A cost allocation model for optimizing the inventory of a supply chain with controllable lead time

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Abstract: In most Supply chain management researches, lead time is considered as a predetermined and fixed parameter. In recent years, some researchers are motivated to consider controllable lead time as a decision variable but they have only considered single product and two-echelon supply chains. This paper proposes a cost allocation model considering the elements of a three echelon supply chain consists of a retailer, a manufacturer and a distributor with multiple products and controllable lead time. After presenting a case with independent decision making and ordering policy by each member, a model will be proposed in which all of the elements of the supply chain are cooperating with each other and have a unique ordering policy. The proposed model will determine the optimal order quantity in the mentioned situation. Finally, we will examine the proportion of the crashing lead time related costs that each part of the chain should pay in order to make the group decision making beneficiary for all of the chain members.

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1. Introduction

In real supply chain problems, lead time has an important effect on managers' decisions about when to order and how many products should be ordered in each ordering interval. In some problems, lead time is controllable; it means that by paying a penalty cost, this time would be decreased. For example assume that lead time contains elements such as time for receiving raw material, setup time, processing time and transportation time. Each of these components can be shortened if its related penalty cost is paid. In this area there are some manuscripts in the literature. Chandra and Grabis (2008) considered a single stage variable lead-time inventory system with lead-time dependent procurement cost. Jha and Shanker (2009) proposed a model of an inventory system with a vendor and a buyer but they inserted a service level constraint into their model instead of computing a cost function. Leng and Parlar (2009) proposed a game theory model to distinguish that which one of the retailer or the manufacturer should be responsible to pay the penalty cost of crashing lead-time. Sajadieh et al. (2009) proposed a model that allows shortage in customer demands and they considered the lead time as a stochastic parameter.

Ye and Xu (2010) proposed a cost allocation model in a two echelon supply chain with controllable lead time. Yang (2010) proved that the computational time needed to solve a model with present value and dependent crashing lead time is polynomial and as a result, such a model could be NP-hard. In this paper, a three-echelon supply chain consists of a manufacturer, a distributor and a retailer with multiple products is considered. First we will explain the model in which any member of the chain decides separately and has its own inventory system. Then a model will be proposed in which all of the members participate in group decision making and determining ordering policy. Finally, circumstances will be discussed under which all of supply chain members are satisfied and benefits from cooperating in group decision making. The rest of this paper is classified as follows: Section 2 illustrates the notations and after that, models for separate decision making and group decision making will be introduced. Section 3 is dedicated to a cost allocation model using Nash equilibrium to satisfy every component of the supply chain to participate in the group decision making. A numerical example will illustrate the proposed method in Section 4. Finally, Section 5 is for conclusions and suggestions for future research.

2. Model Constructions

2.1 Notations

The notations which are used in this paper are as follows:

Q_i	Ordering quantity for i^{th} product
L_i	Lead time for i^{th} product
k	Safety factor
m_i	Number of lots ordered for i^{th} product
θ	Proportion of the penalty cost that the manufacturer should pay
θ'	Proportion of the penalty cost that the

	distributer should pay
θ''	Proportion of the penalty cost that the retailer should pay
D_i	Annual demand for i^{th} product
p_i	Production capability for i^{th} product
A_i	Ordering cost of i^{th} product for retailer
A_i'	Ordering cost of i^{th} product for distributer
S_i	Setup cost for i^{th} product (manufacturer)
h_i	Holding cost of i^{th} product for retailer
h_i'	Holding cost of i^{th} product for manufacturer
h_i''	Holding cost of i^{th} product for distributer
λ	Manufacturer's bargaining power
λ'	Distributer bargaining power
γ_i	Retailer's marginal profit per unit
γ_i'	Distributer's marginal profit per unit
π_i	Shortage cost per unit for i^{th} product
β	Fraction of demand that would be backordered
TC_m	Total cost of manufacturer
TC_D	Total cost of distributer
TC_c	Total cost of retailer
$R(L_i)$	Crashing lead time penalty cost for i^{th} product

As mentioned before, there is a model in the literature which has discussed about the same problem with two levels and only one product in the supply chain (Ye & Xu, 2010). The final model of the mentioned manuscript has been showed in Equation 1.

$$Max\left\{(S_m)^{\lambda}\times(S_c)^{1-\lambda}\right\}$$
$$st:\quad TC_m\geq 0 \qquad (1)$$
$$TC_c\geq 0,$$

where S_m and S_c are satisfaction functions for the manufacturer and the retailer, respectively. In next sections, we want to develop this model for considering three levels and multiple products in the supply chain.

2.2 Model for Independent Ordering

It is assumed that the inventory level is being monitored with Fixed Order Size (FOS) system. In this system, each time that inventory reaches to a predetermined level, system will order a batch of Q_i products. The aforementioned level is called Re Order Point (ROP) and can be computed by Equation 2.

$$ROP_i=\mu_i L_i+k\sigma_i\sqrt{L_i} \qquad (2)$$

In Equation 2, $\mu_i L_i$ represents the demand for i^{th} product during its lead time and $k\sigma_i\sqrt{L_i}$ is the safety stock that should be stored because of that the demand is probabilistic. It is assumed that demand for i^{th} product follows a normal distribution with mean μ_i and standard deviation σ_i.

We will consider a case in which the retailer, the distributer and the manufacturer orders and decides about the quantity of ordering and safety stock independently from other parts of the chain. Next subsections are dedicated to formulation of the total cost for each part of the chain.

2.2.1 Manufacturer's Related Cost

Manufacturer's cost contains cost of ordering, cost for holding inventory and a proportion of the penalty cost for crashing lead time that he should pay. So:
TC_m = ordering cost+ holding cost+ manufacturer's proportion of crashing lead time penalty.

Manufacturer produces a batch of m_iQ_i from the i^{th} product and thereupon, the ordering cost of the manufacturer is $\left(\frac{D_iS_i}{m_iQ_i}\right)$ because total number of orders per year is $\left(\frac{D_i}{m_iQ_i}\right)$ (Tersine, 1994). According to Quyang et al. (2004), the expected average inventory for the manufacturer is $\frac{Q_i}{2}\left[m_i(1-\frac{D_i}{p_i})-1+\frac{2D_i}{p_i}\right]$. So the expected cost for holding the inventory will be $h_i'\frac{Q_i}{2}\left[m_i(1-\frac{D_i}{p_i})-1+\frac{2D_i}{p_i}\right]$. Besides, the portion of the crashing lead time that manufacturer would pay is $\frac{\theta D_iR(L_i)}{Q_i}$. Total cost for the manufacturer can be computed by Equation 3.

$$TC_m=\sum_{i=1}^{n}\left(\begin{array}{l}\left(\frac{D_iS_i}{m_iQ_i}\right)\\ +h_i'\frac{Q_i}{2}\left[m_i(1-\frac{D_i}{p_i})-1+\frac{2D_i}{p_i}\right]\\ +\frac{\theta D_iR(L_i)}{Q_i}\end{array}\right) \qquad (3)$$

2.1.1. Distributor's Related Cost

Distributor's cost contains cost of ordering, cost of holding inventory and a proportion of the penalty cost for crashing lead time he should pay. So:

TC_D = ordering cost + holding cost + distributor's proportion of decreasing lead time penalty.

It is assumed that the distributer orders the same batch size as manufacturer produces and doesn't hold any safety inventory. Thereupon, the average inventory for the distributor is $\frac{m_i Q_i}{2}$. Distributor orders $\frac{D_i}{m_i Q_i}$ batches from i^{th} product. So the ordering cost is $\frac{D_i A_i'}{m_i Q_i}$. Finally the proportion of crashing lead time penalty that the distributor should pay is $\frac{\theta' D_i R(L_i)}{Q_i}$. Total cost of the distributor can be computed by Equation 4.

$$TC_D = \sum_{i=1}^{n}\left(\frac{D_i A_i'}{m_i Q_i} + h_i'' \frac{m_i Q_i}{2} + \frac{\theta' D_i R(L_i)}{Q_i}\right) \quad (4)$$

2.2.3 Retailer's Related Cost

Retailer's cost contains shortage cost, cost of ordering, cost for holding inventory and a proportion of the penalty cost that retailer should pay and. So:

TC_R = shortage cost + ordering cost + holding cost + retailer's proportion of decreasing lead time penalty.

The expected shortage per order can be calculated by the Equation.5

$$E(x > r) = \int_{r}^{+\infty} (x-r)dF(x) = \sigma\sqrt{L}\psi(k) \quad (5)$$

In the above equation, $\psi(k)$ is equal to $\phi(k) - k[1-\varphi(k)]$. φ and ϕ are the probability density function and cumulative distribution function of the standard normal distribution, respectively. So the expected number of backorders per cycle with fraction of back ordered demand of β is $\beta\sigma_i\sqrt{L}\psi(k)$. Thus, the retailer's average inventory level is $\frac{Q_i}{2} + k\sigma_i\sqrt{L_i} + (1-\beta)\sigma_i\sqrt{L_i}\psi(k)$. So it can be concluded that the expected annual holding cost for retailer is $h_i\left[\frac{Q_i}{2} + k\sigma_i\sqrt{L_i} + (1-\beta)\sigma_i\sqrt{L_i}\psi(k)\right]$.

Total number of orders per year is $\frac{D_i}{Q_i}$ and the proportion of penalty cost for decreasing lead time in any ordering interval is $(1-\theta-\theta')R(L_i)$. So the retailer's proportion of annual crashing lead time penalty cost is $\frac{(1-\theta-\theta')D_i R(L_i)}{Q_i}$.

The expected lost sale cost in each ordering interval is $(1-\beta)\sigma_i\sqrt{L_i}\psi(k)$, so the expected shortage cost per year is $\frac{D_i(\pi_i + \gamma_i(1-\beta)\sigma_i\sqrt{L_i}\psi(k))}{Q_i}$. The expected annual ordering cost is $\frac{D_i}{Q_i}A_i$.

So, total retailer's expected cost for all of products can be computed by Equation 6.

$$TC_c = \sum_{i=1}^{n}\left(\begin{array}{l}\frac{D_i}{Q_i}A_i \\ +h_i\left[\frac{Q_i}{2} + k\sigma_i\sqrt{L_i} + (1-\beta)\sigma_i\sqrt{L_i}\psi(k)\right] \\ +\frac{(1-\theta-\theta')D_i R(L_i)}{Q_i} \\ +\frac{D_i(\pi_i + \gamma_i(1-\beta)\sigma_i\sqrt{L_i}\psi(k))}{Q_i}\end{array}\right) \quad (6)$$

Next section will present an integrated model for decentralized decision making in which each part of the supply chain decides separately and has its own ordering policy.

2.2.4 Model for Decentralized Decision Making

In this part of the manuscript, a model for finding the optimal value for Q -the order quantity for i^{th} product- will be illustrated and the algorithm for finding the solution is introduced. It is assumed that the manufacturer and the distributor agree about the number of batches and order quantity and then the retailer should decide about the quantity he should order. In other words, the buyer should pay all of the crashing lead time penalty cost and as a result $\theta = \theta' = 0$. The model is as follows:

$$MinTC_m = \sum_{i=1}^{n}\left(\begin{array}{l}\left(\frac{D_i S_i}{m_i Q_I}\right) + h_i' \frac{Q_i}{2}\left[m_i(1-\frac{D_i}{p_i}) - 1 + \frac{2D_i}{p_i}\right] \\ +\frac{\theta D_i R(L_i)}{Q_i}\end{array}\right)$$

$st:$

$$TC_c = \sum_{i=1}^{n}\left(\frac{D_i}{Q_i}A_i + h_i\left[\frac{Q_i}{2} + k\sigma_i\sqrt{L_i} + (1-\beta)\sigma_i\sqrt{L_i}\psi(k)\right]\right.$$
$$\left. + \frac{\theta'' D_i R(L_i)}{Q_i} + \frac{D_i(\pi_i + \gamma_i(1-\beta)\sigma_i\sqrt{L_i}\psi(k)}{Q_i}\right)$$

$$TC_D = \sum_{i=1}^{n}\left(\frac{D_i A_i'}{m_i Q_i} + h_i'' \frac{m_i Q_i}{2} + \frac{\theta' D_i R(L_i)}{Q_i}\right)$$

The above model will minimize the total cost of manufacturer subjected to the cost of distributor and retailer follow Equations 4 and 6, respectively.

Based on Quyang et al. (2004) and extending their work to adapt a 3 level supply chain, the optimal value for order quantity and safety factor can be estimated by Equations 7 and 8, respectively.

$$Q_i^* = \sqrt{\frac{2D_i(A_i + R(L_i) + (\pi_i + \gamma_i(1-\beta))\sigma_i\sqrt{L_i}\psi(k))}{h_i}} \quad (7)$$

$$\phi(k^*) = 1 - \sum_{i=1}^{n} \frac{h_i}{h_i(1-\beta) + \frac{D_i}{Q_i^*}(\pi_i + \gamma_i(1-\beta))} \quad (8)$$

In order to solve such a model we need an algorithm. The basic idea of this algorithm is derived from Ye and Xu (2010). The algorithm is as follows:

Step 1: For each L_{ij} (lead time for i^{th} product) start from k=0 and compute Q by using Equation 7 and then find $\phi(k)$ by Equation 8. k can be calculated using normal cumulative distribution function. Replace the resulted k in Equation 7 and find the value of Q^*. Do this process again until convergence in the resulted values of Q. If this occurs, then go to step 2.

Step 2: By using the results of step 1, set the triple (Q_j, k_j, L_j) and calculate TC_c. So we should compute TC_c for each j (j=1,2,…,J)

Step 3: Set $TC_c = \min_j\{TC_c\}$. The values for Q_j and k_j resulted from step 2 are the optimal values.

Step 4: m is the first integer number that satisfies the Equation 9.

$$m(m-1) \le \frac{2D_i(S_i + A_i')}{h_i'(Q_i^*)^2\left(1-\frac{D_i}{p_i}\right) + h_i''(Q_i^*)^2} \le m(m+1) \quad (9)$$

2.2 Model for Centralized Decision Making

In this situation, the 3 levels of the supply chain – manufacturer, distributor and retailer- negotiate about their decisions like order quantity, number of batches, lead time for each product, safety factor, etc and make an integrated decision about these items. The model that has been extended for this situation is shown in Equation 10.

$$TC_{totall} = \sum_{i=1}^{n}\left(\begin{array}{l} \frac{D_i(A_i + R(L_i^{**}) + \pi_i + \gamma_i(1-\beta)\sigma_i\sqrt{L_i^{**}}\psi(k^{**}))}{Q_i^{**}} \\ + \frac{D_iA_i'}{m_i^{**}Q_i^{**}} + h_i''\frac{m_i^{**}Q_i^{**}}{2} \\ + h_i\left(\frac{Q_i^{**}}{2} + k^{**}\sigma_i\sqrt{L_i^{**}} + (1-\beta)\sigma_i\sqrt{L_i^{**}}\psi(k^{**})\right) \\ + \frac{D_iS_i}{m_i^{**}Q_i^{**}} + h_i'\frac{Q_i^{**}}{2}(m_i\left(1-\frac{D_i}{p_i}\right) - 1 + \frac{2D_i}{p_i}) \end{array}\right) \quad (10)$$

Equation 10 is the result of aggregation of equations 3, 4 and 6. In this equation, the crashing lead time penalty cost is integrated for all elements of the chain because the decisions are made for total chain rather than each element.

On the other hand, Equation 10 is convex toward Q_i and k (Quyang et al., 2004). So the optimal values for order quantity and safety factor can be obtained by derivation from Equation 10 to Q_i and k. The results are shown in Equations 11 and 12.

$$Q_i^{**} = \sqrt{\frac{2D_i(A_i + A_i' + R(L_i) + \frac{S}{m_i}) + (\pi_i + \gamma_i(1-\beta)\sigma_i\sqrt{L_i}\psi(k))}{h_i + h_i'' + h_i'(m_i\left(1-\frac{D_i}{p_i}\right) - 1 + \frac{2D_i}{p_i})}} \quad (11)$$

$$\phi(k^{**}) = 1 - \sum_{i=1}^{n} \frac{h_i}{h_i(1-\beta) + \frac{D_i}{Q_i^{**}}(\pi_i + \gamma_i(1-\beta))} \quad (12)$$

Theorem: TC_{total} is concave to m and this concavity has upward orientation.

Proof: We can derivate from Equation 10 regards to m for 2 times. The resulted equation is as follows:

$$\frac{\partial^2 TC_{total}}{\partial m^2} = \frac{2D_iS_i}{m_i^3 Q_i} \ge 0$$

It is obvious that this equation is always positive. So the optimal value for m-number of batches for each order- should satisfy the equations below:

$$TC_{total}(Q_i^{**}, k^{**}, L_i^{**}, m_i^{**}) \le TC_{total}(Q_i^{**}, k^{**}, L_i^{**}, m_i^{**} - 1)$$

$$TC_{total}(Q_i^{**}, k^{**}, L_i^{**}, m_i^{**}) \le TC_{total}(Q_i^{**}, k^{**}, L_i^{**}, m_i^{**} + 1)$$

In other words, the optimal m will minimize the value of TC_{total}. So, we can find the optimal values for order quantity, safety factor and number of batches by the algorithm below. The algorithm's basic idea is derived from Ye and Xu (2010).

Step 1: Start from m_i=1

Step 2: For each L_{ij} and for each j (j=1,2,…,J), start from k_j=0 and use three sub steps below.

a) Use k_j and Equation 11 to compute Q.

b) Use Equation 12 and the resulted Q to compute $\varphi(k)$ and k.

c) Do sub steps "a" and "b" over and over with updated value for k in each iteration and resume this procedure up to convergence.

Step 3: for each group of (L_{ij}, Q_j, k_j, m), compute the value of TC_{total}.

Step 4: the least value of TC for various values of j is set as the optimal answer for the given m_i.

Step 5: Set m_i=m_i+1 and perform steps 2 to 4 again and find the optimal value for TC_{total}.

Step 6: If the result of step 5 is less than the one computed in before iteration, then we should go back to step 5 and resume the procedure. Else, we reached to the global minimum of the total cost function and must go to step 7.

Step 7: Values for $(L_{ij}^{**}, Q_j^{**}, k_j^{**}, m^{**})$ for which TC_{Total} is minimized are considered as optimal values.

3. Cost Allocation Model Using Nash Equilibrium

In this part of the paper, a model will be introduced that uses Nash equilibrium to determine the proportion of decreasing lead time penalty that each part of the chain should pay in order to benefit from participating in centralized decision making procedure instead of separate decision making. Each part of the supply chain benefits by using the centralized procedure only if this participation decreases the total cost of it. As mentioned in Section 2, the cost for each part of the chain under decentralized model is as follows:

Manufacture
$$TC_m = \sum_{i=1}^{n}\left(\frac{D_i S_i}{m_i Q_i^*} + h_i' \frac{Q_i^*}{2}\left[m_i(1-\frac{D_i}{p_i}) - 1 + \frac{2D_i}{p_i}\right]\right)$$

Distributor
$$TC_D = \sum_{i=1}^{n}\left(\frac{D_i A_i'}{m_i Q_i^*} + h_i'' \frac{m_i Q_i^*}{2}\right)$$

Retailer
$$TC_c = \sum_{i=1}^{n}\left(\frac{D_i}{Q_i^*}A_i + h_i\left[\frac{Q_i^*}{2} + k^*\sigma_i\sqrt{L_i^*} + (1-\beta)\sigma_i\sqrt{L_i^*}\psi(k^*)\right] + \frac{D_i R(L_i^*)}{Q_i^*} + \frac{D_i(\pi_i + \gamma_i(1-\beta)\sigma_i\sqrt{L_i^*}\psi(k^*))}{Q_i^*}\right)$$

The costs under centralized model will be summarized below:

Manufacturer
$$\sum_{i=1}^{n}\left(\frac{D_i S}{m_i^{**} Q_i^{**}} + h_i' \frac{Q_i^{**}}{2}(m_i^{**}(1-\frac{D_i}{p_i}) - 1 + \frac{2D_i}{p_i}) + \frac{\theta D_i R(L_i^{**})}{Q_i^{**}}\right) \tag{13}$$

Distributor
$$\sum_{i=1}^{n}\left(\frac{D_i A_i'}{m_i^{**} Q_i^{**}} + h_i'' \frac{m_i Q_i^{**}}{2} + \frac{\theta' D_i R(L_i^{**})}{Q_i^{**}}\right) \tag{14}$$

Retailer
$$\sum_{i=1}^{n}\left(\frac{D_i(A_i + \pi_i + \gamma_i(1-\beta)\sigma_i\sqrt{L_i^{**}}\psi(k^{**}))}{Q_i^{**}} + h_i\left(\frac{Q_i^{**}}{2} + k^{**}\sigma_i\sqrt{L_i^{**}} + (1-\beta)\sigma_i\sqrt{L_i^{**}}\psi(k^{**})\right) + \frac{(1-\theta-\theta')D_i R(L_i^{**})}{Q_i^{**}}\right) \tag{15}$$

So the difference between the costs of each element in decentralized and centralized decision making model can be formulated as below:

$$\Delta TC_m = TC_{m_{decentralized}} - TC_{m_{centralized}}$$
$$\Delta TC_D = TC_{D_{decentralized}} - TC_{D_{centralized}}$$
$$\Delta TC_c = TC_{c_{decentralized}} - TC_{c_{centralized}}$$

The major problem is to find values for θ and θ' in such a manner that the differences between costs for each element of the chain be positive. In other words, we want to find the proportion of crashing lead time penalty cost related to each element in a way that every part of the supply chain benefit from participating in centralized decision making. In many practical problems, the elements of the supply chain negotiate to agree on values for θ and θ'. For this reason we can use a bargaining model based on asymmetric Nash equilibrium. To parameterize this model, one should define the satisfaction function for each part of the supply chain as equations below:

Manufacturer
$$S_m(\theta, \theta', \theta'') = \frac{\Delta TC_m}{\max \Delta TC_m} \tag{16}$$

Distributor
$$S_D(\theta, \theta', \theta'') = \frac{\Delta TC_D}{\max \Delta TC_D} \tag{17}$$

Retailer
$$S_c(\theta, \theta', \theta'') = \frac{\Delta TC_c}{\max \Delta TC_c} \tag{18}$$

In the aforementioned equations, the values for $\max \Delta TC_m$, $\max \Delta TC_D$ and $\max \Delta TC_c$ are deduced by setting values of θ_i, θ_i' and θ'' equal to zero in equations related to their costs in centralized model, respectively. In other words, $\max \Delta TC$ for each part of the chain is occurred by assuming that the mentioned part shouldn't pay any crashing lead time penalty cost. Under these circumstances one can allocate costs to the elements of the chain using this model:

$$Max\left\{(S_m)^{\lambda} \times (S_D)^{\lambda'} \times (S_c)^{1-\lambda-\lambda'}\right\}$$

$st: \quad TC_m \geq 0$

$TC_D \geq 0$

$TC_c \geq 0$

After solving the above model, the satisfaction of each part of the chain will be maximized and with respect to constraints, every part of the chain benefits from participating in centralized decision making procedure.

4. Numerical Example

Consider an inventory system with two kinds of product and three levels. Data for these products and elements of the chain are shown below:

$D_1 \approx N(500,8^2)$ $P_1 = 1800$ $A_1 = 150$ $A_1' = 150$

$D_2 \approx N(600,7^2)$ $P_2 = 2000$ $A_2 = 200$ $A_2' = 215$

$S_1 = 1300$ $h_1 = 30$ $h_1' = 20$ $h_1'' = 25$

$S_2 = 1500$ $h_2 = 20$ $h_2' = 15$ $h_2'' = 18$

$\pi_1 = 45$ $\pi_2 = 50$ $\beta = 0.5$

It is assumed that lead time contains three components as Table. 1 (Ye and Xu, 2010).

Table 1 Lead time components information

Lead time component	Normal duration (days)	Minimum duration (days)	Deference (week)	Unit penalty cost per day
1	20	6	2	0.4
2	20	6	2	1.2
3	16	9	1	5

So, the summarized lead time components information is as Table 2.

Table 2 Summarized lead time data

Lead time (week)	$R(L)$
8	0
6	5.6
4	22.4
3	57.4

The results for decentralized model are summarized in table 3.

Table 3 Results for decentralized model

L_i	Q_1	Q_2	TC_c
8	70.72	109.56	7102.48
6	72.03	111.08	6862.05
4	75.81	115.52	6619.52**
3	83.15	124.28	6798.94

After solving the decentralized model, the results are $m_1=m_2=3$

The extended results for centralized model are summarized in table 4.

For modeling the Nash equilibrium model we must first compute the satisfaction function for each element of the supply chain.

Table 4 Results for centralized model

m \ L	8			6			4			3		
	Q_1	Q_2	TC_{total}	Q_1	Q_2	TC_{total}	Q_1	Q_2	TC_{total}	Q_1	Q_2	TC_{total}
1	163.56	235.55	21843.4	163.85	232.87	21637.73	164.69	233.89	21445.46	166.43	235.99	21463.76
2	121.13	172.47	20763.1	121.48	172.88	20554	122.52	174.12	20392.43	124.66	176.67	20522.15
3	102.53	146.02	21371.99	102.91	146.46	21170.26	104.04	147.79	20136.28	106.36	150.53	21830.6**

Table 5 computing the satisfaction function

Element	Decentralized cost	Centralized cost	Difference	Maximum Difference	Satisfaction function
Manufacturer	8569.74	$7788.53+198.57\theta$	$781-198.57\theta$	781	$\frac{781-198.579\theta}{781}$
Distributor	7921.41	$7010.7+198.57\theta'$	$911.34-198.57\theta'$	911.34	$\frac{911.3476-198.579\theta'}{911.3476}$
Retailer	6619.52	$6520+198.57\theta''$	$298.103-198.57\theta''$	99.52	$\frac{99.52-198.579\theta''}{99.52}$

As mentioned in section 3, for determining the portion of decreasing lead time penalty cost that each member should pay we must solve the following model:

$$\max\left\{\left(\frac{781-198.579\theta}{781}\right)^{0.2}\times\left(\frac{911.3476-198.579\theta'}{911.3476}\right)^{0.3}\times\left(\frac{99.52-198.579\theta''}{99.52}\right)^{0.5}\right\}$$

$$st:$$

$$781-198.579\theta\geq 0$$

$$911.3476-198.579\theta'\geq 0$$

$$99.52-198.579\theta''\geq 0$$

$$\theta+\theta'+\theta''=1$$

$$0\leq\theta,\theta',\theta''\leq 1$$

This model is solved by GAMS optimization software and the results are as follows:

$$\theta=0.281$$
$$\theta'=0.253$$
$$\theta''=0.466$$

So, the manufacturer, the distributor and the retailer should pay respectively %28, %0.25 and %47 of the total penalty cost to maximize their benefits from participating in group decision making.

5. Conclusions and Future Researches

Lead time plays an important role in supply chain decisions. In many cases, instead of considering the lead time for each product as a fixed parameter, it can be defined as a decision variable which can vary in a predetermined range. In such problems, each part of the supply chain must burden a proportion of total penalty cost that should be paid by the chain to decrease the lead time. The proposed method in this manuscript is for determination of the proportion that
should be allocated to each element of the supply chain in order to makes the participation in centralized decision making procedure beneficiary to all elements.

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Applying the conclusions of this article in real case problems, determining the satisfaction function in a more realistic manner and extending the bargaining model to non-asymmetric Nash equilibrium can be as future research suggestions in this area.

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