,G)	(B,G,B)	(F,G,G)
C_3	(F,B,B)	(F,F,G)	(G,F,G)
C_4	(69,73,78)	(69,75,80)	(76,79,81)

Step 4: According to Eqs.(1) - (4), convert the linguistic variables into triangular fuzzy numbers (a_1, a_2, a_3) as well aggregate the fuzzy weight of criteria as Table 5. And, aggregate fuzzy ratings of five candidates to construct the aggregated fuzzy decision matrix (addressed in Table 6).

Table 5: Aggregated Fuzzy Weight of Criteria

Criteria	Aggregated Fuzzy Weights
C_1	(0.67,0.92,1.00)
C_2	(0.58,0.83,1.00)
C_3	(0.08,0.33,0.58)
C_4	(0.67,0.92,1.00)

Table 6: Aggregated Fuzzy Decision Matrix

Criteria	A_1	A_2	A_3
C_1	(4.1,6.6,9.1)	(3.3,5.8,8.3)	(5,7.5,8.3)
C_2	(3.3,5.8,8.3)	(6.6,9.1,10)	(4.1,6.6,9)
C_3	(5.8,8.3,9.1)	(3.3,5.8,8.3)	(4.1,6.6,9.1)
C_4	(69,73,78)	(69,75,80)	(76,79,81)

Step 5: Determine the fuzzy best value (FBV, $\tilde{f}_j^*$) and fuzzy worst value (FWV, $\tilde{f}_j^-$). Investigations of the aggregated fuzzy decision matrix with Eq.(5), the $\tilde{f}_j^*$ and $\tilde{f}_j^-$ are listed in Table 7.

Table 7: Fuzzy Best Value (FBV) and Fuzzy Worst Value (FWV)

Criterion	FBV	FWV
C_1	(5,7.5,8.33)	(3.33,5.83,8.33)
C_2	(6.67,9.17,10)	(3.33,5.83,8.33)
C_3	(5.83,8.33,10)	(3.33,5.83,8.33)
C_4	(76,79,81)	(69,73,78)

Step 6: As stated in Eqs.(6)-(7), the values $\tilde{S}_i$ and $\tilde{R}_i$ are computed respectively as shown in Table 8.

Table 8: Index $\tilde{S}_i$ and $\tilde{R}_i$

	A_1	A_2	A_3
$\tilde{S}_i$	(0.75,1.13,0.92)	(0.75,1.5,2.19)	(0.87,1.4,2.38)
$\tilde{R}_i$	(0.42,0.67,0.92)	(0.67,0.92,1)	(0.5,0.75,1.00)

Step 7: By Eq.(8), the $\tilde{S}^*$, $\tilde{S}^-$, $\tilde{R}^*$, can be seen in Table 9

Table 9: $\tilde{S}^*$, $\tilde{S}^-$, $\tilde{R}^*$

	Value
$\tilde{S}^*$	(0.87,1.47,2.38)
$\tilde{S}^-$	(0.75,1.13,0.92)
$\tilde{R}^*$	(0.67,0.92,1.00)

Step 8: Calculate the $\tilde{Q}_i$ for each candidate with Eq.(9), and the result is shown in Table 10.

Table 10: Index $\tilde{Q}_i$, Q_i and rank for candidates

alternative	Q	U_t	Rank
A_1	(0.00,0.00,0.00)	0.00	1
A_2	(0.5,0.83,0.83)	0.83	3
A_3	(0.67,0.69,0.69)	0.69	2

Step 9: Determine a compromise solution (a') by the index Q in double conditions and determines the best alternative.

Condition 1 is satisfied:

$Q(a^{(2)}) - Q(a^{(1)}) = 0.69 > 0.333$

Condition 2 is satisfied:
Alternative 1 is the best alternative by ranking either S_i , R_i or Q_i in ascending order.

5. Conclusion

Decision-making process is getting harder in today's complex environment. Decision makers face up to the uncertainty and vagueness from subjective perceptions and experiences in the decision-making process. Multi-criteria decision systems need experts in different areas. Fuzzy decision making theory can be used in many decision making areas like that. The aim of this study is to propose fuzzy VIKOR approach for selecting plant location. Skilled workers, expansion possibility, availability of acquirement material, and investment cost factors were evaluated to obtain the preference degree associated with each alternative for selecting the most appropriate one. By the help of the fuzzy approach, the ambiguities involved in the assessment data could be effectively represented and processed to make a more effective decision. As a result of the fuzzy VIKOR method Alternative 1 is the best location.

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