Study of nuclear shape and alignments in the odd-proton Holmium isotopes

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Abstract: We report here in the present work the properties of high spin spectroscopy and the alignment effects in the $^{153-163}$Ho isotopes which characterized by smooth increase of the transition energies with respect to spin. The level structure has been established up to high energy and spin values. The systematic behavior of the level pattern of the odd-proton Ho isotopes is discussed. An interesting nuclear features emerging from this study concerns the evaluation of the moment of inertia and the yrast line yields conclusions about the nuclear shape. Also a Comparison study of the results in studied isotopes has been done. Accurate description of back-bending phenomena and band crossing in the Ho isotopes are discussed in the frame of the systematic found in this mass region The change in deformation can be explored through the so- called gauge plots.

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1-Introduction

Nuclei with N=86-96 are spherical in the ground states, but become oblate at high spin along the yrast line, as successive high- j nuclei align their spins along the symmetry axis. On the other hand, nuclei with N ≥ 96 are prolate up to the highest observed spins. One illustration of this point is the absence of nano-second isomers. It appears that the shell effects which drive nuclei prolate are not overcome at high spin. The fascinating progress in this high spin field has been made possible by the essential development in experimental on exciting and detecting the high spin states in nuclei. A part from such interesting expectations as the existence of a super-backbending or super-deformation [1], the research in this new field can also be considered as providing a tool for testing the validity of nuclear models under extreme conditions. As an example, the back-bending phenomena discovered in 1970 [2] is now interpreted as being due to the crossing of the ground state band with an aligned quasiparticle band [3, 4, 5].

In the present work we study the properties of the bands at high spins and alignment effects in the lighter $^{153-163}$Ho isotopes [6-17]. They are located on the border of the strongly deformed region and are of interest for an investigation of the transition towards more spherical nuclei. In this mass region the study of the nuclear states offers interesting nuclear features. The evaluation of the moment of inertia and the yrast line yields conclusions about the nuclear shape. The strong staggering in ^{153}Ho shows this nucleus may be soft rotor. A comparison data for the $^{153-163}$Ho isotopes has been done which show that these isotopes may be hard rotor. Also the back-bending phenomena for ^{157,159,161}Ho nuclei [10, 11] show a dramatic changes through a plot of the moment of inertia ($2\Im / \hbar^2$) versus the square of rotational frequency ($\hbar^2 \omega^2$).

2- Calculations and discussions

Yrast line

The alignment which has been computed for the studied nuclei is always decreasing when the masses increase (Table 1 and Fig. 1). The explanation is similar in all nuclei**.** Comparison study of the calculations in all studies isotopes well be done. It is interesting to calculate and plotted the moment of inertia as a function of the square of the rotational frequency and to investigate how they change as the rotational angular velocity of the nucleus varies (Fig. 1). In this figure the increase in the moment of inertia at $\hbar^2 \omega^2 \cong 0.11$. MeV^2 is so rapid that the rotational frequency actually decreased as higher spin states are reached. The following interesting features can be observed: (i) With decreasing mass number (decreasing deformation and decreasing moments of inertia see Table 2, Fig. 2), the back-bending become sharper, which reflects a decreasing interaction energy. (ii) In general, the calculated backbends $I_{crit.} \cong A^{7/12}$ where $I_{crit.}$ correspond to the point at which the moment of inertia changing at the maximum rate [4, 5], which corresponding to the value of $\hbar^2\omega^2 \cong 0.11$ MeV^2, the calculations predict the back bending to occur at $I_{crit.} \cong 40/2$. (iii) At higher frequencies the behaviour of the three nuclei is approximately the

(17/2 ,21/2), (25/2 ,29/2), is predicted in models using γ- unstable potentials [21,22]. In Fig. (3, 4), Table (1) also included the corresponding data for the $^{153\text{-}163}$Ho isotopes [10 – 17], which show that these isotopes may be hard rotor, also clear in Fig. (7). The comparison demonstrates that both the oscillation and the compression strongly increase from ^{163}Ho to ^{153}Ho which in turn indicates that the coriolis mixing becomes much stronge

same. In the range between 0.138 MeV ^{2} and 0.228 MeV ^{2} , $2\mathfrak{I} / \hbar^2$ depends linearly on $\hbar^2\omega^2$. A more revealing display the yrast band properties are giving Fig. (3), where the moment of inertia parameter as calculated from the level spacing is plotted versus I. In ^{153}Hothe plot show strong oscillations of $\hbar^2/2\mathfrak{I}$, which can be traced to the admixture in the ground, band its decoupling parameters. Also this strong staggering shows that the ^{153}Ho may be soft rotor gainst γ-deformation, since the clustering of levels as

Table (1): Excitation energy, moment of inertia, square of rotational frequency and the inertial parameters for the Ho-isotopes.

		^{157}Ho				^{163}Ho				^{165}Ho			
I	I(I+1)	E	$2\vartheta/^2\hbar$	$(\hbar\omega)^2$	$(E_I\text{-}E_{I\text{-}2})/2I$	E	$2\vartheta/^2\hbar$	$(\hbar\omega)^2$	$(E_I\text{-}E_{I\text{-}2})/2I$	E	$2\vartheta/2\hbar$	$(\hbar\omega)^2$	$(E_I\text{-}E_{I\text{-}2})/2I$
13/2	48	.954	39	.19	75	-	-	-	-	-	-	-	-
17/2	81	1.59	60	.11	47	.255	136	.026	25	.335	159	.022	23
21/2	121	1.86	158	.028	23	.690	102	.058	31	.710	115	.047	28
25/2	169	2.706	67	.188	44	1.254	95	.090	33	1.225	104	.076	30
29/2	225	3.292	106	.095	30	1.912	95	.119	33	1.826	103	.101	31
33/2	289	3.986	161	.055	31	2.642	98	.143	32	2.846	107	.119	30
37/2	361	4.225	311	.024	17	3.405	104	.157	31	3.168	116	.126	28
41/2	441	5.020	315	.027	29	4.120	122	.138	27	3.843	129	.123	27
43/2	484	5.416	222	.049	19	-	-	-	-	-	-	-	-
45/2	529	-	-	-	-	4.813	137	.130	25	4.533	138	.128	25
47/2	576	5.615	472	.01	14	-	-	-	-	-	-	-	-
49/2	625	6.107	205	.071	13	5.554	140	.147	25	5.272	140	.147	25
51/2	676	6.500	265	.049	27	-	-	-	-	-	-	-	-
53/2	729	-	-	-	-	6.360	139	.173	25	6.079	139	.173	25
57/2	841	-	-	-	-	7.234	138	.202	25	6.692	137	.205	25
61/2	961	-	-	-	-	-	-	-	-	7.915	136	.237	25

Table (1): Cont.

		^{167}Ho				^{171}Ho				^{175}Ho			
I	I(I+1)	E	$2\vartheta/^2\hbar$	$(\hbar\omega)^2$	$(E_I\text{-}E_{I\text{-}2})/2I$	E	$2\vartheta/^2\hbar$	$(\hbar\omega)^2$	$(E_I\text{-}E_{I\text{-}})/2I$	E	$2\vartheta/2\hbar$	$(\hbar\omega)^2$	$(E_I\text{-}E_{I\text{-}2})/2I$
13/2	-	-	-	-	-	.245	141	.019	-	.437	145	.018	-
17/2	81	.350	166	.021	22	.513	129	.028	26	.713	126	.029	26
21/2	121	.693	127	.049	26	.867	123	.042	27	1.078	120	.043	27
25/2	169	1.152	115	0.063	28	1.307	119	.059	28	1.550	112	.066	29
29/2	225	1.705	112	0.086	29	1.828	118	.078	28	2.006	133	.062	26
33/2	289	2.333	112	.109	29	2.426	117	.100	28	2.579	122	.092	27

37/2	361	3.007	117	.124	28	3.093	118	.121	28	2.952	203	.045	20
41/2	441	3.668	131	.120	26	3.820	120	.143	28	3.804	104	.191	31
43/2	484	-	-	-	-	-	-	-	-	-	-	-	-
45/2	529	4.336	142	.121	25	4.594	124	.159	27	4.727	127	.152	31
47/2	576	-	-	-	-	-	-	-	-	-	-	-	-
49/2	625	5.085	138	.151	25	5.378	132	.165	26	5.531	129	.173	26
51/2	676	-	-	-	-	-	-	-	-	-	-	-	-
53/2	729	5.923	134	.186	25	6.178	140	.170	25	6.371	234	.186	26
57/2	841	6.838	132	.221	26	-	-	-	-	7.455	113	.356	29
61/2	961	-	-	-	-	-	-	-	-	-	-	-	-

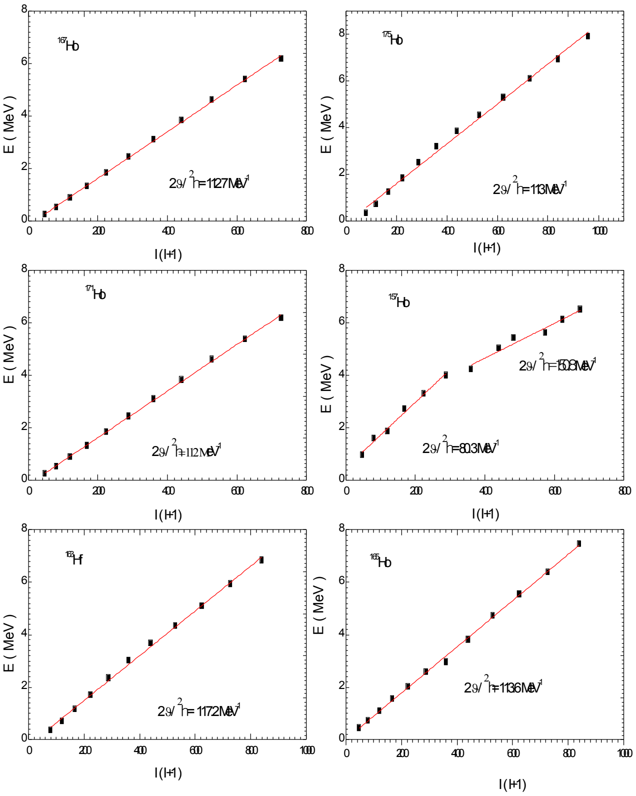

Fig. (1): Moment of inertia as a function of the square of rotational frequency for ^{157,163,167}Ho isotopes.

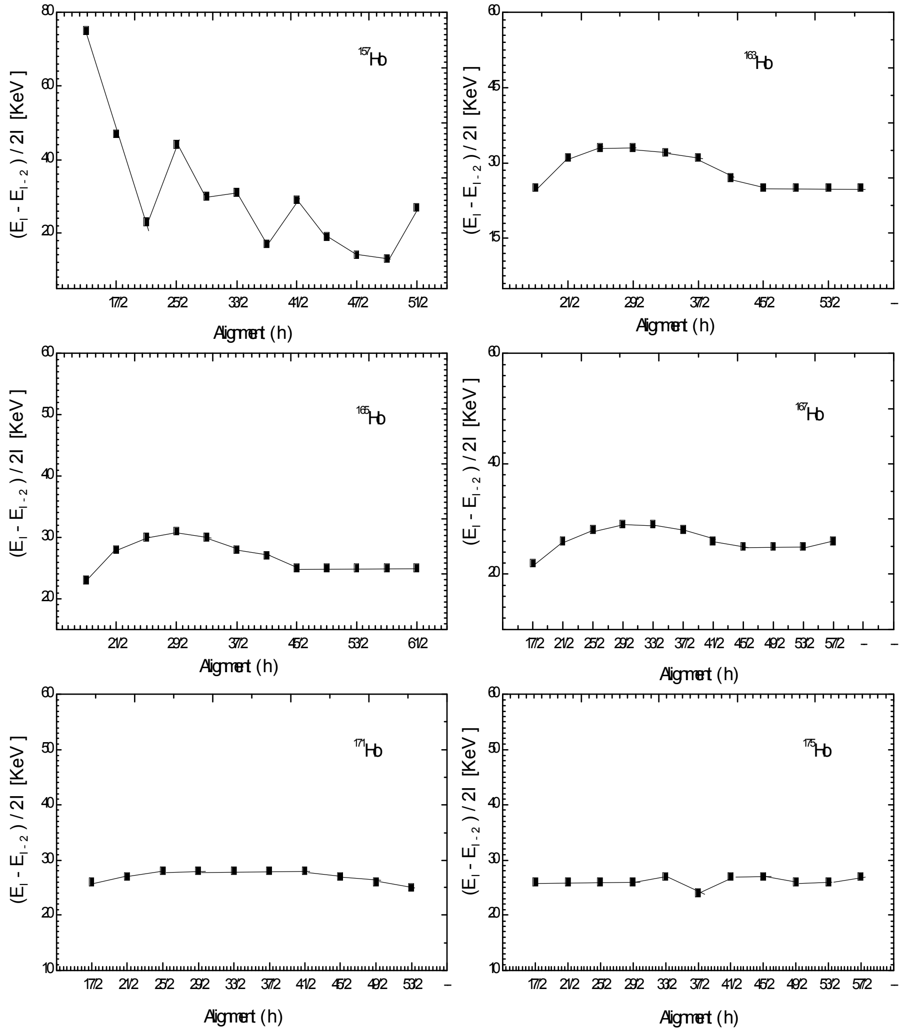

Fig. (2): Excitation energies of the yrast lines of Ho isotopes versus I (I+1). The average moment of inertia are indicated.

isotopes with lower N. The energy displacement relation is given by Ref. [18] in the form:

$$\delta E\ (I) = E\ (I)-[(I+1)\ E\ (I-1) + IE\ (I+1)]\ /\ (2I+1)$$

and plotted against the neutron number Fig. (5) for Ho isotopes. The δE (I) - value of the ^{153}Ho is very much larger than that of the other isotopes and for well-deformed nuclei it approaches zero [18]. Figure (5) clearly shows how stretching (positive δE (I) values) evidenced in the case of soft rotors goes over into the shrinkage (negative δE (I) values) for hard rotors exactly the same in Figure (4). The large δE (13/2, 17/2) value of the ^{153}Ho nucleus probably reflected their γ-softness, which is exactly the same we discussed before, since ^{153}Ho lies on the border between prolate and oblate Ho nuclei.

3- Comparison study of the results in studied isotopes:

I. Moment of inertia:

Nuclei in the mass region of A ≈ 150 are reported to belong to a so-called island of isomerism. A comparison of the low energetic level patterns of the Ho isotopes can be done by comparing the E_x versus I (I+1) plots [see Fig. (2)] or the effective moments of inertia deduced there form. If the $\Im_E$ values of even –odd nuclei are compared directly a significant is observed which is caused by the discrepancy is observed which is caused by the different spins of the ground states. To overcome this difficulty and to enable a realistic comparison, the ground-state spin was subtracted , so that all spins are normalized to ground-state spin. Table (2) shows the deduced moments of inertia whereby $\Im_R$ was calculated using the " reduced spin".

The sharp increase of the moment of inertia can be found to occur as a bend in the E_x versus I (I+1) plots of ^{153}Ho (Fig. 2) at a state with a reduced spin value of around $(29/2)\hbar$ which is an isomeric state in this nucleus.

Furthermore a significant discrepancy is observed when comparing the $\Im_E$ values directly for the N = 85 isotones ^{145} Nd , ^{147}Sm , ^{149} Gd , ^{150}Tb , ^{151}Dy and ^{153}Er Ref.(19 – 23), due the different ground-state spins. To eliminate this problem, reduce moments of inertia $\Im_R$ are introduce, corresponding to a reduce spin (see text). All $\Im_R$ are equal with a 90% confidence level.

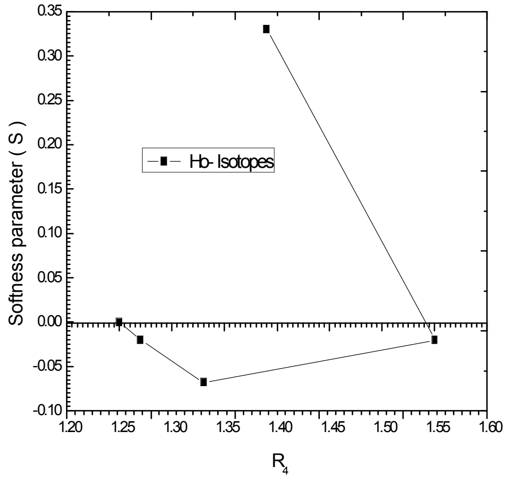

Fig. (3): The moment of inertia as a function of the angular momentum of the yrast bands in Ho isotopes

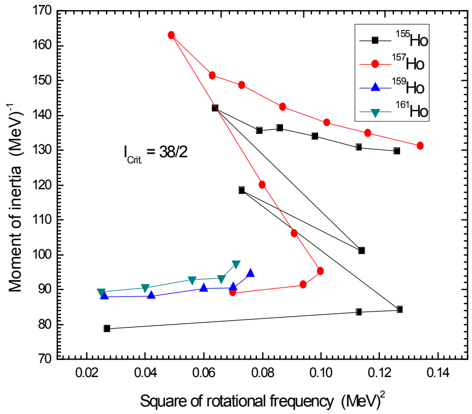

Fig. (4): Softness parameter as a function of the energy ratio R_4 for the nuclides in the isotopic sequence of holmium isotopes.

Table (2): The experimental moment of inertia for the N = 85 isotones.

Nuclide	**$2\Im_E / \hbar^2$ (MeV)$^{-1}$**	**$2\Im_R / \hbar^2$ (MeV)$^{-1}$**
$^{145}Nd_{85}$	98 ± 11	83 ± 10
$^{147}Sm_{85}$	122 ± 13	100 ± 8
$^{149}Gd_{85}$	126 ± 9	106 ± 5
$^{150}Tb_{85}$	164 ± 31	114± 11
$^{151}Dy_{85}$	141 ± 16	121± 12
$^{152}Ho_{85}$	149 ± 22	124± 16
$^{153}Er_{85}$	85 ± 22	65± 16

II. Gauge plots and deformation changes:

It is more elucidating to examine our results in a gauge –backbending plot as recently proposed by Bengtsson et al [24] who pointed out the analogy between backbending in ordinary space and in gauge space. An ordinary backbending plot, as in Fig. (7), based on transition energies reveals the angular momentum gain caused by the alignment of the spin vectors of a nucleon pair. Analogously, the gauge space plot will show a backbending behavior when a change in deformation occurs. In such a plot one examines the difference, together with the two-nucleon separation energy $S_{2n,}$ specifies the Fermi energy λ as afunction of spin which is calculated from the relation:
$\lambda (N,I) = (1/2) [E_x (N+1, I) - E_x (N-1, I) - S^{N+1}]$,

It has been suggested [24] that evidence for a reduction of pairing correlations can be obtained from the analysis of rotations in Gauge space by plotting the neutron number as a function of the Fermi energy Fig. (6), in analogy to the plots of I_x versus ħω which are frequently used for investigation rotational bands in ordinary space. Since the slope of the N (λ) curve reflects the level density at the Fermi surface, an up bend (for example) in this curve (i.e. an increase of slope in a small energy region) indicates a sudden increase in level density.

Table (3): The Fermi energy Vs. Neutron number for the Ho isotopes.

	λ (MeV)						
I / Isotopes	**^{161}Ho**	**^{163}Ho**	**^{165}Ho**	**^{167}Ho**	**^{169}Ho**	**^{171}Ho**	**^{173}Ho**
2	-9.357	-8.575	-8.800	-8.450	-8.125	-7.775	-7.400
4	-9.350	-8.550	-8.775	-8.425	-8.100	-7.750	-7.350
6	-9.325	-8.450	-8.750	-8.400	-8.075	-7.725	-
8	-9.100	-8.425	-8.700	-8.325	-8.050	-	-
10	-8.875	-8.400	-8.600	-8.250	-	-	-
12	-8.825	-8.375	-8.500	-8.150	-	-	-
14	-	-	-8.350	-	-	-	-
16	-	-	-8.225	-	-	-	-

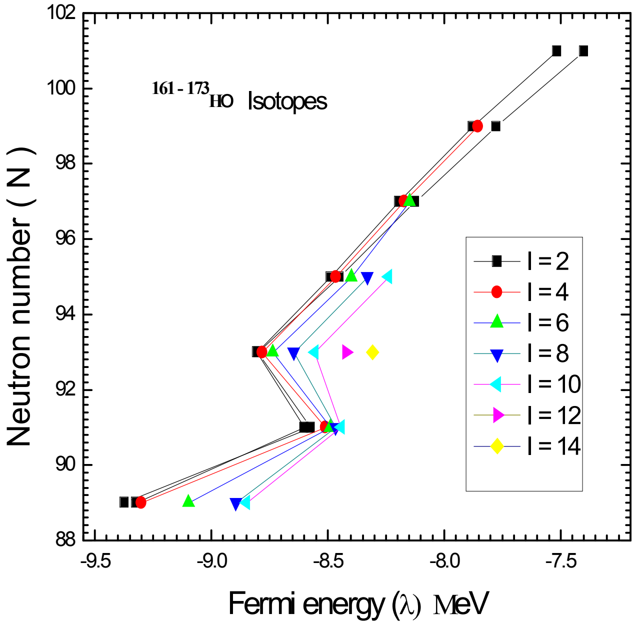

Fig. (6): Neutron numbers as a function of the Fermi energy for Ho isotopes.

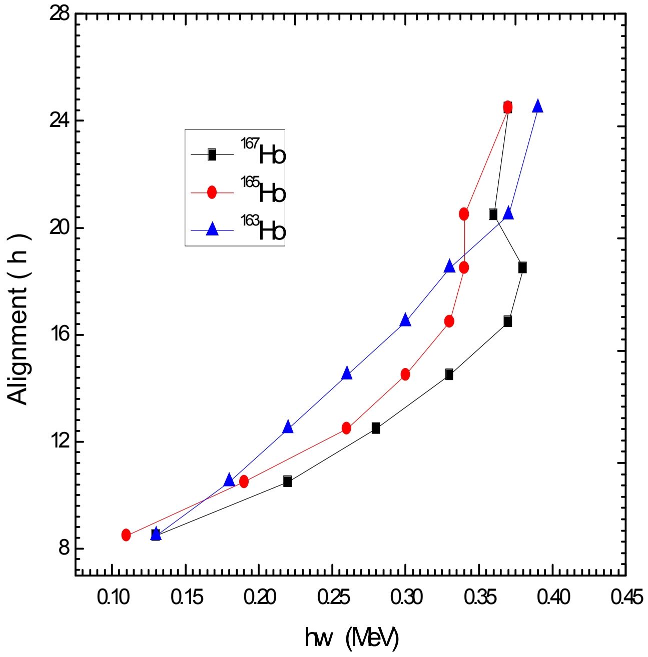

Fig. (7): Experimental I versus ħω plot for Ho isotopes.

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4- Conclusions

Analysis of the ground band energies results in the $^{153-163}$Ho isotopes has been done. They are located on the border of the strongly deformed region, a self-consistent calculations are required to test the suggestion of the γ-deformation changes. From the results of the alignment which has been computed is always decreasing when the masses increase (Fig. 1). In ^{153}Ho isotope a strong oscillation of the moment of inertia which can be traced to the admixture in the ground band its decoupling parameter, the oscillation and compression strong by increase from ^{163}Ho to ^{153}Ho which means that the coriolis mixing becomes much stronger with lower N. The larger value of δE (I) for ^{153}Ho nucleus gives rise to a clear backbending and probably reflected their γ-softness. Using the evaluation of the moment of inertia we obtain a very accurate description of back-bending in ^{159,161,163}Ho-isotopes.

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