Fuzzy image retrieval systems using intuitionistic fuzzy sets

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Abstract: Intuitionistic fuzzy sets are a generalization of fuzzy sets. These sets have a greater ability for displaying uncertainty and are therefore more suitable for representing the content of color images. In this paper we first present methods for converting fuzzy sets into intuitionistic fuzzy sets. We then show how intuitionistic fuzzy sets may be used to improve precision and recall of fuzzy image retrieval systems. For a verification of the method we test it on a database containing 1000 images.

Keywords: Content-based retrieval; color image database; fuzzy number; intuitionistic fuzzy set

1. Introduction

In recent years, the growth of computer technology, multimedia information and very rapid growth of World Wide Web, has led to the creation of large archives of multimedia data in various fields such as remote sensing, medical and online information services. The massive volume of electronic data a large part of which is in the form of images, make the search by users difficult. Thus the need for efficient and automated search tools to index and retrieve information from these visual databases is necessary.

The subject of image retrieval has been studied since the early 1970’s with two different approaches: text-based and visual-based retrieval. In the traditional text-based approach for searching and image indexing, manual notes by humans are used. This method is very slow, intensive and expensive. Also textual notes by various people have different results and encoding all image information is not possible. Hence, a new approach to content based image retrieval to solve the problems of texture indexing was introduced in the early 1990’s.

In content based image retrieval (CBIR), the user describes an image in terms of its visual features and the system retrieves the closest images to what the user has described. Therefore, image retrieval based on content, is a recovery based on similarity. Performance of a CBIR system depends on several factors, of which the method of extraction of a set of characteristics that describe the image and the selection of an appropriate similarity measure are the most important.

The content of an image may be presented at different levels: pixel, feature and semantic. The feature level has many applications uses color, textures, and shape of objects. Recently, much attention has been paid to color image retrieval. One of the most important systems in this field is query by image content (QBIC) (Flickner et al., 1995, Hafner et al., 1995).

Prewitt is possibly the first person to use the concept of a fuzzy set-theoretic approach to image processing (Prewitt, 1970). The main reason for selecting fuzzy logic (Gasteratos et al., 2004, Zimmerman, 1987, Zadeh, 1965) is that because of its flexibility and tolerance of imprecise data, it has proven effective in many applications such as automatic control and image understanding.

Since then there has been a steady progress on research using fuzzy set theory in image processing. The reasons for the use of fuzzy notions in image processing are as follows: (a) Fuzziness is inherently embedded in nature and is reflected in the images. (b) Images are the 2D projections of a 3D world and so information is lost during mapping. (c) Gray levels are considered as imprecise constants. (d) Many definitions such as image boundaries and edges are vague in nature.

Fuzzy similarity based color retrieval has also been proposed by the Santini et al. (1999), Vertan et al. (2000), and Frigui (2001). They have used various fuzzy histograms and fuzzy similarity measures for calculating similarity. Frigui also has used the fuzzy integral as a dissimilarity measure.

Intuitionistic fuzzy sets (IFSs) was presented by Atanassov (1986), as a generalization of fuzzy sets, and vague sets were proposed by Gau et al. (1993). Bustince et al. (1996) pointed out that the notion of vague sets was the same as that of IFSs. Atanassov published a number of additional studies (Atanassov, 1989, 1995, 1999). IFSs sets make descriptions of the objective world more realistic, practical, and accurate, making it very promising. They have been widely applied in decision making (Szmidt et al., 1996), logic programming (Atanassov et al., 1990), pattern recognition (Hung et al., 2004)
and in recent years, seem to be more popular than fuzzy sets technology.

This paper focuses on demonstrating that intuitionistic fuzzy sets are more efficient than fuzzy sets in color image retrieval systems.

2. Material and Methods
2.1. Fuzzy Sets And Intuitionistic Fuzzy Sets

A fuzzy set \( A \) (Zadeh, 1965) in a universe of discourse \( X = \{x_1, x_2, \ldots, x_n\} \) is defined as follows:

\[
A = \{(x, \mu_A(x)) | x \in X\},
\]

Where \( \mu_A(x) : X \rightarrow [0,1] \), indicates the membership function of set \( A \). The non-membership function of the element \( x \) in the set \( A \) is \( \nu_A(x) = 1 - \mu_A(x) \). \( \mu(X) \) is the class of all fuzzy sets of universal \( X \).

Atanassov extended fuzzy sets to IFSs as follows:

An IFS \( A \) (Atanassov, 1986) in \( X \) is defined by:

\[
A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\},
\]

where \( \mu_A(x) : X \rightarrow [0,1] \) and \( \nu_A(x) : X \rightarrow [0,1] \) such that \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \). The numbers \( \mu_A(x) \) and \( \nu_A(x) \) respectively represent the degrees of membership and non-membership degree of the element \( x \) in the set \( A \). Let IFSs \( (X) \) denote the set of all IFSs in \( X \).

2.2. Similarity Measures

For image retrieval, the similarity between two sets of features, extracted from the database image and the query image has been used as a similarity measure. The similarity measure has been used to retrieve images in a database, which are similar to the query image.

Until now many similarity measures have been presented for calculating the similarity of fuzzy sets and intuitionistic fuzzy sets. Now we present similarity measures for using in color image retrieval systems.

Denote \( S : F(X) \times F(X) \rightarrow [0,1] \). \( S(A,B) \) is said to be a degree of similarity between \( A \in F(X) \) and \( B \in F(X) \) if \( S(A,B) \) has the following properties (Fan (1999)):

1) \( 0 \leq S(A,B) \leq 1 \).

2) \( S(A,B) = S(B,A) \).

3) \( S(A,A) = 1 \).

4) \( S(A, \emptyset) = 0 \).

5) \( S(A,C) \leq S(A,B) \) and \( S(A,C) \leq S(B,C) \) if \( A \subseteq B \subseteq C, C \in F(X) \).

One of properties which satisfy these properties is given by Chaire et al. (2003) as follows:

\[
S(A,B) = \frac{\sum_{i=1}^{n} \min(\mu_A(x_i), \mu_B(x_i))}{\sum_{i=1}^{n} \max(\mu_A(x_i), \mu_B(x_i))}
\]

Where \( \mu_A(x_i) \) and \( \mu_B(x_i) \) are respectively the degrees of membership of the \( n \)-th member of the universe in the two sets \( A \) and \( B \). An intuitionistic fuzzy similarity measure is defined as follows and must also have features 1, 2, 3 and 5 (Liang et al., 2003):

Let \( A \) and \( B \) be two IFSs in the universe of discourse \( X = \{x_1, x_2, \ldots, x_n\} \). Then we may write:

\[
A = \sum_{i=1}^{n} \left( \mu_A(x_i) + \nu_A(x_i) \right) \frac{1}{2}, \quad B = \sum_{i=1}^{n} \left( \mu_B(x_i) + \nu_B(x_i) \right) \frac{1}{2}
\]

Assume that:

\[
\varphi_1 = \frac{\mu_A(x_i) + \nu_B(x_i)}{2}, \quad \varphi_2 = \frac{\mu_B(x_i) + \nu_A(x_i)}{2},
\]

\[
\eta_{1i} = \frac{\mu_A(x_i) + \nu_B(x_i)}{2}, \quad \eta_{2i} = \frac{\mu_B(x_i) + \nu_A(x_i)}{2}.
\]

Then we define a similarity measure for two IFSs \( A \) and \( B \) as follows:

\[
S(A,B) = 1 - \frac{1}{\sum_{i=1}^{n} \varphi_1(x_i) \eta_{1i}(x_i) \varphi_2(x_i) \eta_{2i}(x_i)}
\]

where \( 0 \leq w_{\varphi_1} \leq 1 \) and \( \sum_{i=1}^{n} w_{\varphi_1} = 1 \).

2.3. A Sample Fuzzy Content Based Image Retrieval System

In this section we describe an image retrieval system in which we use the HSI color space instead of conventional RGB color space. Since of only two dimensions of HSI space are enough to model images and the size of feature database is reduced (Nachtegael et al., 2007).

HSI color space is based on intuitive description and direct perception of images and can be modeled with cylindrical coordinates. The hue component describes the color according to wavelength and is presented with an angle which can be changed from \( 0^\circ \) to \( 360^\circ \). The saturation component is the amount of color which is displayed and depends on the radius of a cylinder which varies from 0 to 1 (0 or 0 to 255). The intensity component indicates the amount of light. This component varies along the z axis and is coded so that black equals 0 and white is 1 (or 255).
In image modeling, we limit ourselves to eight main colors where each of which is modeled by a trapezoidal fuzzy number. Thus we obtain a fuzzy classification for the hue component, as shown in Figure 1. A classification of the intensity component is shown in Figure 2.

2.3.1. Calculation of Similarity

We shall consider only two components for each image: Hue and intensity. To calculate the similarity of two images we first obtain the amount of similarity of each component and then average them.

Thus in each image for each pixel, according to the amount of hue and intensity component, we obtain its membership degree in each of the eight functions modeled in figures 1 and 2.

$$S_{c}(A_{x}, A_{j}) = \frac{\sum_{x} \min \left(\mu_{x}^{A_{x}}, \mu_{x}^{A_{j}}\right)}{\sum_{x} \max \left(\mu_{x}^{A_{x}}, \mu_{x}^{A_{j}}\right)}$$

where $S_{c}$ is in the interval [0, 1]. We also use this function to calculate the similarity between two membership functions $\mu_{x}^{A_{x}}$ and $\mu_{x}^{A_{j}}$. As follows:

$$S_{c}(A_{x}, A_{j}) = \frac{1}{10} \sum_{x} S_{c}(\mu_{x}^{A_{x}}, \mu_{x}^{A_{j}})$$

when $x$ is either $c$ or $d$.

2.4. Proposed System

In this section we present the architecture of the proposed system, the methods which is used and the improved system.

2.4.1. System Architecture

The architecture of the proposed CBIR system is shown in figures 3 and 4. This system, similar to many CBIR systems, is divided into two phases.

![Figure 3. First phase of system](http://www.americanscience.org)

![Figure 4. Second phase of system](http://www.americanscience.org)
In the first phase of the system, all primary database images are modeled in the form of intuitionistic fuzzy sets. To extract these intuitionistic sets we first obtain the membership degree for all the pixels in each color and then extract the non-membership degree for each pixel.

These intuitionistic fuzzy sets are stored in the indexed image database as features which describe the images. In the second phase of system, the fuzzy sets which describe the query image are extracted and these features are compared with all the features of database images. Thus, using similarity measures that show the amount of similarity of fuzzy sets, the most similar images to the query image are extracted.

2.4.2. Conversion of Fuzzy Sets to Intuitionistic Fuzzy Sets

Each fuzzy set has a membership function. For every member in the domain of a fuzzy function, the sum of membership and non-membership degree will be equal to 1. However the sum of membership and non-membership degree of intuitionistic fuzzy set can be less than 1. Thus when we convert fuzzy sets to intuitionistic fuzzy sets, we use the same membership degrees but different non-membership degrees. Clearly, the non-membership degrees of the intuitionistic fuzzy set members are always less than or equal to the non-membership degrees of their counterparts members in the equivalent fuzzy sets:

\[ F_S: \mu_A(x) + \nu_A(x) = 1 \rightarrow \nu_A(x) = 1 - \mu_A(x) \quad (1) \]

\[ IFS: \theta_A(x) + \nu_A(x) \leq 1 \rightarrow \nu_A(x) \leq 1 - \theta_A(x) \quad (2) \]

So far for modeling the subject in the form of a fuzzy set, it was enough to model its membership function. Its non-membership function can be automatically calculated and is equal to (1-membership function value) for each member. But for modeling the same subject by intuitionistic fuzzy set in addition to the membership function, a function for the non-membership degree must also be designed. The values of this function are smaller or equal to (1-membership function value) for each member.

In the following we present a way to design non-membership functions based on the membership functions for an intuitionistic fuzzy set. In general, the Sugeno fuzzy complement operator can be used. The Sugeno class is a class of fuzzy complements and is defined as follows:

\[ C_A(x) = \frac{1-x}{1+y} , \quad x \in X \]

If \( A = 0 \) the standard fuzzy complement is obtained. In different problems depending on their type, it is possible to use different amounts for \( A \) that can be determined empirically through trial and error.

Since in many cases fuzzy numbers are used to model color images, we will define a specific method for converting fuzzy sets based on the concept of fuzzy numbers. A fuzzy number is a fuzzy subset of the real line whose highest membership values are clustered around a given real number called the mean value; the membership function is monotonic on both sides of this mean value.

Suppose in a system for primary indexing of images, fuzzy numbers are used. A trapezoidal fuzzy number \( A \) is a fuzzy set with a trapezoidal-formed membership function. The trapezoidal membership function usually depends on four scalar parameters \( a, b, c \) and \( d \) as follows:

\[ \mu_A(x; a, b, c, d) = \begin{cases} 
0 & \text{if } x \leq a, \\
\frac{x-a}{b-a} & \text{if } a < x < b, \\
1 & \text{if } b \leq x \leq c, \\
\frac{d-x}{d-c} & \text{if } c < x \leq d, \\
0 & \text{if } d \leq x.
\end{cases} \]

We now define the corresponding non-membership function \( \theta_A \) as follows:

\[ \theta_A(x; a, b, c, d) = \begin{cases} 
1 & \text{if } x \leq a, \\
\frac{b-x}{b-a} & \text{if } a < x < b, \\
0 & \text{if } b \leq x \leq c, \\
\frac{c-x}{d-c} & \text{if } c < x \leq d, \\
1 & \text{if } d \leq x.
\end{cases} \]

Where, for \( 0 \leq \theta \leq 1 \), \( a \) and \( b \) satisfy

\[ a = (b - c) \times \theta , \quad b = (d - c) \times \theta , \]

\( \theta \) is determined empirically through trial and error. A membership function together with the corresponding non-membership function is shown in figure 5.

![Figure 5. membership to non-membership converting method](http://www.americanscience.org)
2.4.3. Improved Fuzzy Image Retrieval System

Using intuitionistic fuzzy sets instead of ordinary fuzzy sets for image retrieval seems more appropriate because color is an intuitive characteristic.

We shall use the fuzzy number approach discussed in section 2.2, with \( \theta = 0.5 \) to convert our fuzzy sets to intuitionistic fuzzy sets. We now present an image retrieval system based on intuitionistic fuzzy sets. We again use the HSI color space. The focus is on hue and intensity components membership functions of which are modeled by using trapezoidal numbers. We suppose that intuitionistic fuzzy membership function is the fuzzy membership function described in section 1.3. Hence we have 16 trapezoidal membership functions for which we obtain corresponding non-membership functions. We now model an image using intuitionistic fuzzy sets.

This time we have sixteen histograms for each component. After normalizing these histograms for each image \( A_j \) and color \( c \) in the hue component we obtain the membership and non-membership functions \( \mu_{A_j}^{c} \) and \( \nu_{A_j}^{c} \) respectively. The equivalent functions for intensity degree \( d \) are denoted by \( \mu_{A_j}^{d} \) and \( \nu_{A_j}^{d} \). These functions are also extracted for the query image \( A_2 \). The similarity of these functions \( S(\mu_{A_j}^{c}, \mu_{A_j}^{d}) \) and \( S(\nu_{A_j}^{c}, \nu_{A_j}^{d}) \) are calculated using the similarity measures for intuitionistic fuzzy sets discussed in section 1.2. The average of these values is the similarity of the two images.

3. Results

The most common measures in information retrieval are precision and recall. These measures are calculated as follows:

\[
\text{Precision} = \frac{\text{Number of related retrieved documents}}{\text{Total number of retrieved documents}}
\]

\[
\text{Recall} = \frac{\text{Number of related retrieved documents}}{\text{Total number of related documents}}
\]

In this section we present the results of implementing two fuzzy and intuitionistic fuzzy systems on a database of 1000 color images (http://wang.ist.psu.edu/docs/related/). This images may be divided into ten groups each contains hundred similar images. To implement the intuitionistic fuzzy system, we first extract and store sixteen intuitionistic fuzzy sets for the color content of each of the 1000 images. We then consider each image as a query image and calculate the similarity of its describing sets to each of the other 999 images. We do the same for a fuzzy system, this time extracting sixteen fuzzy sets instead of intuitionistic sets.

We then retrieve a number of similar images for each query image and obtain the precision and recall of these systems. Finally, we calculate the average precision and recall for the 1000 images. The results are shown in table 1.

Table 1. The comparison of the ordinary fuzzy image retrieval system with the intuitionistic fuzzy image retrieval system

<table>
<thead>
<tr>
<th>number of retrieved images</th>
<th>Recall of fuzzy image retrieval system</th>
<th>Recall of intuitionistic fuzzy image retrieval system</th>
<th>Precision of fuzzy image retrieval system</th>
<th>Precision of intuitionistic fuzzy image retrieval system</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0211</td>
<td>0.0217</td>
<td>0.634</td>
<td>0.652</td>
</tr>
<tr>
<td>10</td>
<td>0.1887</td>
<td>0.1924</td>
<td>0.5661</td>
<td>0.5772</td>
</tr>
<tr>
<td>20</td>
<td>0.3535</td>
<td>0.3626</td>
<td>0.53035</td>
<td>0.54395</td>
</tr>
<tr>
<td>30</td>
<td>0.5107</td>
<td>0.5243</td>
<td>0.51076</td>
<td>0.5243</td>
</tr>
<tr>
<td>40</td>
<td>0.6571</td>
<td>0.6730</td>
<td>0.49287</td>
<td>0.5048</td>
</tr>
<tr>
<td>50</td>
<td>0.7894</td>
<td>0.8152</td>
<td>0.47366</td>
<td>0.48912</td>
</tr>
</tbody>
</table>

As the results show, in all cases the precision and recall of intuitionistic system are greater and therefore this system is better than the ordinary system.

4. Discussions

In this paper we studied the features and applications of intuitionistic fuzzy sets. One of the main applications of these sets is in color based image retrieval systems. Because of uncertainty in color description intuitionistic fuzzy sets are more suitable than fuzzy sets for modeling such systems. So far, several fuzzy image retrieval systems have been designed.

There are several ways of converting fuzzy sets to equivalent intuitionistic fuzzy sets. In this paper, we study a specific retrieval system using intuitionistic fuzzy sets and show that this is an improvement on a model using ordinary fuzzy sets.

An open problem is to design a complete and exact method of converting fuzzy sets to intuitionistic fuzzy sets. The design of such methods leads to greater efficiency in systems.

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