Performance Investigation of Classic and Heuristic Methods in Portfolio Optimization

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Abstract: Contrary to the growing use of portfolios and in spite of the rich literature on the subject, yet there are some problems and unanswered questions. The aim of this work is to be a useful instrument for helping finance practitioners and researchers with the portfolio selection problem. This study reviews Modern Portfolio Theory (MPT) literature and describes the problems and solutions, which have been put forward in the literature. In this paper, selection of a portfolio is optimized via two different methods from two major optimization approaches, Heuristic and Classic. Heuristic methods are supposed not to "get stuck" in local optima, in which classics often do get stuck. Heuristic algorithms perform a wide random search; consequently, the chance of being trapped in local optima is deeply decreased. Therefore, in this study, Genetic Algorithm, a heuristic evolutionary method, and a classic solver are applied to construct and optimize portfolios in a sample market of five stocks. The research findings indicate that Genetic Algorithm, in contrast to classic methods, is more adaptable to the portfolio selection problem and has a better performance in contrast to its classic optimization counterparts.
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1. Introduction

The rationale for stock market diversification is that the overall risk from owning many stocks is lower than the risk of holding a few stocks (Hagin, 1979). In a metaphor *"Money is like manure – when it stacks up, it stinks; when you spread it around, it makes things grow."* Texan Clint Murchison, Sr., one of the wealthiest investors ever. Effects of diversification on risk reduction have been studied extensively (Brealey, 1969, Evans, 1968, Gaumnitz, 1967), but the importance of portfolio management lies not in the number of holdings, but rather in both the nature and degree of the combined risk of the underlying stocks (Hagin, 1979). Brealey (Brealey, 1969) has shown quite dramatically what can happen when one considers both the nature and degree of risk in portfolio composition. He reported that a portfolio containing *only* 11 securities, which were carefully selected for their risk-diversifying characteristics, would be less risky than a portfolio of 2000 securities, which were selected without regard to risk!

Thus, the question is *"How should investment risk be considered?"*. Furthermore, *"Once the investment risk is considered, how should portfolios be optimized?"* This paper intends to summarize the answers to these questions efficiently, and also to make new viewpoints on the topic.

Modern portfolio theory (MPT) takes its origin from the pioneering work of Markowitz. Since Markowitz's 1952 revolutionary article on portfolio selection (Markowitz, 1952), there has been many contributions to this important field of financial studies, and within last 60 years considerable progress has been made. Markowitz's work has permanently changed the course of investment-related thought. Before Markowitz's article, it was more or less taken for granted that the proper way to construct an investment portfolio was just to select the best securities. It was erroneously assumed that this technique would maximize the expected return of the resultant portfolio (Hagin, 2004).

Markowitz model has received remarkable attention of math and computer science experts, because finding the optimum solution to the model equations has always been a challenging issue. Experts have tried different optimization methods to go through this problem, many of whom have tried classic methods and many have tried heuristic methods. This research is also carried out to evaluate the performance of both approaches and make a comparison between them in order to identify the more promising approach.

The rest of the paper is organized as follows: In section 2, a review of the literature of Markowitz portfolio theory and its shortcomings is given. Section 3 is devoted to an investigation of portfolio optimization approaches, methods and programs in brief. In section 4, methodology and data are presented. A portfolio selection problem is optimized via two different methods and discussed in the next section, section 5. Finally, section 6 summarizes the paper and talks about the conclusions.

2. Literature Review

2.1. Markowitz Model

The insight for which Harry Markowitz received the Nobel Prize was first published in 1952 in an article entitled *"Portfolio Selection"* (Markowitz, 1952) and more extensively in his 1959 book, *Portfolio Selection: Efficient Diversification of Investments* (Markowitz, 1959). When published in 1952, Markowitz's ideas scarcely took the investment profession by storm. As with insights from other researchers, his equation-filled presentation is over the heads of most investors. Without the benefit of Markowitz's insights, dangerous homilies such as *"put all of your eggs in one basket, and watch the basket"* still prevail (Hagin, 2004).

Markowitz pointed out that the goal of portfolio management is not solely to maximize the expected rate of return, but instead to maximize *"expected utility"* – setting complexities aside, *"utility"* can be viewed as being synonymous with *"satisfaction"*.

Markowitz began with the valid premise that all investors want a combination of high returns and low risk. In other words, rational investors maximize their utility by seeking either;

a. The highest available rate of return for a given level of risk. Or

b. The lowest level of available risk for a given rate of return (Hagin, 1979).

One of Markowitz's most important innovations was using the variance (or its square root, the standard deviation) of a distribution of likely returns (possible expected returns) as a measure of Risk. With his insight, Markowitz reduced the complicated and multidimensional problem of portfolio construction with respect to a large number of different assets, all with varying properties, to a conceptually simple two-dimensional problem known as *"mean-variance"* analysis (Hagin, 2004).

According to Markowitz formulation, the selection of an efficient portfolio begins with an analysis of three estimates:

a. The expected return for each security.

b. The variance of the expected return for each security.

c. The possibly offsetting, or possibly complementary, interactions, or covariance, of return with every other security consideration (Hagin, 1979).

The calculation of the expected return for an aggregate of portfolio of securities is relatively easy; it is merely the weighted average of the expected return of the individual securities. Nothing else is relevant:

$$E\ (\text{Rportfolio}) = \sum_{i=1}^{n} E(R_i)\,.\,W_i \qquad \textbf{(1)}$$

Where:

E (Rportfolio) = Portfolio's expected rate of return.

E (Ri) = Security i's expected rate of return.

Wi = the proportion of the portfolio's value invested in security i (Markowitz, 1952).

The calculation of the combined variance is more complicated. The point to be considered is that the risk of a portfolio is *not* typically equal to the weighted average of the risks of its component securities. The risk of a portfolio depends not only on the risks of its securities, considered in isolation, but also on the extent to which they are affected similarly by underlying events (Sharpe, 1978). Therefore, the relationship between the risk of a portfolio, consisting of n securities, and the relevant variables could be shown as follows:

$$\text{VAR(Rportfolio)} = \sum_{i=1}^{n} \sum_{j=1}^{n} \text{Wi.Wj.COV (Ri,Rj)} \qquad \textbf{(2)}$$

Using relevant mathematical equations, eq. 2 could be transformed to eq. 3 shown below:

$$\text{VAR(Rportfolio)} = \sum_{i=1}^{n} \sum_{j=1}^{n} \text{Wi.Wj.SRi.SRj.rij} \qquad \textbf{(3)}$$

Where in both above equations:

VAR (Rportfolio) = Variance of portfolio's rate of return.

VAR (Ri) =Variance of security i's rate of return.

COV (Ri,Rj)= Covariance between securities i,j's rate of return.

n = the number of securities.

Wi or Wj =the proportion of the portfolio's value invested in security i or j.

rij = Coefficient of correlation between securities i and j.

Si = Standard deviation for security i.

And Sj = Standard deviation for security j (Markowitz, 1952).

Investigating risk equation leads to some interesting results:

1. In portfolios with perfectly positively correlated returns, diversification does not help. *In such cases, diversification does not provide risk reduction, only risk averaging.*
2. In portfolios with perfectly negatively correlated returns, *diversification can eliminate the risk.* This principle motivates all hedging strategies.
3. A special case of extreme importance arises when a cross-plot of security returns shows no pattern that can be presented even approximately by an upward-sloping or downward-sloping line, known as uncorrelated returns. *In this case, the risk of the portfolio is less than the risk of its component securities.* Diversification has indeed helped and has surely provided substantial risk reduction. This case provides the basis for insurance, or risk pooling. This is why insurance companies attempt to write many individual uncorrelated policies and spread their coverage so as to minimize overall risk (Sharpe, 1978). Imagine a portfolio of equal parts of N securities, each with an equal risk of S%. Then, by using eq.2 (or eq.3), and simplifying, following results would be achieved:

$$\text{Total Risk of the portfolio} = S_{\text{portfolio}} = \frac{S\%}{\sqrt{N}} \quad (4)$$

And

$$\lim_{N\to\infty} S_{\text{portfolio}} = \frac{S\%}{\sqrt{N}} = 0 \quad (5)$$

2.1.1. Basic assumptions of Markowitz model

a. The investor does (or should) maximize the discounted (or capitalized) value of future returns (Markowitz, 1952, Williams, 1938).

b. Investors behave rational in investment decisions, which means they choose to hold efficient portfolios- portfolios, which maximize each investor's utility (Markowitz, 1952).

2.2. Shortcomings of Markowitz Model and Solutions

This section reviews the most important criticisms of Markowitz model and solutions or suggestions made over different periods by experienced practitioners and researchers.

2.2.1. The alternatives of lending and borrowing, which are very general alternatives in stock markets, are omitted and ignored in Markowitz original model.

William Sharpe (Sharpe, 1964) broadened Markowitz' analysis to include riskless asset (such as short-term government securities) and the possibility of borrowing; the approach is known as Asset allocation line and in a more general sense, capital asset pricing model (CAPM). Besides, Arbitrage theory of capital asset pricing, which is similar to CAPM and a substitute for that, was developed by Ross et al. in 1980's (Roll, 1984, Ross, 1976). Similar to CAPM, APT is based on the rationale that unsystematic risk is diversifiable and therefore, should have a zero price in the market with no arbitrage opportunities. It is out of the scope of this paper to discuss them here, we only mention that in his broadened model, Sharpe demonstrated: *"When a risky security or portfolio is combined with a riskless one, the risk of the combination is proportional to the amount invested in the risky component (Sharpe, 1967)."*

2.2.2. Markowitz efficient portfolios are based on the analysis of estimates of future risk and expected returns, hence these estimates and therefore, the efficient set can be expected to change over time. As a result, the amounts of both human and computational resources to resolve the model, increases enormously.

To use quantitative expressions, for a market of N stocks, Markowitz model needs $\frac{N(N+3)}{2}$ different estimates. Aside from the sheer volume of the data, practical problems face experts who are to estimate the co-movements between different stocks from different industries. Furthermore, even if the experts prepared such estimates, it would be difficult to place any confidence in their accuracy (Hagin, 1979). To bridge the gap between his theoretical solution and the practical problems, Markowitz suggested using the relationship between each security's rate of return and the rate of return on a market index as a substitute for explicit data on the covariance of each pair of securities under study. Brealey justifies this approach artistically: *"when the wind of recession blows, there are few companies that do not lean with it".* Sharpe (Sharpe, 1964) pursued the approach with single-index model. This model is driven with three estimates:

a. The amount of specific, or non-market, return (Alpha).

b. A measure of responsiveness to market movements (beta).

c. The variance of non-market return.

To solve some disquieting assumptions reflected by Sharpe's model, limited multi-index approach, which still minimized data collection requirements and had advantages like *"fine-tuning* *and explicitly estimating the comovements"*, was introduced and evaluated by several tests for performance investigation in contrast to other portfolio construction alternatives (King, 1966, Cohen, 1967, Elton, 1973).

2.2.3. Markowitz normative model has found relatively little application in practice when some additional features, such as fixed costs and minimum transaction lots, are relevant in the portfolio selection problem.

Hans Kellerer et al. (Kellerer, 2000) introduced some different mixed-integer linear programming models dealing with fixed costs and possibly minimum lots to solve this shortcoming. Three main conclusions can be drawn of their research:

a. The fixed costs increase a portfolio's risk by reducing the number of securities held.

b. The introduction of fixed costs produces a reduction in portfolio diversification much greater than the increase of their level.

c. The application of minimum lots strengthens the effects described at the previous point.

2.2.4. The combination of a large number of stocks and a large number of transactions (trading tickets) increases the costs of trading because both custodial fees and transaction costs increase.

It is usually the case in Markowitz models. To overcome this problem, Dimitris Bertsimas et al. (Bertsimas, 1999) developed a method to construct portfolios through nonlinear mixed-integer programming. Their portfolio is close to a target portfolio that is constructed using quadratic programming. The constructed portfolio has the same liquidity, turnover and expected return as the target portfolio; it controls frictional costs and does so with fewer stocks and fewer tickets.

2.2.5. In efficient and semi-efficient markets, the future returns of each security cannot be correctly reflected by the securities' data in the past. Therefore, the statistical techniques and the experts' judgment and experience are combined together to estimate the security returns in the future.

To be more realistic in estimations, Jun Li and Jiuping Xu (Li, 2009) assumed the returns of securities to be fuzzy random variables; they concluded that the proposed model can provide more flexible results. They also showed that the proposed portfolio selection model can generate an efficient frontier according to the investor's degree of optimism.

2.2.6. MPT relies on the assumptions that markets are efficient and investors are rational. However, many market anomalies have been identified that question these assumptions.

Behavioral finance provides several explanations for these anomalies and has provided theories that explain the inefficiency of markets and the apparent irrationality of investors. Bart Frijns et al. (Frijns, 2008) showed that mean-variance variables in Markowitz model can be extended by behavioral concepts and socio-demographic variables. They showed that *the level of the risk-free rate, an individual's risk aversion, market sentiment, self-assessed financial expertise, age and gender* are other determining factors of portfolio choice.

3. Portfolio optimization approaches, methods and programs

Markowitz demonstrated that once prepared, the foregoing security descriptions could be manipulated by portfolio optimization programs to produce an explicit definition of the efficient portfolio in terms of:

a. The securities to be held.
b. The proportion of available funds to be allocated to each (Hagin, 1979).

Since then, many methods have been applied to solve Markowitz equations. One of the earliest methods used to solve the equations is known as *"quadratic programming"* Other classic solvers have also been applied to the model; authors can refer to programs like *LINDO, LINGO* and *Microsoft Office Excel* as some sample programs that use classic solvers as optimizers. The problem with classic methods is that they can usually just find the local optima, *in other words*, in a complicated problem with nonlinear equations, like that of Markowitz, classic solvers stop solving as soon as reaching a local optimum!

To overcome this problem, heuristic (also referred to as Metaheuristic) methods have been applied to the portfolio selection problem. Although still there is no guarantee, heuristic methods have *usually* turned out to achieve better results and reach higher performances in contrast to their classic counterparts; especially it is so in complicated problems (e.g. engineering problems in electrical, communications and flood mechanics fields). This better performance goes back to their nature of design; they have been created to *"jump out"* of local optima to reach the global optimum. They are supposed not to *"get stuck"* in local optima. *In other words*, because heuristic algorithms perform a wide random search, the chance of being trapped in local optima is deeply decreased. Fig. 1 illustrates the concept of local and global optimum.

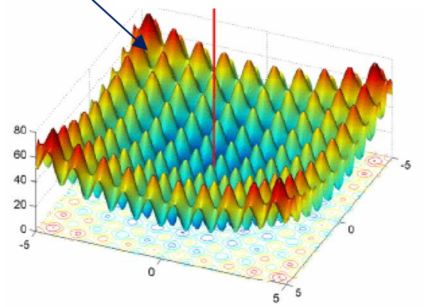

Figure 1. The concept of local and global optimum
Source: Example of Rastrigin's Function, MATLAB2007a[21].

4. Methodology and Data

4.1. Methodology

The authors have examined different metaheuristic methods to solve the portfolio selection problem, from which the followings have outstanding performances and could be mentioned here: Genetic Algorithm, Particle Swarm Optimization (PSO) (Talebi Arash, 2010), Simulated Annealing, and Imperialist Competitive Algorithm (ICA).

In this section, portfolio selection problem is optimized using the Genetic Algorithm (GA), *which will be described in following paragraphs briefly*, and a classic solver. For GA, MATLAB software (The MathWorks, 1984-2009) is used and Microsoft Office Excel (Microsoftcorporation, 2007) is also invoked as a classic optimizer.

4.1.1 Genetic algorithm

The basic principles of genetic algorithms (GAs) were first proposed by Holland (Holland, 1975). GAs are general-purpose search algorithms, which use principles inspired by natural genetics to evolve solutions to problems (G´omez, 1999). A basic element of the Biological Genetics is the chromosomes. Chromosomes cross over each other. Mutate itself and new set of chromosomes is generated. Based on the requirement, some of the chromosomes survive. This is the cycle of one generation in Biological Genetics. The above process is repeated for many generations and finally best set of chromosomes based on the requirement will be available. The Mathematical algorithm equivalent to the above behavior used as an optimization technique is called *"Artificial Genetic Algorithm"* (Gopi, 2007).

The basic idea is to maintain a population of chromosomes (representing candidate solutions to the concrete problem being solved) that evolves over time through a process of competition and controlled variation. A GA starts with a population of randomly generated chromosomes, and advances toward better chromosomes by applying genetic operators modeled on the genetic processes occurring in nature. The population undergoes evolution in a form of natural selection. During successive iterations, called generations, chromosomes in the population are rated for their adaptation as solutions, and based on these evaluations, a new population of chromosomes is formed using a selection mechanism and specific genetic operators such as crossover and mutation (Pourzeynali, 2006).

An evaluation or fitness function must be devised for each problem to be solved. Given a particular chromosome, a possible solution, the fitness function returns a single numerical fitness, which is supposed to be proportional to the utility or adaptation of the solution represented by that chromosome.

Although there are many possible variants of the basic GA, the fundamental underlying mechanism consists of three operations: evaluation of individual fitness, formation of a gene pool (intermediate population) through a selection mechanism, and recombination through crossover and mutation operators. The operators invoked by Gas are described in here:

(1) Chromosome representation

Each design is represented by an n0-bit-long chromosome, where n0 is the sum of the length required to represent each design variable.

(2) Initial population

The GA starts from a population of chromosomes as a set of initial designs. The initial population is chosen randomly.

(3) Fitness function

The GA uses a function value for the selection of an operator; this function reflects the objective and a penalty for constraint violation.

(4) Crossover

The crossover operator is used to produce two offsprings from the selected parents.

(5) Mutation

In order to maintain variability of population, operation is also performed on certain individuals. The mutation is performed on a bit-by bit basis, with a certain probability of mutation (Back, 2000, Chambers, 2001, David Davis, 1999, Back, 1996). The GA process is shown in Fig. 2.

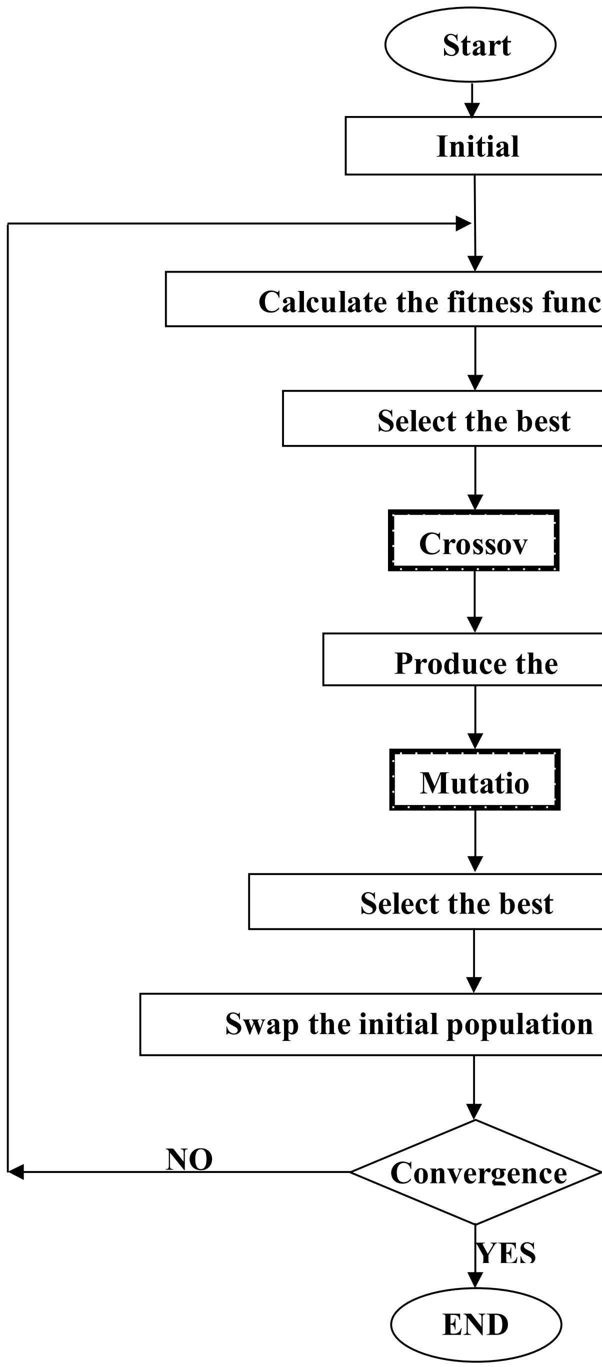

Figure 2. Flowchart of Genetic Algorithms
Source: "Adaptation in natural and artificial systems" by JH. Holland [24]

4.2. Data

Table 1 shows the data for five stocks rate of return during ten periods, note that data presented here is semi-real data and is extracted compatibly from real stock market sources. Using related equations, matrix 1 (table 2) is approximated, which is the covariance matrix between each pair of stocks.

5. Findings and Discussion

5.1. Calculations and Results

Using Excel solver and above tables as inputs, and by solving for the optimum weights, which minimize portfolios risk while maximizing its return, w_i*'s* would be calculated as follows:

$W_1 = 0.25$ $W_2 = 0.06$
$W_3 = 0.25$ $W_4 = 0.25$ $W_5 = 0.19$

Where W_i *is the proportion of the portfolio's value invested in security i.*

Substituting the above weights in eq.1 leads to below expected return for the constructed portfolio:

$ER_{portfolio} = 0.3606711$ *or* 36.06711%

And their substitution in eq.2 leads to the following portfolio risk:

VAR (Rportfolio) = 0.013949 or 1.3949%, so

Risk (Rportfolio) = $\sqrt{VAR(R_{portfolio})} = 0.118105$ *or* 11.8105%.

Excel sensitivity report, limits report and answer report are presented in tables 3 to 5.

Solving for the optimum weights using MATLAB's Genetic Algorithms toolbox and the related M-files prepared by authors, results in the following proportions for w_i*'s*:

W1 = 0.25 *W2 = 0.1072*
W3 = 0.25 *W4 = 0.25* *W5 = 0.1428*

This, in turn, leads to the following results:
$ER_{portfolio} = 0.3632$ *or* 36.32% *and*
VAR (Rportfolio) = 0.0134
or 1.34% and thus, the risk would be calculated as below:

Risk (Rportfolio) = 0.1156 or 11.56%.

Table 6 illustrates GA's optimum parameters required to achieve the best set of solutions in the portfolio selection problem.

In figures 3 and 4, MATLAB generated figures are illustrated. Best Fitness diagram, fig.3, plots the best function value in each generation versus the iteration number. Note that this figure presents the process GA went through to reach the optimum answer. Best individual column diagram, fig.4, plots the vector entries of the individual with the best fitness function value in each generation.

Note that in both methods, the very same goal function is optimized and the same constraints are applied[i].

Table 1. Data for five stocks rate of return during ten periods (The numbers are expressed as the percentage)

Period/Stock	**A**	**B**	**C**	**D**	**E**
P1		40	50		-20
P2	¶	37	25		-32
P3	62	45	-30		-15
P4	25	38	-15	95	40
P5	30	-62	35	19	55
P6	37	40	35	18	14
P7	40	94	35	54	85
P8	85	39	55	35	55
P9	42	32	38	40	28
P10	20	15	45	65	55

¶ Blank in the table means that stock was not present in the stock exchange market during that period, blanks are included to represent more general and real cases.

Source: Extracted from a real stock market by authors.

Table 2. The covariance matrix between stocks (Matrix 1)

Stock/Stock	**A**	**B**	**C**	**D**	**E**
A	0.039898438	0.022579688	0.003015625	-0.018106122	-0.011539063
B	0.022579688	0.133756	-0.011204	0.038857143	-0.0056
C	0.003015625	-0.011204	0.069661	-0.035546939	0.027695
D	-0.018106122	0.038857143	-0.035546939	0.064195918	0.01207551
E	-0.011539063	-0.0056	0.027695	0.01207551	0.135065

Source: Calculated by Authors.

Table 3. Excel Sensitivity report

Cell	Name	Final Value	Reduced Gradient
B16	Stock A weight	25%	-13%
C16	Stock B weight	6%	0%
D$16	Stock C weight	25%	-5%
E16	Stock D weight	25%	-11%
F16	Stock E weight	19%	11%

Constraints

Cell	Name	Final Value	Lagrange Multiplier
H16	Total	100%	15%
H18	Total	36.06710714	0

Source: Calculated by Authors.

Table 4. Excel Limit report

Cell	Target Name	Value
J41	To be opt. function	0.098067294

Table 5. Excel Answer report

Population type	Double Vector	**Mutation function**	Gaussian
Population size	50	**Mutation scale**	1
Selection function	Roulette	**Number of generation**	500
Crossover fraction	0.8	**Stall time limit**	Infinite
Crossover function	Scattered	**Stall generations**	50

Table 6. Best parameters for GA solver in portfolio optimization problem †.

Constraints Cell	Name	Cell Value	Formula	Status	Slack
H16	Total	100%	H16=1	Not Binding	0
H18	Total	36.06710714	H18>=0	Not Binding	36.06710714
F16	Stock E weight	19%	F16<=0.25	Not Binding	0.06
B16	Stock A weight	25%	B16>=0	Not Binding	25%
C16	Stock B weight	6%	C16>=0	Not Binding	6%
D16	Stock C weight	25%	D16>=0	Not Binding	25%
E16	Stock D weight	25%	E16>=0	Not Binding	25%
E16	Stock D weight	25%	E16<=0.25	Binding	0
B16	Stock A weight	25%	B16<=0.25	Binding	0
C16	Stock B weight	6%	C16<=0.25	Not Binding	19%
D16	Stock C weight	25%	D16<=0.25	Binding	0
F16	Stock E weight	19%	F16>=0	Binding	0%
Constraints Cell	Name	Cell Value	Formula	Status	Slack
H16	Total	100%	H16=1	Not Binding	0
H18	Total	36.06710714	H18>=0	Not Binding	36.06710714
F16	Stock E weight	19%	F16<=0.25	Not Binding	0.06
B16	Stock A weight	25%	B16>=0	Not Binding	25%
C16	Stock B weight	6%	C16>=0	Not Binding	6%
D16	Stock C weight	25%	D16>=0	Not Binding	25%
E16	Stock D weight	25%	E16>=0	Not Binding	25%
E16	Stock D weight	25%	E16<=0.25	Binding	0
B16	Stock A weight	25%	B16<=0.25	Binding	0
C16	Stock B weight	6%	C16<=0.25	Not Binding	19%
D16	Stock C weight	25%	D16<=0.25	Binding	0
F16	Stock E weight	19%	F16>=0	Binding	0%

Source: Calculated by Authors.

† Other parameters that may not be mentioned are best achieved using GA toolbox defaults.

Source: Calculated by Authors

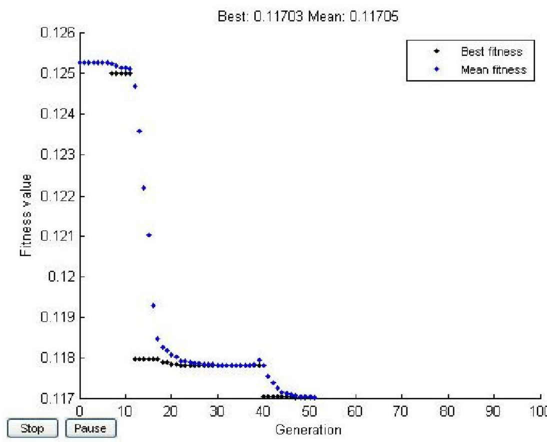

Figure 3. Best Fitness
Source: Calculated by Authors

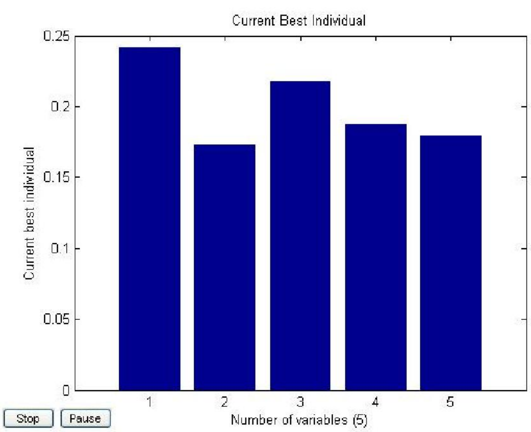

Figure 4. Best Individual
Source: Calculated by Authors

5.2. Discussion

Comparing both applied methods, it is obvious that GA has better performance in both risk and return issues. In quantitative expressions:

$$ERGA._{constructedportfolio} - ERClassic._{constructedportfolio} = 0.25289\%$$

$$Risk(RGA._{constructedportfolio}) - Risk\ (RClassic._{constructedportfolio}) = -\ 0.2505\%$$

At first glance, it might appear it is not much different which method to use, *in other words*, both methods have almost led to the same results and performance. Nevertheless, *deeply looking*, this is not true for at least two main reasons:

1. In real markets, surely there are many stocks more than just five, on which one should make investment decisions. As the dimensions of the problem extend, heuristic methods have better performances (The authors are currently working on a 50 sample model and better performance of heuristics in high dimensions is crystal clear). In other words, the more the number of available stocks, more difficult and time-consuming it will be to solve the problem via classic methods (Application of heuristic methods would be certainly much easier in such cases).

2. Non-individual portfolios -those constructed and managed by investment companies like banks and mutual funds- are usually big enough not to neglect 0.25289% increase in the expected return or -0.2505% decrease in portfolio's risk.

Another clue for not neglecting apparently small differences arises when one considers the fact that the only way the systematic risk can be lowered is to expand the definition of the *"market"* to include dissimilar markets (Hagin, 1979). Internationally diversified portfolios are much less risky than the limited ones (Solnik, 1974a, Solnik, 1974b). In such cases, small percentage of risk or expected return is representative of a relatively large amount of money.

6. Summary and Conclusions

Markowitz demonstrated that the two relevant characteristics of a portfolio are its expected return and some measure of its risk- operationally defined as the dispersion of possible returns around the expected return. Rational investors will choose to hold efficient portfolios. The identification of efficient portfolios would require information on each security's expected return, variance of return and covariance of returns. Finally, once prepared the foregoing security descriptions could be manipulated by portfolio optimization programs.

Markowitz model's main shortcomings are omission and ignorance of borrowing and lending alternatives, the enormous amounts of both human and computational resources it needs, its little application in practice when some additional features are relevant, increases in the costs of trading, poor future returns' prediction, which is to be fed to the model, relying on the questionable assumptions that markets are efficient and investors are rational.

To overcome the shortcomings different solutions are proposed by experienced practitioners and researchers, from which Asset allocation line, Single-index model, mixed-integer linear programming models, portfolio construction through nonlinear mixed- integer programming, fuzzy random variable consideration, and going toward behavioral finance, are the most important ones, *respectively*.

Portfolio optimization methods are divided into two major groups, classics and heuristics. It was shown that in portfolio optimization problem, heuristic methods have better performances in contrast to classic methods and are more adaptable with the portfolio problem. It is so because heuristic methods do not stop solving as soon as finding a local optimum, they continue solving to find the global optimum. In other words, they are supposed not to *"get stuck"* in local optima. Since heuristic algorithms perform a wide random search, the chance of being trapped in local optima is deeply decreased. On the other hand, classics *"get stuck"* in local optima and cannot usually reach the global one, especially in high dimension problems like that of Markowitz in real stock markets. A portfolio selection problem was optimized for five stocks with semi-real input data and via two different methods, a heuristic method named genetic algorithm, and a classic solver. The results indicate that GA was better in both portfolios' expected return and portfolio's risk dimensions, though the difference seems to be small. Portfolio constructed via GA outperformed the classic-constructed portfolio. For *at least* two reasons, the small percent difference between two methods is found to be of high importance: 1. The volumes of portfolios are usually large enough not to neglect a small percentage, and 2. Once the number of stock increases, the difference between heuristic and classic methods' result is increased as well.

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[i] The Goal Function used in both methods is: $\text{Minimizing } K = \text{Risk} - \frac{\text{Expected return}}{18}$

Division by 18 is used to make both terms in the same range to achieve the best possible result. Constraints are to make sure that 0% $\leq$wi' $\leq 25\%$, the constraints are aimed at maximizing rational possible diversification, and also to guarantee that Expected Return $\geq 0\%$.