Thermal-diffusion and diffusion-thermo effects on MHD three-dimensional axisymmetric flow with Hall andion-slip currents

T. Hayat[1] and F. A. Hendi [2]

[1]Department of Mathematics, Quaid-I-AzamUniversity, Islamabad, Pakistan
[2]Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, SaudiArabia
pensy_t@yahoo.com

Abstract: This paper investigates the effects of thermal-diffusion and diffusion-thermo on MHD three-dimensional axisymmetric flow of a viscous fluid between radially stretching sheets in the presence of Hall and ion-slip currents, viscous dissipation, Joule heating and first order chemical reaction. Governing partial differential equations are obtained through four laws of conservation, Maxwell's equations and generalized Ohm's law. Obtained partial differential equations are made dimensionless by using similarity transformation. The re- sulting problems are solved byhomotopy analysis method (HAM). Convergence of analytic solutions is ensured. Effects of emerging parameters on dimensionless velocities, temperature and concentration fields are seen through plots. Behavior of different physical parameters on skin friction coefficients, Nusselt number and Sherwood number is analyzed.

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Key words: Soret and Dufour effects, skin friction coefficient, Nusseltnumber andSher-wood numbers.

1. Introduction

Stretching flows are important type of flows induced by continuously moving boundary and has gained tremendous interest because of their applications in industry and engineering such as aerodynamic extrusion of plastic sheets, cooling of an infinite plate in a cooling bath, liquid film in condensation process, continuous filament extrusion from a dye, the fluid dynamics of a long thread traveling between a feed roll and wind-up roll etc. **Crane** [1] was first to consider the two-dimensional flow driven by stretching elastic flat sheet which stretches in its own plane with a velocity varying linearly with a distance from a fixed point.

The pioneering work of **Crane** is subsequently extended by many researchers. Axisymmetric flow driven by radially stretching sheet has been considered by many investigators [3−10].

These studies are restricted to axisymmetric flow induced by a stretching sheet.

Most of the researchers neglected the **Dufour and Soret** effects on heat and mass transfer under the assumption that they are of smaller magnitude than that described by **Fourier's and Fick**'slaw. Recent advancements show that **Dufour** effect are important in heat transport and **Soret** effects are influential in mass transfer phenomenon. Some recent contributions can be mentioned through refs.[11−14]. In spite of these studied, **Dufour and Soret** effects on MHD three-dimensional axisymmetric flow induced by radially stretching sheets have not yet bee studies. The present work is an attempt in this direction. In this paper we use homotopy analysis method (HAM)[15−29] to solve the nonlinear problem describing MHD axisymmetric flow of a viscous fluid between two radially stretching sheets in the presence of Hall and ion-slip currents, viscous dissipation and Joule heating. In section two, the mathematical formulation and definition of skin friction coefficients, Nusselt number and Sherwood number are presented. Section three extends the application of HAM to construct series solutions of the governing nonlinear problem. Section four analyzes the convergence of HAM solutions. In section five, the results and discussion are given. The conclusions are summarized in section six.

2. Formulation of the problem

Let us consider three dimensional axisymmetric flow of an electrically conducting fluid be- tween two infinite parallel radially stretching sheets placed at $z=\pm L$. The flow is induced by the sheets stretching radially with same rate. The flow is considered symmetric about $z=0$. A uniform magnetic field B_0 perpendicular to the planes of sheets is applied i.e. in the z-direction. It is assumed that the magnetic Reynolds number is very small and induced magnetic field is neglected. There is no external electric field. Both the sheets have constant temperature T_w and constant concentration C_w. Flow fields are defined by the following expressions

$$V=[u(r,z),v(r,z),w(r,z)],T=T(r,z),C=C(r,z). \qquad (1)$$

The relevant governing equations are

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0, \tag{2}$$

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left[2\frac{\partial^2 u}{\partial r^2}+\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 w}{\partial r\partial z}+\frac{2}{r}\frac{\partial u}{\partial r}-\frac{\partial u}{r^2}\right]$$

$$-\frac{\sigma B_0^2}{\rho[(1+\beta_e\beta_i)^2+\beta_e^2]}[(1+\beta_e\beta_i)u-\beta_e v], \tag{3}$$

$$u\frac{\partial v}{\partial r}+w\frac{\partial u}{\partial z}+\frac{vu}{r}=\nu\left[\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}\frac{\partial u}{\partial r}+\frac{\partial^2 v}{\partial z^2}\right]$$

$$-\frac{\sigma B_0^2}{\rho[(1+\beta_e\beta_i)^2+\beta_e^2]}[(1+\beta_e\beta_i)u-\beta_e v], \tag{4}$$

$$u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left[2\frac{\partial^2 w}{\partial z^2}+\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 u}{\partial r\partial z}+\frac{1}{r}\frac{\partial u}{\partial r}\right], \tag{5}$$

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\frac{K}{\rho c_p}\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right]+\frac{DK_T}{\rho C_s c_p}\left[\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}+\frac{\partial^2 C}{\partial z^2}\right]$$

$$+\frac{\sigma B_0^2}{\rho c_p[(1+\beta_e\beta_i)^2+\beta_e^2]}[u^2+v^2]$$

$$+\frac{\mu}{\rho c_p}\left[2\left(\frac{\partial u}{\partial r}\right)^2+\left(\frac{\partial v}{\partial r}\right)^2-\frac{2}{r}v\frac{\partial v}{\partial r}+2\frac{\partial u}{\partial z}\frac{\partial w}{\partial r}\right]$$

$$+\frac{\mu}{\rho c_p}\left[\left(\frac{\partial w}{\partial r}\right)^2+\frac{v^2}{r^2}+\frac{2}{r^2}u^2+\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2+2\left(\frac{\partial w}{\partial z}\right)^2\right], \tag{6}$$

$$u\frac{\partial C}{\partial r}+w\frac{\partial C}{\partial z}=D\left[\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}+\frac{\partial^2 C}{\partial z^2}\right]+\frac{DK_T}{T_m}\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right]-K_1 C, \tag{7}$$

where T denotes the temperaturefield, C the concentrationfield, σ the densityof fluid, $\sigma\left(=\frac{e^2 n_e \tau_e}{m_e}\right)$ theelectrical conductivity, $\beta_e\ (=w_e\tau_e)$ the Hall parameter, $\beta_i\ (=en_e B_0/(1+n_e/n_a)K_{ai})$ istheion-slipparameter, τ_e the electron collisiontime, m_e themassofelectron, w_e the cyclotron frequency, n_e isthe electron number density and n_a the neutral particle density, K_{ai} the friction coefficient between ionsand neutralparticles, K thethermal conductivity, K_T theconcentration susceptibility, T_m themeantemperature, K_1 thechemicalreaction constant, D thecoefficientofmassdiffusivity and c_p thespecificheat offluid. Theboundaryconditions thatcorrespond totheflowunder considerationare

$$\frac{\partial u}{\partial z}=0,\ v=0,\ w=0,\ \frac{\partial T}{\partial z}=0,\ \frac{\partial C}{\partial z}=0,\ \text{at}\ z=0,$$
$$u=ar,\ v=0,\ T=T_w,\ C=C_w\ \text{at}\ z=L,\ a>0. \tag{8}$$

Introducing similarity variables

$$u=raf'(\eta),\ v=arg,\ w=-2aLf(\eta),\ \theta=\frac{T}{T_w},\ \phi=\frac{C}{C_w},\ \eta=\frac{z}{L}. \tag{9}$$

In Eqs. (2) − (7), one obtains

$$f'''(\eta)-\frac{M^2 Re}{(1+\beta_e\beta_i)^2+\beta_e^2}[(1+\beta_e\beta_i)f''-\beta_e g']+2Re[ff'''(\eta)+gg']=0,$$

$$f(0)=0,\ f(1)=0,\ f'(1)=1,\ f''(0)=0, \tag{10}$$

$$g''(\eta)-\frac{M\,Re}{(1+\beta_e\beta_i)^2+\beta_e^2}[(1+\beta_e\beta_i)g+\beta_e f']-2Re[f'g-fg']=0,$$

$$g(0)=0,\ g(1)=0 \tag{11}$$

$$\theta''+2\,Re\,Pr\,f\theta'+\frac{Pr\,M\,Re\,Ec}{(1+\beta_e\beta_i)^2+\beta_e^2}[(f')^2+(g)^2]$$
$$+Pr\,Ec\left[\frac{12}{\delta}(f')^2+(f'')^2+(g')^2\right]+Du\,Pr\,\phi''=0,$$

$$\theta'(0)=0,\ \theta(1)=1 \tag{12}$$

$$\varphi''+2\,Re\,Sc\,f\varphi'+ScSr\theta''-Re\,Sc\gamma\varphi=0,$$

$$\phi'(0)=0,\ \phi(1)=1 \tag{13}$$

Where

$$Re=\frac{aL^2}{\nu},\ M=\frac{\sigma B_0^2}{\rho a},\ Pr=\frac{\mu c_p}{K},\ Du=\frac{DK_T C_w}{\nu C_s c_p T_w},$$
$$Sc=\frac{\nu}{D},\ Sr=\frac{DK_T T_w}{\nu T_m C_w},\ Ec=\frac{a^2 r^2}{c_p T_w},\ \gamma=\frac{K_1}{a},\ \delta=\frac{r^2}{L^2}$$

respectively denote the Reynolds number, Hartman number, Prandtl number, Dufournum- ber, Schmidt number, Soret number, local Eckert number, and first order chemical reaction parameter. Furthermore, $\gamma>0$ corresponds to generative chemical reaction and $\gamma<0$ indicates generative chemical reaction.

Definitionsofskinfrictioncoefficients C_{fr} and C_{gr} in radial and azimuthal directions,

Nusselt number Nu and Sherwood number Sh are givenbelow

$$C_{fr}=\frac{\tau_{rz}|_{z=0}}{\rho(ar)^2}=\frac{\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right)\Big|_{z=0}}{\rho(ar)^2}=\frac{1}{Re_r}f''(1),$$

$$C_{gr}=\frac{\tau_{\theta z}|_{z=0}}{\rho(ar)^2}=\frac{\mu\frac{\partial v}{\partial z}\Big|_{z=0}}{\rho(ar)^2}=\frac{1}{Re_r}g'(1)$$

$$Nu=-\frac{Lq_w}{KT_w}=-\frac{LK\frac{\partial T}{\partial z}\Big|_{z=0}}{KT_w}=-\theta'(1),$$

$$Sh=-\frac{LM_w}{DC_w}=-\frac{LD\frac{\partial C}{\partial z}\Big|_{z=0}}{DC_w}=-\phi'(1), \tag{14}$$

inwhich$Re_r(= arL/\nu)$isthelocalReynolds number.

3 Solutions by homotopy analysis method

$F(\eta)$, $\theta(\eta)$and$\phi(\eta)$ in the form of base functions

$$\{\eta^{2n}, n \geq 0\}, \tag{15}$$

can be written as

$$f(\eta) = \sum_{n=0}^{\infty} a_n \eta^{2n+1}, g(\eta) = \sum_{n=0}^{\infty} b_n \eta^{2n+1}$$

$$\theta(\eta) = \sum_{n=0}^{\infty} c_n \eta^{2n}, \phi(\eta) = \sum_{n=0}^{\infty} d_n \eta^{2n}, \tag{16}$$

in whicha_n, b_n, $c_n$and d_n are the coefficients to be determined. The initial guesses $f_0(\eta)$, $g_0(\eta)$, $\theta_0(\eta)$, $\phi_0(\eta)$and linear operators$\mathcal{L}_i(i = 1-3)$are chosen in the following forms

$$f_0(\eta) = \frac{1}{2}\eta(\eta^2 - 1), g_0(\eta) = 0,$$
$$\theta_0(\eta) = \eta^2, \phi_0(\eta) = \eta^2, \tag{17}$$

$$\mathcal{L}_1[f(\eta)] = \frac{d^4 f}{d\eta^4}, \mathcal{L}_2[g(\eta)] = \frac{d^2 g}{d\eta^2},$$
$$\mathcal{L}_3[\theta(\eta)] = \frac{d^2\theta}{d\eta^2}, \mathcal{L}_4[\phi(\eta)] = \frac{d^2\phi}{d\eta^2}, \tag{18}$$

whence

$$\mathcal{L}_1[C_1 + C_2\eta + C_3\eta^2 + C_4\eta^3] = 0, \mathcal{L}_2[C_5 + C_6\eta] = 0,$$
$$\mathcal{L}_3[C_7 + C_8\eta] = 0, \mathcal{L}_4[C_9 + C_{10}\eta] = 0 \tag{19}$$

andC_i $(i = 1-10)$ are the constants.

3.1 Zeroth-order formation problems

The zeroth order deformation problems are constructed as follows

$$(1-q)\mathcal{L}_1[\Phi(\eta;q) - f_0(\eta)] = q\hbar_1 N_1[\Phi(\eta;q), \Psi(\eta;q)],$$
$$(1-q)\mathcal{L}_2[\Phi(\eta;q) - g_0(\eta)] = q\hbar_2 N_2[\Phi(\eta;q), \Psi(\eta;q)],$$
$$(1-q)\mathcal{L}_3[\Theta(\eta;q) - \theta_0(\eta)] = q\hbar_3 N_3[\Theta(\eta;q), \Phi(\eta;q), \Psi(\eta;q), \Lambda(\eta;q)],$$
$$(1-q)\mathcal{L}_4[\Lambda(\eta;q) - \phi_0(\eta)] = q\hbar_4 N_4[\Lambda(\eta;q), \Theta(\eta;q), \Phi(\eta;q), \Psi(\eta;q), (\eta;q)], \tag{20}$$

$$\Phi(0;q) = 0, \Phi(1;q) = 0, \left.\frac{\partial\Phi(\eta;q)}{\partial\eta}\right|_{\eta=1} = 1, \left.\frac{\partial^2\Phi(\eta;q)}{\partial\eta^2}\right|_{\eta=0} = 0,$$

$$\Psi(0;q) = 0, \Psi(1;q) = 0$$

$$\Theta(1;q) = 1, \left.\frac{\partial\Theta(\eta;q)}{\partial\eta}\right|_{\eta=0} = 0,$$

$$\Lambda(1;q) - 1, \left.\frac{\partial\Lambda(\eta;q)}{\partial\eta}\right|_{\eta=0} - 0, \tag{21}$$

In above expressions $q \in [0,1]$ and $\hbar_i \neq 0 (i = 1-3)$ are respectively the embeddingandauxiliary parameters and

$\Phi(\eta;0) = f_0(\eta)$, $\Psi(\eta;0) = g_0(\eta)$, $\Theta(\eta;0) = \theta_0(\eta)$, $\Lambda(\eta;0) = \phi_0(\eta)$

and

$\Phi(\eta;1) = f(\eta)$, $\Psi(\eta;1) = g(\eta)$, $\Theta(\eta;1) = \theta(\eta)$, $\Lambda(\eta;1) = \phi(\eta)$.

When$q$ varies from 0 to 1, then $\Phi(\eta;q)$ variesfrom the initial guess $f_0(\eta)$ to $f(\eta)$, $g(\eta;q)$ varies from the initial guess $g_0(\eta)$ to $g(\eta)$, $\Theta(\eta;q)$ varies from the initial guess $\theta_0(\eta)$ to $\theta(\eta)$ and $\Lambda(\eta;q)$ varies from the initial guess $\phi_0(\eta)$ to $\phi(\eta)$. The non linear operatorsN_{0i} $(i = 1-4)$ are given below

$$N_1[\Phi(\eta;q), \Psi(\eta;q)] = \frac{\partial^4\Phi(\eta;q)}{\partial\eta^4} - \frac{M\,Re}{(1+\beta_e\beta_i)^2 + \beta_e^2}\left[(1+\beta_e\beta_i)\frac{\partial^2\Phi(\eta;q)}{\partial\eta^2} - \beta_e\frac{\partial\Psi(\eta;q)}{\partial\eta}\right] + 2Re\left[\Phi(\eta;q)\frac{\partial^3\Phi(\eta;q)}{\partial\eta^3} + \Psi(\eta;q)\frac{\partial\Psi(\eta;q)}{\partial\eta}\right], \tag{22}$$

$$N_2[\Phi(\eta;q), \Psi(\eta;q)] = \frac{\partial^2\Psi(\eta;q)}{\partial\eta^2} - \frac{M\,Re}{(1+\beta_e\beta_i)^2 + \beta_e^2}\left[\beta_e\frac{\partial\Phi(\eta;q)}{\partial\eta} + (1+\beta_e\beta_i)\Psi(\eta;q)\right] - 2Re\left[\Psi(\eta;q)\frac{\partial\Phi(\eta;q)}{\partial\eta} - \Phi(\eta;q)\frac{\partial\Psi(\eta;q)}{\partial\eta}\right], \tag{23}$$

$$N_3[\Theta(\eta;q), \Phi(\eta;q), \Psi(\eta;q)] = \frac{\partial^2\Theta}{\partial\eta^2} + 2\,Re\,Pr\,\Phi(\eta;q)\frac{\partial\Theta(\eta;q)}{\partial\eta} + Pr\,Ec\left[\frac{12}{\delta}\frac{\partial\Phi(\eta;q)}{\partial\eta} + \left(\frac{\partial^2\Phi(\eta;q)}{\partial\eta^2}\right)^2 + \left(\frac{\partial\Psi(\eta;q)}{\partial\eta}\right)^2\right] + \frac{Pr\,M\,Re\,Ec}{(1+\beta_e\beta_i)^2 + \beta_e^2}\left[\left(\frac{\partial\Phi(\eta;q)}{\partial\eta}\right)^2 + (\Psi(\eta;q))^2\right] + Du\,Pr\frac{\partial^2\Lambda}{\partial\eta^2}, \tag{24}$$

$$N_4[\Lambda(\eta;q), \Theta(\eta;q), \Phi(\eta;q), \Psi(\eta;q)] = \frac{\partial^2\Lambda}{\partial\eta^2} + 2\,Re\,Sc\frac{\partial\Lambda(\eta;q)}{\partial\eta}\Phi(\eta;q) + Sc\,Sr\frac{\partial^2\Theta}{\partial\eta^2} - Re\,Sc\gamma\Lambda(\eta) \tag{25}$$

Taylor's power series gives

$$\Phi(\eta;q) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta)q^m,$$

$$\Psi(\eta;q) = g_0(\eta) + \sum_{m=1}^{\infty} g_m(\eta)q^m,$$

$$\Theta(\eta;q) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta)q^m,$$

$$\Lambda(\eta;q) = \phi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta)q^m, \tag{26}$$

In which

$$f_m(\eta) = \frac{1}{m!}\frac{\partial^m \Phi(\eta;q)}{\partial \eta^m}\bigg|_{q=0}, \quad g_m(\eta) = \frac{1}{m!}\frac{\partial^m \Psi(\eta;q)}{\partial \eta^m}\bigg|_{q=0},$$
$$\theta_m(\eta) = \frac{1}{m!}\frac{\partial^m \Theta(\eta;q)}{\partial \eta^m}\bigg|_{q=0}, \quad \phi_m(\eta) = \frac{1}{m!}\frac{\partial^m \Lambda(\eta;q)}{\partial \eta^m}\bigg|_{q=0}. \tag{27}$$

3.2 Higher order deformation problems

Writing

$$f_m(\eta) = \{f_0(\eta), f_1(\eta), f_2(\eta), f_3(\eta) \ldots\ldots f_m(\eta)\},$$
$$g_m(\eta) = \{g_0(\eta), g_1(\eta), g_2(\eta), g_3(\eta) \ldots\ldots g_m(\eta)\},$$
$$\theta_m(\eta) = \{\theta_0(\eta), \theta_1(\eta), \theta_2(\eta), \theta_3(\eta) \ldots\ldots \theta_m(\eta)\},$$
$$\phi_m(\eta) = \{\phi_0(\eta), \phi_1(\eta), \phi_2(\eta), \phi_3(\eta) \ldots\ldots \phi_m(\eta)\},$$

the so called mth order deformation problems are

$$\mathcal{L}_1[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar_1 \mathcal{R}_{1m}(f_{m-1}(\eta)),$$
$$f_m(0) = 0, f_m(1) = 0, f'_m(1) = 0, f''_m(0) = 0, \tag{29}$$
$$\mathcal{L}_2[g_m(\eta) - \chi_m g_{m-1}(\eta)] = \hbar_2 \mathcal{R}_{2m}(g_{m-1}(\eta)),$$
$$g_m(0) = 0, g_m(1) = 0, \tag{30}$$

$$\mathcal{L}_3[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = \hbar_3 \mathcal{R}_{3m}(\theta_{m-1}(\eta)),$$
$$\theta'_m(0) = 0, \vartheta_m(1) = 0, \tag{31}$$
$$\mathcal{L}_4[\phi_m(\eta) - \chi_m \phi_{m-1}(\eta)] = \hbar_4 \mathcal{R}_{4m}(\phi_{m-1}(\eta)),$$
$$\phi'_m(0) = 0, \phi_m(1) = 0, \tag{32}$$
$$\chi_m = \begin{cases} 0, & m \le 1, \\ 1, & m > 1, \end{cases}$$

$$\mathcal{R}_{1m}(f_{m-1}(\eta), g_{m-1}(\eta)) = f''''_{m-1}(\eta) - \frac{M\,Re}{(1+\beta_e\beta_i)^2+\beta_e^2}[(1+\beta_e\beta_i)f''_{m-1}(\eta) - \beta_e g'_{m-1}(\eta)] + 2Re\sum_{n=0}^{m-1}[f_n(\eta)f'''_{m-1-n}(\eta) + g_n(\eta)g'_{m-1-n}(\eta)], \tag{33}$$

$$\mathcal{R}_{2m}(f_{m-1}(\eta), g_{m-1}(\eta)) = f''''_{m-1}(\eta) - \frac{M\,Re}{(1+\beta_e\beta_i)^2+\beta_e^2}[\beta_e f'_{m-1}(\eta) + (1+\beta_e\beta_i)g_{m-1}(\eta)] - 2Re\sum_{n=0}^{m-1}[g_n(\eta)f'_{m-1-n}(\eta) + f_n(\eta)g'_{m-1-n}(\eta)], \tag{34}$$

$$\mathcal{R}_{3m}(\theta_{m-1}(\eta), f_{m-1}(\eta), g_{m-1}(\eta), \phi_{m-1}(\eta)) = \theta''_{m-1}(\eta) + 2Re\,Pr\sum_{n=0}^{m-1} f_n(\eta)\theta'_{m-1-n}(\eta) + Pr\,Ec\sum_{n=0}^{m-1}[f''_n(\eta)f''_{m-1-n}(\eta) + g'_n(\eta)g'_{m-1-n}(\eta) + \frac{12}{\delta}f'_n(\eta)f'_{m-1-n}(\eta)] + \frac{M\,Re}{(1+\beta_e\beta_i)^2+\beta_e^2}\sum_{n=0}^{m-1}[f'_n(\eta)f'_{m-1-n}(\eta) + g_n(\eta)g_{m-1-n}(\eta)] + Du\,Pr\,\phi''_{m-1}(\eta), \tag{35}$$

$$\mathcal{R}_{4m}(\theta_{m-1}(\eta), f_{m-1}(\eta), g_{m-1}(\eta), \phi_{m-1}(\eta)) = \phi''_{m-1}(\eta) + Sc\,Sr\,\theta''_{m-1}(\eta) - Re\,Sc\gamma\,\phi_{m-1}(\eta) + 2Re\,Sc\sum_{n=0}^{m-1} f_n(\eta)\phi'_{m-1-n}(\eta). \tag{36}$$

It is found that problems (26) − (28) have the following general solutions

$$f(\eta) = f^*(\eta) + C_1^m + C_2^m\eta + C_3^m\eta^2 + C_4^m\eta^3,$$
$$g(\eta) = g^*(\eta) + C_5^m + C_6^m\eta,$$
$$\theta(\eta) = \theta^*(\eta) + C_7^m + C_8^m\eta,$$
$$\phi(\eta) = \phi^*(\eta) + C_9^m + C_{10}^m\eta, \tag{37}$$

where $f^*(\eta), g^*(\eta), \theta^*(\eta)$ and $\phi^*(\eta)$ are the corresponding particular solutions.

4 Convergence of solutions

(28)

The convergence and rate of approximation of series solutions (32)- (35) strongly depend upon the values of auxiliary parameters. For this purpose the $\hbar_i$ − curves are plotted through figures 1 – 3.These figures show that the admissible ranges for $\hbar_i$ are $-1.2 \le \hbar_1 \le -0.5$, $-1.1 \le \hbar_2 \le -0.7$ and $-1 \le \hbar_3 \le -0.3$. However the whole forthcoming calculations are performed when $\hbar_1 = \hbar_2 = \hbar_3 = -1$. In order to ensure the convergence of solutions, Table 1 is constructed. From this table it is evident that the convergence is achieved at 20^{th} order of approximations up to 10^{th} decimal places.

5 Results and Discussion

Figs. 5 − 7 describe the behavior of Hartman number on magnitude of radial velocity $f'(\eta)$, axial velocity $f(\eta)$ and azimuthal velocity $g(\eta)$. These figures show that $f'(\eta)$ and $f(\eta)$ decrease with an increase in M while $g(\eta)$ increases by increasing M. It is noted from Figs. 8 – 10 that an increase in Reynolds number leads to decrease in the magnitude of radial velocity $f'(\eta)$ and axial velocity $f(\eta)$. However azimuthal velocity $g(\eta)$ increases when Reynolds number Re isincreased. Figs. 11and 12 illustrate that the magnitude of radial velocity $f'(\eta)$ and axial velocity $f(\eta)$ increase by increasing Hall parameter β_e whereas azimuthal velocity $g(\eta)$ decreases when Hall parameter is increased (Fig-13). Figs. 14 − 16 demonstrate the influence of ion-slip parameter β_i on magnitude of radial velocity $f'(\eta)$, axial velocity $f(\eta)$ and azimuthal velocity $g(\eta)$. These figures depict that the magnitude of radial velocity $f'(\eta)$ and axial velocity $f(\eta)$ increase whereas azimuthalvelocity $g(\eta)$ decreases when ion-slipparameter β_i is increased. ComparisonofFigs.5 − 16indicatesthateffectsof M and Re are opposite tothose of β_e and β_i onthemagnitudeofradial velocity $f'(\eta)$, axial velocity $f(\eta)$ and azimuthalvelocity $g(\eta)$. From Figs. 17 and 18, it is evident that the reisoppositeeffect of Pr ondimensionlesstemperature $\theta(\eta)$ and dimensionless concentration $\phi(\eta)$. The dimensionlesstemperature $\theta(\eta)$ increases but dimensionlessconcentration $\phi(\eta)$ decreasesbyincreasing Schamidt Sc (Figs. 19 − 20).Figs.21 − 22reveal thatthe dimensionless temperature $\theta(\eta)$ and $\phi(\eta)$ showopposite behaviorwithanincrease in Soretnumber Sr. Thedimensionless temperature $\theta(\eta)$ isanincreasing functionof Du, Re and Ec and $\phi(\eta)$ decreases when Du, Re and Ec are increased asshowninFigs. 23 − 30. Table 2showsthevariationofskinfriction coefficients $Re_r C_{fr}$ .and $Re_r C_{gr}$. inradial andazimuthaldirection. From this table, itisobvious thatskinfrictioncoefficients $Re_r C_{fr}$ .and $Re_r C_{gr}$ isanincreasingfunctionof Re ,and M whereasthesedecreaseby decreasing β_e and β_i .Table 3ispreparedfortheinfluence ofphysical parametersonNusseltnumber Nu andSherwoodnumber Sh. ThisTable showsthat the Nusseltnumber Nu andSherwoodnumber Sh areincreasing functions of Re, M, Sc, Sr, Pr, Ec and γ and decreasingfunctions of β_e and β_i.

Table 1. Convergence of HAM solutions for different order of approximations when $Re = M = \beta_e = \beta_i = 2, Sc = Sr = Du = Pr = Ec = \gamma = 0.5, \delta = 12.$

Order of approximations	$f''(1)$	$g'(1)$	$\theta'(1)$	$\phi'(1)$
1	3.504433498	0.06896551724	-1.747126437	0.06666666667
5	3.502994846	0.05721071880	-1.846520547	0.9214152609
8	3.502976934	0.05720645042	-1.849694145	0.9214767184
12	3.502976863	0.05720646969	-1.849691882	0.9215289842
15	3.502976863	0.05720646967	-1.849692097	0.9215288080
20	3.502976863	0.05720646967	-1.849692094	0.9215288131
25	3.502976863	0.05720646967	-1.849692094	0.9215288131

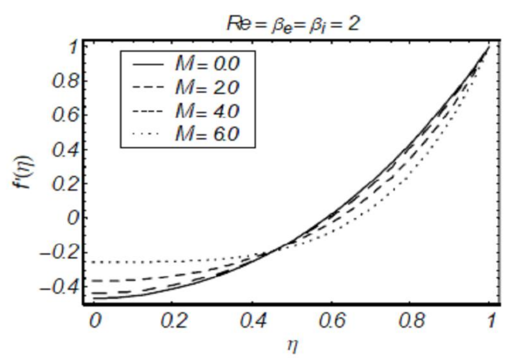

Fig. 5. Influence of M on $f'(\eta)$.

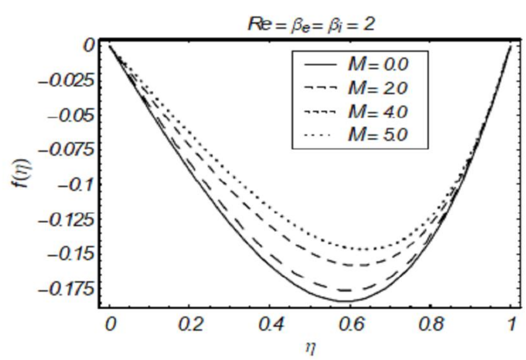

Fig. 6. Influence of M on $f(\eta)$.

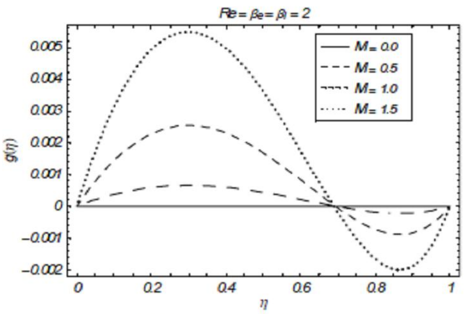

Fig. 7. Influence of M on $g(\eta)$.

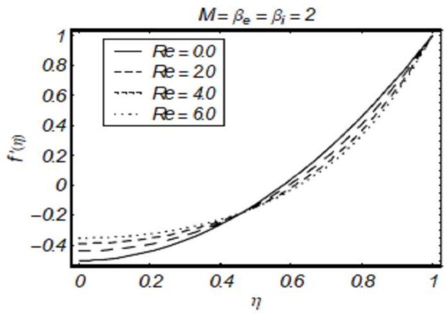

Fig. 8. Influence of Re on $f'(\eta)$.

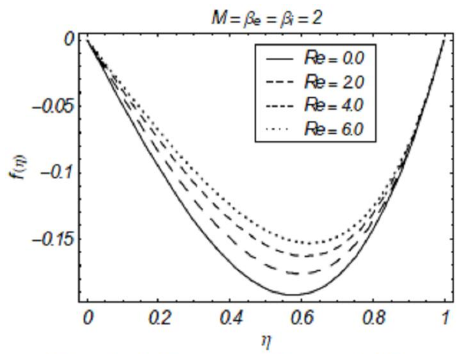

Fig. 9. Influence of Re on $f(\eta)$.

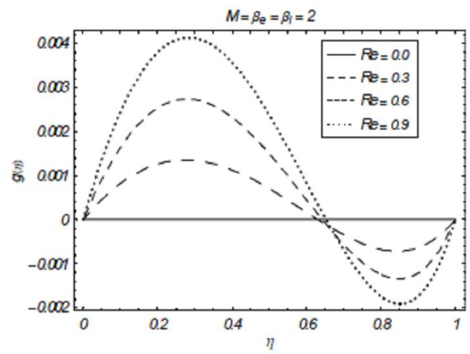

Fig. 10. Influence of Re on $g(\eta)$.

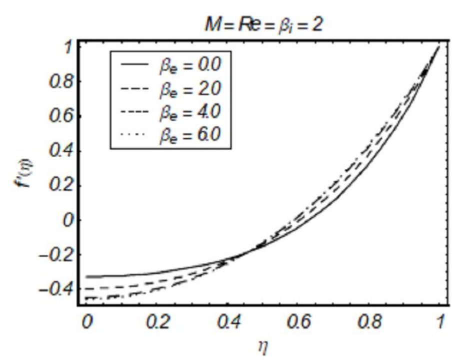

Fig. 11. Influence of β_e on $f'(\eta)$.

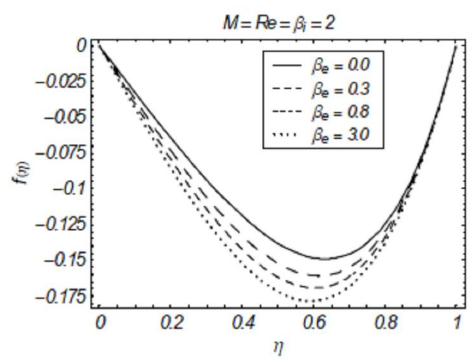

Fig. 12. Influence of β_e on $f(\eta)$.

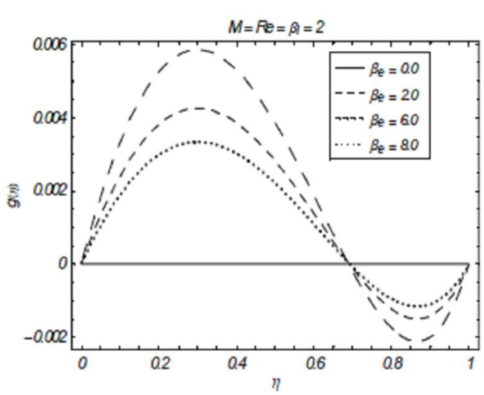

Fig. 13. Influence of β_e on $g(\eta)$.

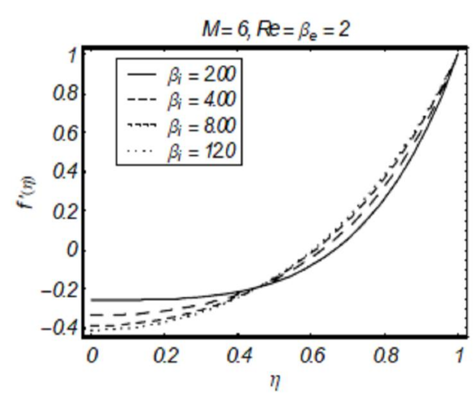

Fig. 14. Influence of β_i on $f'(\eta)$.

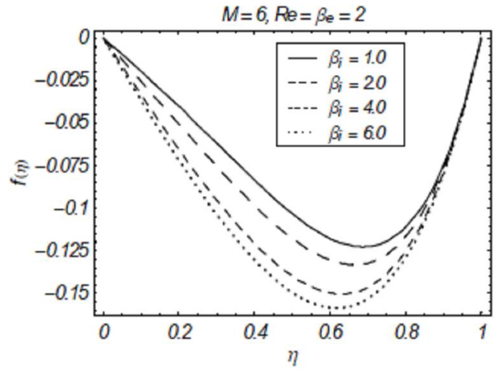

Fig. 15. Influence of β_e on $f(\eta)$.

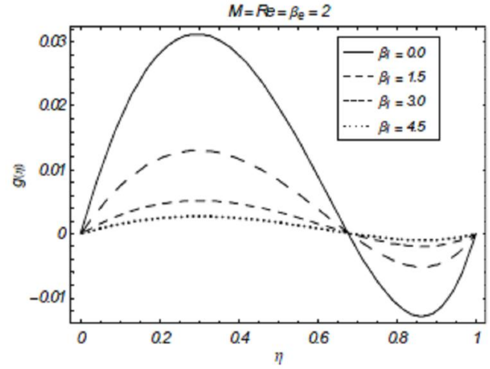

Fig. 16. Influence of β_e on (η).

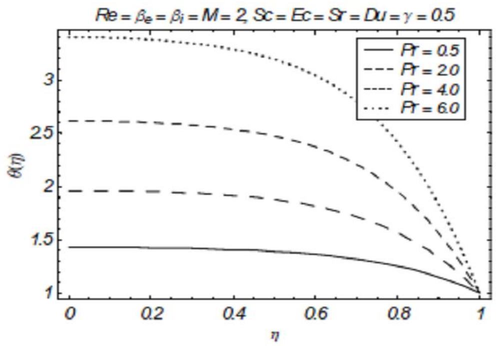

Fig. 17. Influence of Pr on $\theta(\eta)$.

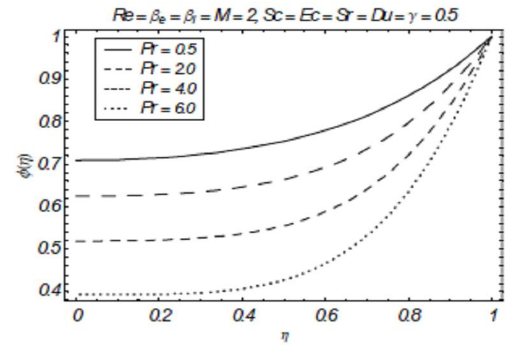

Fig. 18. Influence of Pr on $\phi(\eta)$.

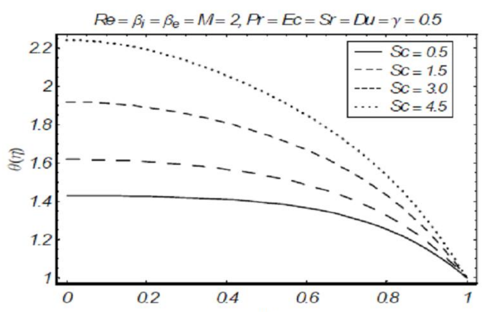

Fig. 19. Influence of Sc on $\theta(\eta)$.

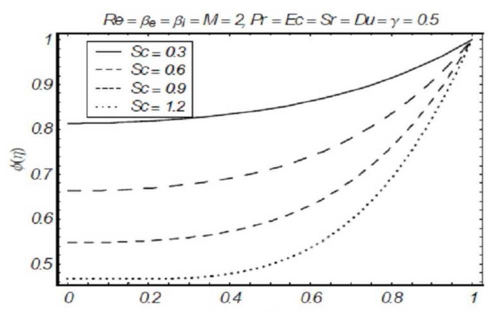

Fig. 20. Influence of Sc on $\phi(\eta)$.

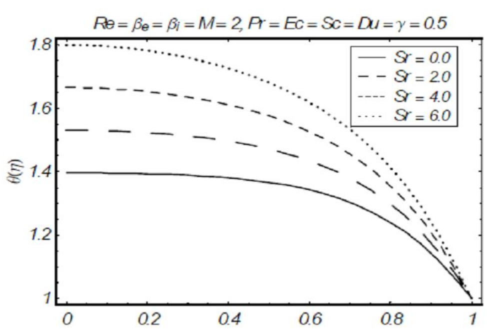

Fig. 21. Influence of Sr on $\theta(\eta)$.

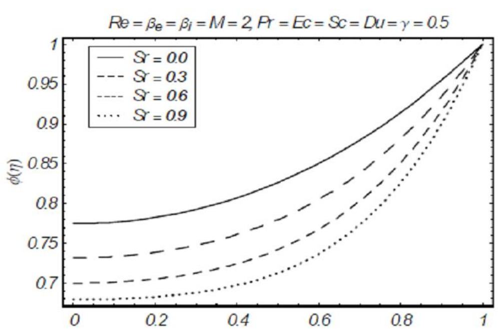

Fig. 22. Influence of Sr on $\phi(\eta)$.

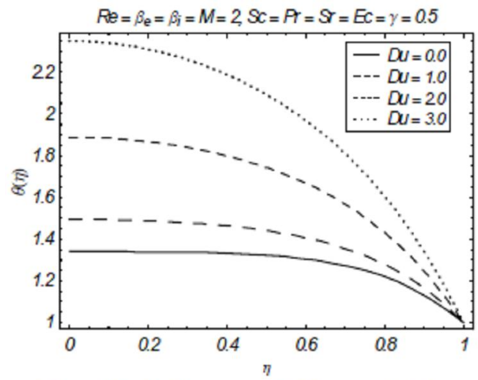

Fig. 23. Influence of Du on $\theta(\eta)$.

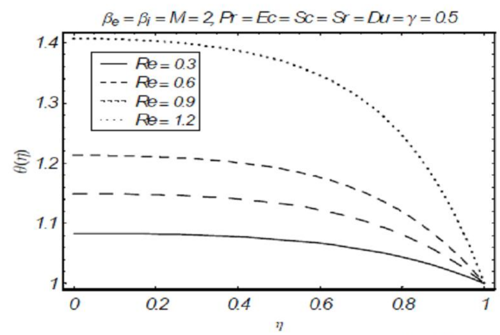

Fig. 25. Influence of Re on $\theta(\eta)$.

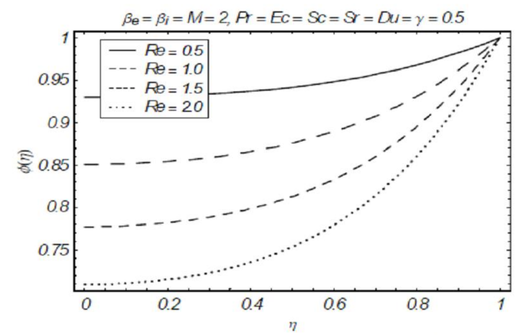

Fig. 26. Influence of Re on $\phi(\eta)$.

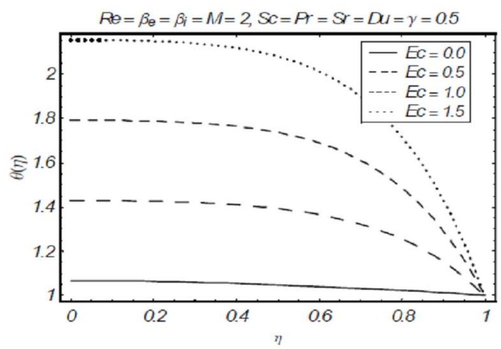

Fig. 27. Influence of Ec on $\theta(\eta)$.

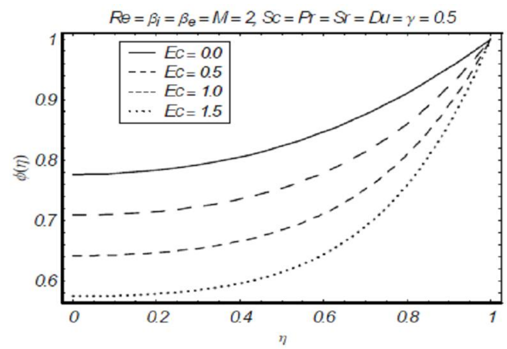

Fig. 28. Influence of Ec on $\phi(\eta)$.

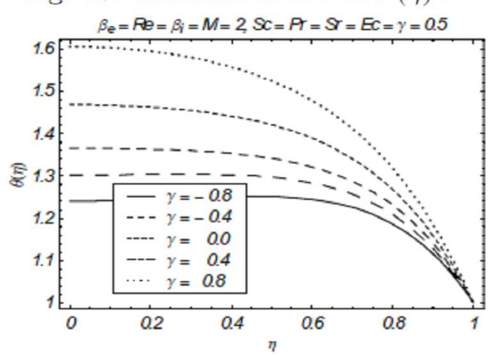

Fig. 29. Influence of γ on $\theta(\eta)$.

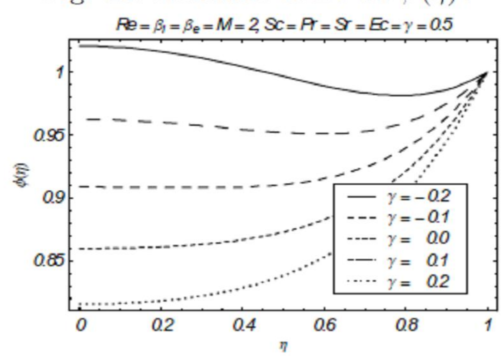

Fig. 30. Influence of γ on$\phi(\eta)$.

Table 2.Variationofskinfriction coefficientC_{fr} fordifferentvaluesofphysical parame- ters.

M	Re	β_i	β_e	$\mathrm{Re}_r C_{fr}$	$\mathrm{Re}_r C_{gr}$
0.0	2.0	2.0	5.0	3.239833418	0.00000000000
1.0				3.307407692	0.01447375808
2.0				3.502976863	0.05720646967
3.0				3.807821207	0.12548753550
2.0	0.5	2.0	2.0	3.126210064	0.01639036384
	1.5			3.378034991	0.04478752612
	2.5			3.626922424	0.06866388835
	3.5			3.871079696	0.08922423007
2.0	2.0	0.0	2.0	3.572561447	0.3295342724
		1.0		3.590800897	0.1267575429
		2.0		3.502976863	0.05720646967
		3.0		3.442796692	0.03143095076
2.0	2.0	2.0	0.0	4.545163164	0.00000000000
			1.0	3.685601974	0.08176778980
			2.0	3.502976863	0.05720646966
			3.0	3.426056360	0.04312757989

Table 3.VariationofNusseltnumber Nu andSherwoodnumber Sh fordifferentvalues Ofemergingparameters.

M	Re	β_i	β_e	Du	Sr	Sc	Pr	Ec	γ	$-Nu$	Sh
0.0	2.0	2.0	2.0	0.5	0.5	0.5	0.5	0.5	0.5	1.833614372	0.9194098210
1.0										1.836739998	0.9196930955
2.0										1.849692094	0.9215288131
3.0										1.880238090	0.9270849609
2.0	0.5	2.0	2.0	0.5	0.5	0.5	0.5	0.5	0.5	0.4435706585	0.2335825756
	1.5									1.368747745	0.6947597913
	2.5									2.342580862	1.145526913
	3.5									3.363687149	1.585348291
2.0	2.0	0.0		0.5	0.5	0.5	0.5	0.5	0.5	1.934273446	0.9415203960
		1.0								1.870809809	0.9260833275
		2.0								1.849692150	0.9215287354
		3.0								1.841874791	0.9200254506
2.0	2.0	2.0	0.0	0.5	0.5	0.5	0.5	0.5	0.5	2.228712593	1.0074908610
			1.0							1.881453411	0.9280871833
			2.0							1.849691882	0.9215289842
			3.0							1.841130301	0.9199550261
2.0	2.0	2.0	2.0	0.0	0.5	0.5	0.5	0.5	0.5	1.599872098	0.8618143193
				1.0						2.136225282	0.9900071202
				2.0						2.856661225	1.162131882
				3.0						3.872516224	1.404726281
2.0	2.0	2.0	2.0	0.5	0.0	0.5	0.5	0.5	0.5	1.734395272	0.4787117831
					1.0					1.981219342	1.427285374
					2.0					2.308670501	2.689229983
					3.0					2.762923533	4.446538062

(Continuation of Table 3)

M	Re	β_i	β_e	Du	Sr	Sc	Pr	Ec	γ	$-Nu$	Sh
2.0	2.0	2.0	2.0	0.5	0.5	0.5	0.5	0.5	0.5	1.849692094	0.9215288131
						1.0				2.105039223	1.882504518
						1.5				2.364616134	2.879069271
						2.0				2.628449674	3.911397123
2.0	2.0	2.0	2.0	0.5	0.5	0.5	0.12	0.5	0.5	0.4066730295	0.5761056816
							0.42			1.524377928	0.8436780815
							0.72			2.812261733	1.151812812
							1.02			4.310743857	1.510122706
2.0	2.0	2.0	2.0	0.5	0.5	0.5	0.5	0.0	0.5	0.1440515805	0.5131098729
								0.5		1.849692094	0.9215288131
								1.0		3.555332607	1.329947753
								1.5		5.260973120	1.738366693
2.0	2.0	2.0	2.0	0.5	0.5	0.5	0.5	0.5	-1.0	1.131386786	1.602056379
									-0.5	1.512496082	0.2729019290
									0.0	1.717201144	0.4487913869
									0.5	1.849692094	0.9215288131
									1.0	1.945443553	1.267388131

6. Final Remark

In this investigation, we have discussed the thermal-diffusion (So ret effect) and diffusion-thermo (Dufour effect) on heat and mass transfer of the steady flow ofaviscous fluid between radiallystretchingsheets inthepresence ofHallandion-slip currents, viscous dissipation and Joule heating. The main point soft hepresented analysis aregivenbelow.

•Variationof$Re$and$M$on$f'(\eta)$, $f(\eta)$and $g(\eta)$isopposite tothatof$\beta_e$andβ_i.

•There areopposite effects of Re, Pr, Sc, Du, Sr, Ecandγon$\theta(\eta)$and$\varphi(\eta)$

•The reisnosignificanteffects$\beta_e$and$\beta_i$on$\theta(\eta)$and$\varphi(\eta)$.

•Qualitatively,the effectsof Re and M areopposite β_e and β_i onthe skinfriction coefficients C_{fr}andC_{gr}.

•Variationof$Re, M, Du, Sc, Sr, Pr, Ec, \gamma\, \beta_eand\beta_i$ ontheNusseltnumber $Nu$and Sherwoodnumber$Sh$are qualitatively similar.

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