The primary field of a vertical Hertizan dipole in free space

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Abstract: The paper presents a method which allows the calculation of the atmospheric distortion of radar pulses. Two integrals transformation of the wave equations of Hertizan vector-a Laplace transform in time and a two dimensional Fourier transform in the horizontal coordinates in space are applied. The integral representation determines the electromagnetic field anywhere in the ionosphere; and calculates the primary field of a vertical Hertizan dipole in free Space.

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1. Introduction

In the problem of the electromagnetic radiation from a vertical electric dipole situated at a certain height h above a plane earth all field quantities are usually assumed to vary harmonically in time. One of the two well-known methods for solving this steady-state problem is due is Sommerfeld[1], the other to Weyl[2].

Today, interest is mainly concerned with the effect of the atmosphere on target identification and imaging. An accurate and efficient method for computing Sommerfeld integrals is crucial in the analysis of electromagnetic field due to radiators and scatterers embedded in layered media.

In several recent publications, however, the case is considered where the time dependence of the current in the dipole is impulsive rather than harmonic e.g. the studies by Banos[3]; Lindell and Alanen[4] and Dvorak and Mechaik[5], these techniques can be grouped in the following categories; quasi-analytical solutions which include asymptotic approximations series expansions and image representations, direct numerical integration, and methods which use numerical techniques.

In the present paper it is shown that Cagniard's method can be simplified considerably if the corresponding modification for two-dimensional problems as developed by the present author. Cagniard's[6] is taken a guidance the theoretical study for computing the magnetic field from a Fizragld vector in the ionosphere is presented Abo-seliem[7,8]. Two integral transforms are applied to analyze a Laplace transform in time and two-dimensional Fourier transforms in the horizontal coordinate in the space is applied for the Hertizan vector in the wave equation. This leads to an integral representation of the wave equation in the free space.

2. Formulation of the problem

Fig.1 shows the duct model of Kahan and Eckart[9]. A dielectric layer is assumed of relative permittivity ε_1 over laying an infinitely conducting plane earth which is confined by the plane z=0 of a rectangular coordinate system. The source of the field is assumed to be a vertical electric dipole in the medium 1 at the point x=y=0, z=d>0 whose moment is given by $\vec{p}_e = \{0,0,F(t)\delta(x,y,z-d)\}$, t being the time variable and δ the three-dimensional δ -distributation. Regarding F (t), we make the assumptions F (t) = 0 for t ≤ 0 and $\frac{dF(t)}{dt} = 0$ for t=0.

The starting point is the wave equation for the Z-component of the Hertz vector $\vec{\pi}(x,y,z;t)$ which we denote by $\vec{\pi}_i(x,y,z;t)$, i=1,2, as:

$$[\nabla^2 - v_i^{-2}\partial_t^2]\vec{\pi}_i(x,y,z;t) = \begin{cases} 0 & for\ i=2 \\ -\delta(x,y,z-d)F(t) & for\ i=1 \end{cases} \tag{1}$$

Where v_i denotes the phase velocity of medium i. The electric and magnetic field generated by this Hertizan dipole can be derived from a Hertizan vector $\vec{\pi}$ through the radiations. **grad div**$\vec{\pi}$

$$\vec{E} = \text{grad div}\,\vec{\pi} - \mu\mu_0\varepsilon\varepsilon_0 \frac{\partial^2 \vec{\pi}}{\partial t^2} \tag{2}$$

And

$$\vec{H} = \varepsilon\varepsilon_0 \nabla X \frac{\partial \vec{\pi}}{\partial t} \tag{3}$$

In the region z >h, we write:

$$\vec{\pi} = (\pi_o + \pi_i)\vec{e}_z \tag{4}$$

Where π_o yields the primary simulation, while π_i accounts for the secondary simulations, similarly, in the regions 0<z<h, we write:

$$\vec{\pi} = \pi_2 \vec{e}_z$$

Where π_o yields the reflected and consists of an incident waves from the conducting plane z=0. At any interior point of the appropriate half-space $\vec{\pi}_i = \vec{\pi}_1(x, y, z; t)$ and in medium $\vec{\pi}_2 = \vec{\pi}_2(x, y, z; t)$ are assumed to be continuous together with their first and second other partial derivatives. We get for $h < z < \infty$ [8]

$$\pi(x,y,z;t) = \frac{sf(s)}{8\pi^2}\int\int_{-\infty}^{\infty}\left[\frac{\exp(-s\gamma_1|z-d|)}{\gamma_1} - \exp(-s\gamma_1(z+d))\frac{1+C_{12}\exp(-2s\gamma_1(h-d))}{\gamma_1(1+C_{12}\exp(-2s\gamma_2 h))}\right]e^{i(\alpha x+\beta y)}d\alpha d\beta \tag{6}$$

Where

$$C_{12}(\alpha,\beta) = \frac{\gamma_1(\alpha_1\beta) - \gamma_2(\alpha_1\beta)}{\gamma_1(\alpha_1\beta) + \gamma_2(\alpha_1\beta)} \tag{7}$$

And

$$\gamma_i(\alpha,\beta) = (\alpha^2 + \beta^2 - \nu_i^{-2})^{1/2} \tag{8}$$

With $\Re e(\gamma_i) \geq 0, \quad i = 1,2$. Here α and β are the variables in the transform space of the two-dimensional Fourier transform $f(s)$ is the Laplace transform of $F(t)$. A similar expression can be derived for $\pi_2(x, y, z; s)$

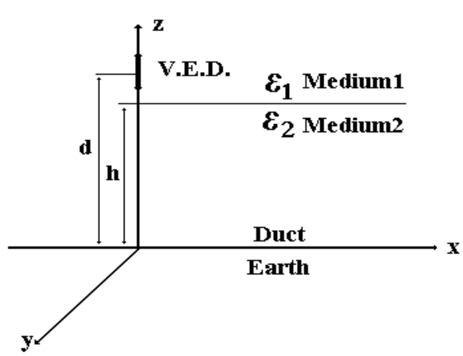

Fig.1: Geometry of the problem.

3. Computation of the primary integral

We shall try to cost the integral on the right-hand side of (6) in such a form that $\pi^{pri}(x, y, z; s)$ can be found more or less by inspection. This is accomplished by introducing new variables of integration ω and q through

$$\alpha = \omega\cos\varphi - q\sin\varphi, \qquad \beta = q\cos\varphi + \omega\sin\varphi \tag{9}$$

Since $d\alpha d\beta = d\omega dq$ we obtain (5)

$$\pi^{pri}(x,y,z;s) = \frac{sf(s)}{4\pi^2}\int_{-\infty}^{\infty}dq\int_{-\infty}^{\infty}\frac{\exp(-s\gamma_1|z-d| + js\omega r)}{2\gamma_1}d\omega \tag{10}$$

In which, as $\alpha^2 + \beta^2 = \omega^2 + q^2$,

$$\gamma_i = (\omega^2 + q^2 - \nu_i^{-2})^{1/2}, \quad (\Re e(\gamma_i) \geq 0) \tag{11}$$

In order to bring the right-hand side of (10) in a form which is analogous to the two-dimensional case; we introduce the variable $p = i\omega$ and regard p as a complex variable in the p-plane. While q kept real. The result is:

$$\pi^{pri}(x,y,z;s) = \frac{sf(s)}{4i\pi^2}\int_{-\infty}^{\infty}dq\int_{-i\infty}^{i\infty}\frac{\exp(spr - \gamma_1|z-d|)}{2\gamma_1}dp \tag{12}$$

In which

$$\gamma_i = (q^2 - \nu_i^{-2} - p^2)^{1/2}, \quad (\Re e(\gamma_i) \geq 0 \tag{13}$$

(8)

By virtue of Cauchy's Theorem and Jordan's Lemma the integration along the imaginary p-axis can be replaced by integration along the branch Γ of a hyperbola, where Γ is given through:

$$\rho = \frac{r}{R}\tau \pm i\frac{|x|}{R^2}[\tau^2 - R^2(q^2 - \nu_i^{-2})]^{1/2}, \quad [R(q^2 - \nu_i^{-2})]^{1/2} < \tau < \infty \tag{14}$$

A long Γ we have

$$\gamma = \frac{|x|}{R^2}\tau \mp i\frac{r}{R^2}[\tau^2 - R^2(q^2 - \nu_i^{-2})]^{1/2} \tag{15}$$

$$\frac{\partial p}{\partial \tau} = \pm\frac{i\gamma}{[\tau^2 - R^2(q^2 - \nu_i^{-2})]^{1/2}} \tag{16}$$

In (14), (15) and (16) the upper and lower signs belong together.

Taking into account the symmetry of the part of integration with respect to the real axis and introducing τ as variable of integration, we obtain:

$$E^{prim}(x,y,z;s) = \frac{sf(s)}{4\pi^2}\int_{-\infty}^{\infty}dq\int_{R(q^2-\nu_1^{-2})^{1/2}}^{i\infty}\exp(-s\tau[\tau^2 - R^2(q^2 - \nu_1^{-2})]^{-1/2})\,d\tau \tag{17}$$

Now we interchange the order of integration, which leads to:

$$E^{prim}(x,y,z,s)=\frac{sf(s)}{4\pi^2}\int_{R/r}^{\infty}\exp(-s\tau)d\tau\int_{-\eta}^{\eta}[\tau^2-R^2(q^2-\nu_1^{-2})]^{-1/2}d\tau$$

$$=\frac{sf(s)}{4\pi R}\int_{R/r}^{\infty}\exp(-s\tau)d\tau \qquad (18)$$

Where $\eta=(\frac{\tau^2}{R^2}-\nu_i^{-2})^{1/2}$

$$\mathbf{E^p(x,y,z;s)=f(s)\frac{\exp(--sR/v)}{4\pi R}} \qquad (19)$$

Application of the shift rule yields the well-known result

$$\mathbf{E^p(x,y,z;s)=\frac{f(t-R/v)}{4\pi R}} \qquad (20)$$

4. Conclusion

We used a method originally given by Cagniard and modified by de Hoop and Frankena extending it to the present case of more complexes and to source position in the medium of greater refractive index. A disadvantage of a method is not it cannot be used to calculate the primary field in the dielectric half-space outside the layer in a similar manner.

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