

**GA-Based Fuzzy State Feedback Controller applied to a Nonlinear Power System**Alireza Alfi<sup>1</sup>, \*S.Ehsan Razavi<sup>2</sup>, Amir Hassannia<sup>1</sup><sup>1</sup>Shahrood University of Technology, Faculty of Electrical and Robotic Engineering, Shahrood 36199-95<sup>2</sup>Department of Electrical Engineering, East Tehran Branch, Islamic Azad University, Tehran, Iran\*Email: [e\\_razavi\\_control@yahoo.com](mailto:e_razavi_control@yahoo.com)

**Abstract:** In this paper, a pole placement problem in a nonlinear system is investigated. Changing the operating point of nonlinear system is effective on its linear model and leads to difficult performance of state feedback that is designed only for one operating point. In this paper, an optimal fuzzy state feedback controller is provided for a special nonlinear power system which aims to improve the performance of state feedback. In the core of this controller, to overcome the key drawback of fuzzy logic controller, i.e., the lack of systematic methods to define fuzzy rules and fuzzy membership functions, the fuzzy state feedback controller are optimised by GA. Simulation results illustrates the effectiveness of the proposed optimal fuzzy state feedback controller.

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**Keywords:** State Feedback, Pole Placement, Fuzzy Control, Genetic Algorithm, Nonlinear Power System

**1. Introduction**

Stability is one of the most important issues in power systems. Different input disturbances to a power system can cause its instability. So, different methods have been proposed to increase the stability margin of these systems. However, many nonlinear systems can be approximated by linear systems using linearization methods. State feedback controllers are appropriate tools for pole placement of a linear system by increase of its stability margin. Though, after linearization in operating point, the state feedback can be also designed for nonlinear systems, but in a nonlinear system, change of operating point is extremely effective on state feedback performance. Since most systems including power systems are generally nonlinear, application of this control method is faced with particular problems. Different solutions to this problem have been opted that generally they are based on a state feedback design on a linear model from nonlinear system with variable parameters [1].

Although various controllers design for these systems has been discussed in several papers [2-4], the complexity of controller and lack of enough flexibility to change of operating point are difficulties in this problem. In recent years, researchers have extensively used the fuzzy logic for modeling, identification, and control of highly nonlinear dynamic systems. Based on this, the goal of this paper is to design of a fuzzy state feedback controller for stabilization of a nonlinear system. In the structure of the proposed fuzzy controller, different controllers' combination is used which are designed for different operating points whereas a fuzzy supervisor has the task of selecting the proper

combination of these controllers for an operating point of the system.

Although there are a number of distinguished advantages of the fuzzy logic controllers over the classical controllers such as they are not so sensitive to the variation of system structure, parameters and operation points as well as can be easily implemented in a large scale nonlinear system. But, one major drawback of them is the lack of systematic methods to define fuzzy rules and fuzzy membership functions. Most fuzzy rules are also based on human knowledge and differ among persons despite the same system performance. On the other hand, it is difficult to assume that the given expert's knowledge captured in the form of the fuzzy controller leads to optimal control. Consequently, the effective approaches for tuning the membership function and control rules without a trial and error method are significantly required. In the problem in hand, optimal design of fuzzy supervisor is relatively difficult due to difficulty of choosing the proper combination of controllers. Because of this, in this paper, the idea of employing Genetic Algorithm (GA) algorithm to solve the combinatorial optimization problems is proposed.

The rest of this paper is organized as follows: At first, the complete modelling of a synchronous generator parallel with turbine, governor, AVR and excitation system is expressed. The model is a nonlinear system with complete 14-order. This model is linearized for a system operating point. It is shown that by designing of a proper state feedback a significant increase in stability margin of this system can be established. But minor changes in the system operating point greatly affect the performance of the

state feedback and may even lead to system instability. To overcome this problem, a new combined structure is provided from state feedback based on fuzzy supervision. First, a common structure for fuzzy supervisor is considered and its performance is evaluated. Then, the membership functions of fuzzy controller will be optimized by genetic algorithm for better performance in a wider range of operating point changes. Finally, fuzzy state feedback controller performance is evaluated by simulation in MATLAB environment.

**2. Power System Modelling**

The system studied in this paper includes synchronous generator, transformer, infinite bus, turbine, governor, Automatic Voltage Regulating (AVR) and excitation system as shown in Fig. 1.

Each component of this system is modeled accurately by dynamic equations. A synchronous generator with 7-order is used in which there are two state variables for rotor dynamics and other five variables related to the stator combined fluxes, dampers and excitation field. In addition, static excitation system (Exciter) is modelled by a first-order function. Turbine mechanical system as a model with 6-order is considered. The state variables are related to high pressure turbine torques, intermediate pressure turbine, low pressure turbine and also time delay due to auxiliary valves. The complete nonlinear model of system is given by [5]:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \omega_0 [F_{HP}x_9 + F_{IP}x_{11} + F_{LP}x_{12} \\ &\quad - x_6(y_{1d}x_4 + y_{4d}x_3 + y_{5d}x_5) \\ &\quad + x_4(y_{1q}x_6 + y_{3q}x_7) - K_d x_2] / 2H \\ \dot{x}_3 &= \omega_0 (x_8 - R_{fd}(y_{4d}x_4 + y_{2d}x_3 + y_{6d}x_5)) \\ \dot{x}_4 &= \omega_0 [V_b \sin x_1 + x_6 \\ &\quad - (R_a + R_e)(y_{1d}x_4 + y_{4d}x_3 + y_{5d}x_5)] + x_2 x_6 \\ \dot{x}_5 &= -\omega_0 R_{kd}(y_{5d}x_4 + y_{6d}x_3 + y_{3d}x_5) \\ \dot{x}_6 &= \omega_0 [V_b \cos x_1 - x_4 \\ &\quad - (R_a + R_e)(y_{1q}x_6 + y_{3q}x_7)] + x_2 x_3 \\ \dot{x}_7 &= -\omega_0 R_{kq}(y_{3q}x_6 + y_{2q}x_7) \\ \dot{x}_8 &= (u_2 - x_8) / \tau_{ex} \\ \dot{x}_9 &= (P_0 x_{13} - x_9) / \tau_{HP} \\ \dot{x}_{10} &= (x_9 - x_{10}) / \tau_{RH} \\ \dot{x}_{11} &= (x_{10} x_{14} - x_{11}) / \tau_{IP} \\ \dot{x}_{12} &= (x_{11} - x_{12}) / \tau_{LP} \\ \dot{x}_{13} &= (u_1 - x_{13}) / \tau_{GVM} \\ \dot{x}_{14} &= (u_1 - x_{14}) / \tau_{GVI} \end{aligned}$$

The nonlinear equations of this system can be linearized in desired operating point of system. The

operating point of system is determined by the active ( $P_t$ ) and the reactive power ( $Q_t$ ) values produced and infinite bus voltage ( $V_b$ ). The nominal values are respectively:

$$\begin{aligned} P_t &= 1 \text{ pu} \\ Q_t &= 0.6 \text{ pu} \\ V_b &= 1 \text{ pu} \end{aligned}$$

**3. Design of the Pole Placement Stabilizer**

All eigen-values of linear system in nominal operating point must be located in the left side of complex plane, i.e. The system is stable in response to disturbances entered from  $u_1$  and  $u_2$ . But the existence of poles near the imaginary axis will lead to a very long damping time due to the disturbance. For example, Fig. 2 indicates the frequency fluctuations of system while step shift in turbine steam valve position.

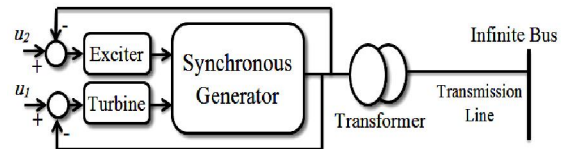


Fig. 1 The system under study

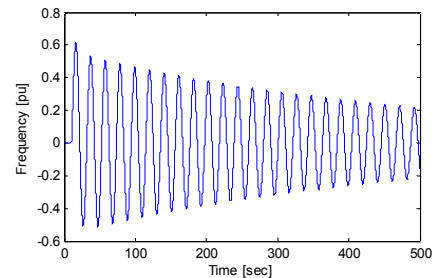


Fig. 2 Frequency fluctuations after change in turbine power

In order to reduce the damping time of system, the following state feedback controller is designed by Ackerman's algorithm [6].

$$K = \begin{bmatrix} -1.2 \times 10^5 & 6 \times 10^5 & 0.013 & 1.8 \times 10^4 & 6 & -4.7 \times 10^4 & -1.12 \times 10^7 \\ -5.16 \times 10^5 & 2.4 \times 10^6 & 1.29 & 6.8 \times 10^4 & 394 & -1.8 \times 10^5 & -4.42 \times 10^7 \\ 9.9 \times 10^{-4} & -2.5 \times 10^4 & -1.01 \times 10^6 & 2.4 \times 10^4 & 3.2 \times 10^4 & -8.5 \times 10^3 & 855 \\ -0.8 & -1 \times 10^5 & -4.06 \times 10^6 & 9.7 \times 10^4 & 1.3 \times 10^5 & -3.4 \times 10^4 & 3.4 \times 10^3 \end{bmatrix}$$

This controller assigns poles near the imaginary axis to farther points and reduces damping time. Table 1 shows the poles of the system without and with using the above state feedback. The system response to the disturbance is also shown in Fig. 3.

Table I. The System Poles assignment using the State Feedback

| Without State Feedback        | With State Feedback                |
|-------------------------------|------------------------------------|
| -0.0133                       | -11.839                            |
| -0.0019 + 0.3038i             | -11.454                            |
| -0.0019 - 0.3038i             | $-2.9063 \times 10^{-12} + 1i$     |
| -0.0466                       | $-2.9063 \times 10^{-12} - 1i$     |
| -0.0022                       | $9.4276 + 6.9378 \times 10^{-1}i$  |
| $-1.8626 \times 10^{-8} + 1i$ | $9.4276 + 6.9378 \times 10^{-1}i$  |
| $-1.8626 \times 10^{-8} - 1i$ | -8.0811                            |
| -100                          | -6.5840                            |
| -1.3889                       | -3.9558                            |
| -3.3333                       | -4.2165                            |
| -0.1                          | -4.6580                            |
| -3.3333                       | $-5.8327 + 1.4433 \times 10^{-1}i$ |
| -10                           | $-5.8327 - 1.4433 \times 10^{-1}i$ |
| -100                          | -5.2894                            |

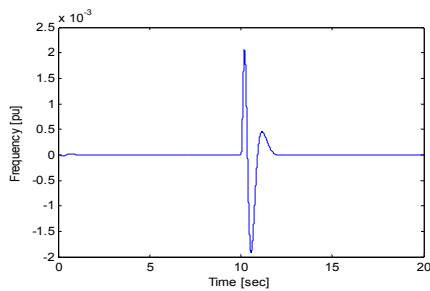


Fig. 3 Frequency fluctuations after applying state feedback

It is obviously that the designed state feedback can play an effective role in the stability of system. But the main problem occurs when the system operating point changes. In fact, the state feedback is designed for a linear system in a specified operating point.

Since the power system under study is nonlinear, the linear approximation with different operating points will be changed. Simulation results show that this change seriously affects the performance of the state feedback. Figs. 4-7 depict the response of linear system in the different operating points to the disturbance input using the state feedback. Due to change of each one of the active and reactive power values produced and infinite bus voltage, the system operating point can change. Here, the change of reactive power produced is only considered. As shown in Figs. 4-7, even a minor change in the system operating point can lead to non-optimal performance of the state feedback and also possibly instability of the system. In fact, it can be concluded that the performance of state feedback is extremely dependent on the parameters of linear system. Hence,

this is one of the major problems in application of the state feedback for stabilization of nonlinear systems.

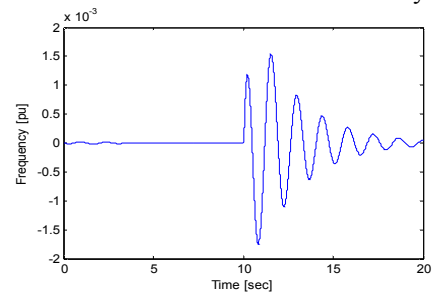


Fig. 4 Frequency fluctuations in non-nominal operating point,  $P_t=1, Q_t=0.604, V_b=1$

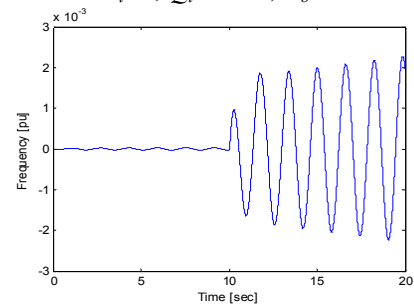


Fig. 5 Frequency fluctuations in non-nominal operating point,  $P_t=1, Q_t=0.607, V_b=1$

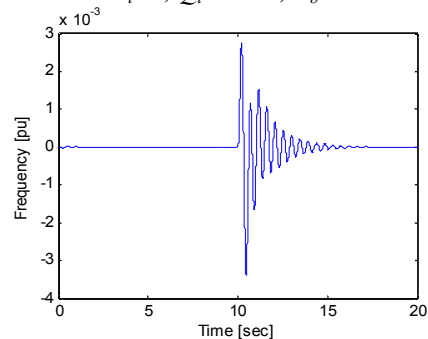


Fig. 6 Frequency fluctuations in non-nominal operating point,  $P_t=1, Q_t=0.5988, V_b=1$

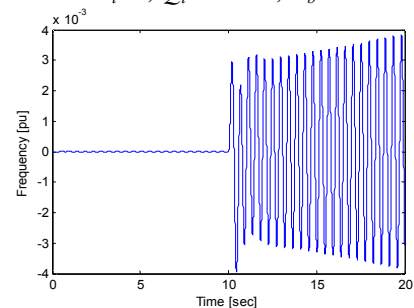


Fig. 7 Frequency fluctuations in non-nominal operating point,  $P_t=1, Q_t=0.5986, V_b=1$

Because of this, in this paper, a fuzzy controller as a supervisor is proposed to assignment of poles of the power system. In the structure of this controller, five state feedbacks are used which are designed for five different system operating points. The final control signal is obtained by incorporating these controllers with appropriate gain. The gain of each controller is determined by a fuzzy supervisor as well as the operating point of system. Fig. 8 shows the general scheme of the proposed fuzzy controller.

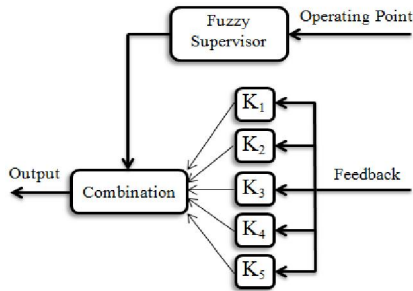


Fig. 8 Structure of the proposed fuzzy controller

At first, for fuzzy supervisor a typical structure with three input signals and five output signals is considered. The membership functions are shown in Figs. 9 and. The input membership functions for the fuzzy sets vsmall, small, normal, large and vlarge are shown in Fig. 9. The output membership functions for the fuzzy sets rule, medium and high are also described in Fig. 10. The input signals include the values of active and reactive power produced and the infinite bus voltage that identify system operating point. Also, five output signals determine the weighting coefficients related to the controllers designed in various system operating points.

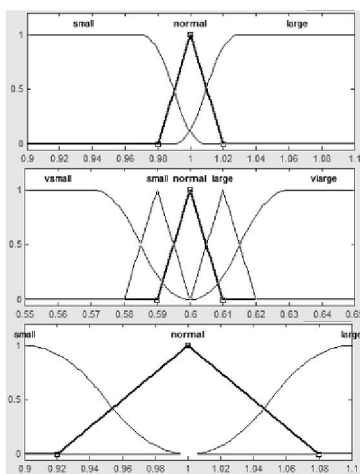


Fig. 9 Membership functions of input signals

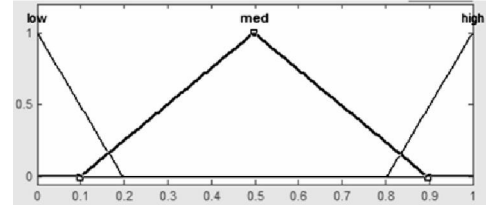


Fig. 10 Membership functions of output signals

There are a set of fuzzy rules that determines every controller gain properly based on the system operating point. The general form of these rules is based on this fact that a controller is designed closer to the operating point with more gain. The some fuzzy IF-THEN rules are defined as follows:

1) If input1 is normal and input2 is very small and input3 is normal Then output1 is high, output2 is medium, output3 is low, output4 is low, output5 is low

2) If input1 is normal and input2 is small and input3 is normal Then output1 is medium, output2 is high, output3 is medium, output4 is low, output5 is low

3) If input1 is normal and input2 is normal and input3 is normal Then output1 is low, output2 is medium, output3 is high, output4 is medium, output5 is low

4) If input1 is normal and input2 is large and input3 is normal Then output1 is low, output2 is low, output3 is medium, output4 is high, output5 is medium

5) If input1 is normal and input2 is very large and input3 is normal Then output1 is low, output2 is low, output 3 is low, output4 is medium, output5 is high

Figs. 11-15 show the system response to the disturbance input in several system operating points. Simulation results show that the proposed controller can guarantee the stability of the system in relatively wider range.

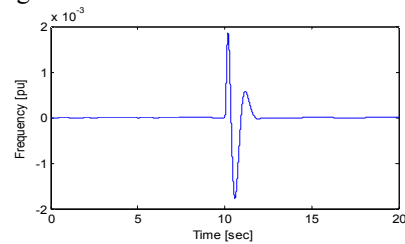


Fig. 11 Frequency fluctuations in nominal operating point,  $P_t=1, Q_t=0.6, V_b=1$

Fig. 12 Frequency fluctuations in non-nominal operating point,  $P_t=1, Q_t=0.607, V_b=1$

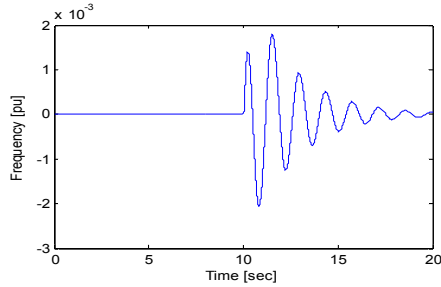


Fig. 13 Frequency fluctuations in non-nominal operating point,  
 $P_t=1, Q_t=0.612, V_b=1$

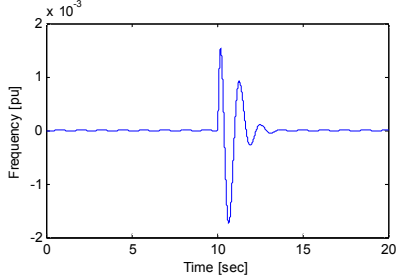


Fig. 14 Frequency fluctuations in non-nominal operating point,  
 $P_t=1, Q_t=0.5986, V_b=1$

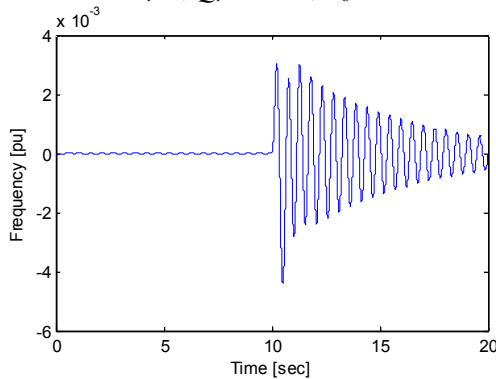
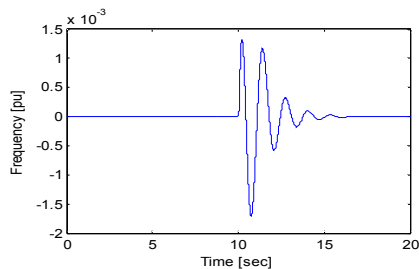


Fig. 15 Frequency fluctuations in non-nominal operating point,  
 $P_t=1, Q_t=0.59, V_b=1$



Moreover, Fig. 16 represents the variation range of the system operating point based on the stability region using state feedback and the proposed fuzzy state feedback. This figure shows that the proper combination of the state feedbacks which is designed in different operating points can increase the range of

changes in system operating point with stability maintenance. To further increase the range of stability, a greater number combination of controller in different operating points of system can be used. Another way is distribution of these five controllers in a wider range. In both cases the task of fuzzy supervisor becomes harder and more accurate composition of the different controllers requires. In this case, the design of fuzzy controller usually can not provide proper combination for system stability in a wide range by a manual and experimental method. Hence, the complexity of fuzzy controller design in this case, the necessity of an intelligent approach to the design specifies.

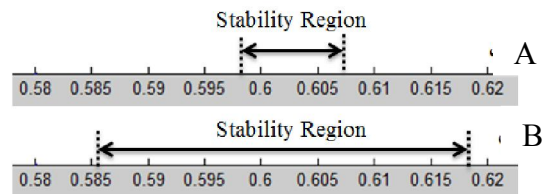


Fig. 16 System stability range with variation in parameter, A) with normal state feedback, B) with fuzzy state feedback.

#### 4. Design of the Proposed Optimal Fuzzy Pole Placement

In this section, to increase the system stability range related to the operating points, the combination of the designed state feedback controllers is used. Previous state feedbacks in five points with same intervals are designed in the range  $Q \in [0.58, 0.62]^{pu}$  while new feedbacks are designed in five points with the same intervals in range  $Q \in [0.56, 0.64]^{pu}$ . An increase interval where the state feedback is designed makes it more difficult to ensure the system stability and requires careful selection of different state feedbacks. In this condition, a fuzzy controller design is very difficult experimentally. On the other hand, it is difficult to assume that the given expert's knowledge captured in the form of the fuzzy controller leads to optimal control. Consequently, the effective approaches for tuning the membership function and control rules without a trial and error method are significantly required. In the problem in hand, optimal design of fuzzy supervisor is relatively difficult due to difficulty of choosing the proper combination of controllers. Therefore, GA is employed to solve the combinatorial optimization problems. This algorithm has been used already for optimization of some fuzzy controllers [7,8,9].

To design GA-based optimal fuzzy system, a typical fuzzy controller similar to the previous state is

considered except that some parameters related to the membership functions of this controller are assumed as a variable. By this assumption, the membership functions related to the first output signal of fuzzy controller are shown in Fig. 17. Also, other output signals have similar membership functions and variables. Thus, for five output signals there are a total of 25 variables. Since the variation of operating point is considered with change in the production of reactive power, the membership functions related to this input signal with 5 variables is also illustrated in Fig. 18.

Totally 30 variables of this problem will be chosen by GA so a specific objective function is minimized. Different objective functions can be selected as a measure of the system stability evaluation. An appropriate objective function should also provide a good indicator of the system stability such that the simplicity in computing and the speed of the algorithm are reasonable. The largest pole in every operating point can be considered an approximate measure of the system stability. Here the maximum summation of the largest poles is chosen at different specific operating points as an objective function. These operating points are opted with sufficient number and in range  $Q \in [0.56, 0.64]^{pu}$ . Obviously, being negative of objective function value is a necessary condition for the system stability in above range. GA operates such that the minimum negative value can be obtained for the objective function. Other parameters of genetic algorithm such as initial population, the number of individuals per generation, rules of composition, genetic mutation, and conditions of algorithm stop and etc are selected typically and due to 30 people population of each gene. Successful implementation of this algorithm will provide the most appropriate selection for membership functions of fuzzy controller. The obtained results after the implementation for 100 generations are shown in Figs. 19 and 20.

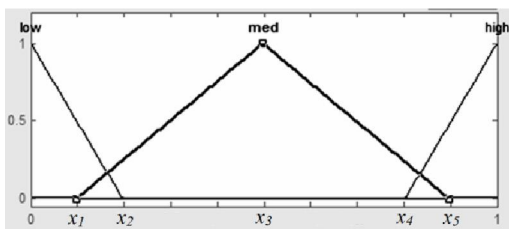


Fig. 17 Membership functions encoded in the first output signal

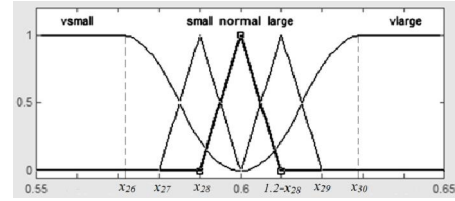


Fig. 18 Membership functions encoded in the second input signal

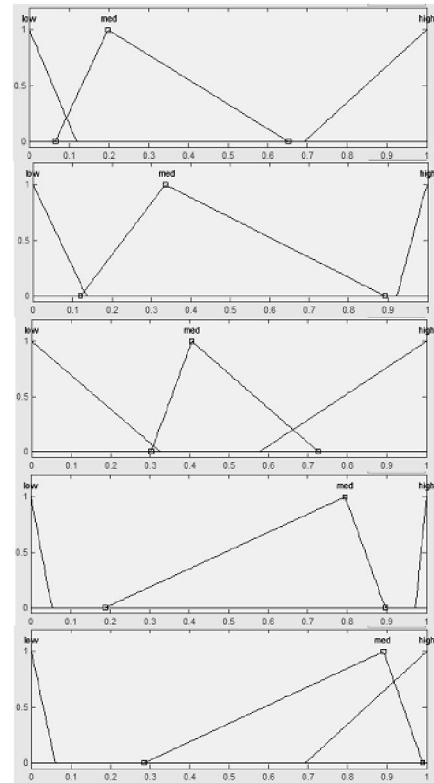


Fig. 19 Optimal membership functions for output signals

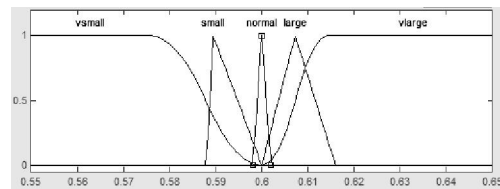


Fig. 20 Optimal membership functions for the second input signal

It should be noted that in this algorithm some limits have been applied for range of allowable changes of each of variables  $x_1$  to  $x_{30}$  so that more suitable forms for membership functions can be obtained. Fuzzy controller can be selected in operating points by this membership functions and can guarantee system stability. If these points are selected by sufficient number and proper distribution, it can be said that the optimal fuzzy state feedback

controller will stabilize the system in range  $Q \in [0.56, 0.64]^{pu}$ . In Fig. 21, the spread of system stability is compared with the previous aforementioned controller.

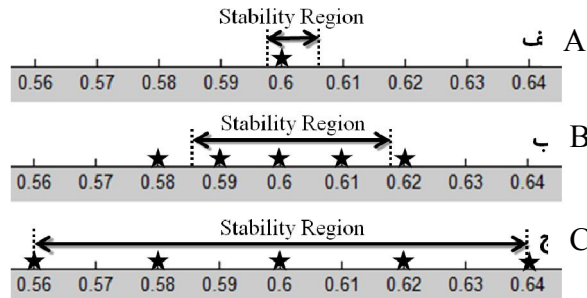


Fig. 21 Range of system stability with variation in parameter  $Q$ , A) with normal state feedback B) with fuzzy state feedback c) with optimal fuzzy state feedback

In this figure, the operating points for designing of state feedbacks are marked with asterisk. When state feedbacks are designed in close points, the system can be stabilized by distances between these points with proper combination of state feedbacks. This appropriate combination is possible with the optimal fuzzy supervisor. It is noticeable that by increasing the distance of these operating points, the design of fuzzy supervisor becomes more difficult and it is probable that the design of fuzzy supervisor is impossible for system stability, even by using GA. In this case, a stronger optimization algorithm should be used. Another solution is also to implement a similar optimization for a set of fuzzy rules. The Change and extension of fuzzy controller structure may be remedial in this case.

## 5. Conclusion

Although the state feedback is a proper and efficient control tool in linear systems, but the performance and efficiency of state feedback in nonlinear systems is highly sensitive to the system operating point. The pervious works have been shown that a state feedback which is designed for a nonlinear system in a specified operating point is effective only in a very small range of the operating point. To expand this range, a new structure of state feedback based on fuzzy supervision was provided in this paper. The performance evaluation of the new structure shows a good performance in a wider range of variation in the operating point. Therefore, the application of the proposed fuzzy state feedback is more suitable in nonlinear systems. Regarding the designing complexity of this controller for proper performance in the different operating points, GA

was employed for optimal design of fuzzy controller. Simulations results represented the feasibility of the proposed controller by guarantee the stability of system in a wider range of the variation of operating point.

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**Appendix**

Table II. the Parameters of the System

Table II. The State Variables of the system

| State Variable | Definition               |
|----------------|--------------------------|
| $x_1$          | Rotor angle              |
| $x_2$          | Rotor speed error        |
| $x_3$          | Field linkage flux       |
| $x_4$          | D-axes linkage flux      |
| $x_5$          | D-damper linkage flux    |
| $x_6$          | Q-axes linkage flux      |
| $x_7$          | Q-damper linkage flux    |
| $x_8$          | Field voltage            |
| $x_9$          | HP steam mass            |
| $x_{10}$       | Reheater steam mass      |
| $x_{11}$       | IP steam mass            |
| $x_{12}$       | LP steam mass            |
| $x_{13}$       | Main guide vane position |
| $x_{14}$       | Guide vane position      |

| Parameter    | Value    | Definition                        |
|--------------|----------|-----------------------------------|
| $F_{HP}$     | 0.24     | Power fraction of HP turbine      |
| $F_{IP}$     | 0.34     | Power fraction of IP turbine      |
| $F_{LP}$     | 0.42     | Power fraction of LP turbine      |
| $\tau_{HP}$  | 0.3      | Time constant of HP turbine       |
| $\tau_{IP}$  | 0.3      | Time constant of IP turbine       |
| $\tau_{LP}$  | 0.72     | Time constant of LP turbine       |
| $\tau_{ex}$  | 0.01     | Time constant of exciter          |
| $\tau_{RH}$  | 10       | Time constant of reheater         |
| $\tau_{GVM}$ | 0.1      | Time constant of main guide vane  |
| $\tau_{GVI}$ | 0.01     | Time constant of guide vane       |
| $R_a$        | 0.005    | Stator resistance                 |
| $R_c$        | 0.063    | Line resistance                   |
| $R_{fd}$     | 0.0015   | Field resistance                  |
| $R_{kd}$     | 0.0078   | D damper resistance               |
| $R_{kq}$     | 0.0084   | Q damper resistance               |
| $y_{1d}$     | 5.6219   | Inverse inductance matrix element |
| $y_{2d}$     | 1.5743   | Inverse inductance matrix element |
| $y_{3d}$     | 5.9413   | Inverse inductance matrix element |
| $y_{4d}$     | -0.6468  | Inverse inductance matrix element |
| $y_{5d}$     | -4.7699  | Inverse inductance matrix element |
| $y_{1q}$     | 1.3131   | Inverse inductance matrix element |
| $y_{2q}$     | 1.5811   | Inverse inductance matrix element |
| $y_{3q}$     | -1.1858  | Inverse inductance matrix element |
| $H$          | 3.25     | Inertia constant                  |
| $K_d$        | 0.025    | Friction factor                   |
| $P_0$        | 1.0      | Internal pressure of the boiler   |
| $\omega_0$   | $100\pi$ | Base frequency                    |