

Effect of Corrugated Bed Shapes on Hydraulic Jump and Downstream Local Scour

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Abstract: Hydraulic jump is generally help in the dissipation of excess kinetic energy downstream of hydraulic structures such as drops, spillways, chutes and gates. The corrugated stilling basin beds decrease the required depth and length of the jump, so, it reduces the cost of energy dissipating stilling basin. Through this research, an experimental study was conducted to study the effect of using three different shapes of corrugated beds on the characteristics of a hydraulic jump and downstream local scour. Forty eight experimental runs were carried out considering wide rang of Froude numbers ranging from 2.0 to 6.5. Five values of the relative roughness of corrugated shapes were investigated. A case of smooth bed is included to estimate the influence of corrugated beds on hydraulic jump parameters and the scour hole dimensions. Obtained results were analyzed and graphically presented and also, simple formulae are developed to estimate the hydraulic jump parameters and the scour hole dimensions. The results of this study confirm the effectiveness of corrugated beds for energy dissipation downstream hydraulic structures and corrugating the stilling bed can decrease the cost of stilling basins.

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1. INTRODUCTION

Flow over spillways or underneath gates is a tremendous amount of potential energy, which is converted into kinetic energy downstream the hydraulic structures. This energy should be dissipated to prevent the possibility of excessive scouring of the downstream river bed, minimize erosion and the undermining of the structures, which endanger the structure safety.

Hydraulic jump can be controlled by sills of various shapes, such as sharp crested, or broad crested weir, abrupt rise or abrupt drop in channel bed. These structures ensure the formation of hydraulic jump and control its position for all probable operating conditions, Peterka (1958). Hydraulic jump is widely produced in stilling basins and a large amount of published investigations was carried out to find the solutions of localize the jump in order to have a stabilized flow condition and to control the erosion phenomena, Hager (1992). The hydraulic jump which occurs in wide rectangular horizontal channels with smooth bed is called classical jump and has been widely studied by Peterka (1958), Rajaratnam (1967), McCorquodale (1986) and Hager (1992). The first study on hydraulic jump over rough bed were carried out by Rajaratnam (1968), and this study was mentioned that, the length of jump upon rough bed is smaller than the length of classical jump, and also Many different corrugated bed shapes have been proposed studied by Ead and

Rajaratnam (2002), Farhad Izadjoo and Shafai (2005) and Ead S. Ali (2007).

Scour downstream a hydraulic jump has been studied by many researchers such as Novak (1961), Catakli O. (1973), Pillai N. (1989), Rice, C.E., and Kadavy, K.C. (1993), Baghdadi (1997), Hoffmans (1998) and El-Abd (2002).

A review of the previous published materials showed that, the roughness of stilling basins bed can effectively decrease the required conjugate depth and length of the jump; thus, it can reduce the cost of energy dissipating stilling basin. The present research was put on track with the objective of investigating the hydraulic jumps occurring on corrugated beds of considered shapes (semi-circular, trapezoidal and triangular) and the effect of corrugated bed shapes on the downstream deformed scour hole.

2. EXPERIMENTAL SET-UP

In this paper, in order to reach the main purpose of this research, a model of a streamlined lip gate with horizontal basin is constructed, to develop the required supercritical flow and initial depth of the jumps. The model is investigated experimentally with smooth bed to use its results as a reference case to the cases of corrugated beds. To create the required roughness of the bed, plastic semi-circular, trapezoidal and triangular sheets were attached on the flume bed, and in order to diminish the effects of cavitations, the crests of corrugation were set at the same level of upstream bed, see Figures1 and 2.

The dimensions of the corrugated beds are:-
The corrugated height, $t = 30\text{mm}$ is kept constant,
The corrugated wave length, S is chosen to be $S = 2t$,
Both the trapezoidal and triangular sections have side slopes of 63.5 and 45 degrees respectively, and corrugated beds start after gate by $5S$, (calculated).

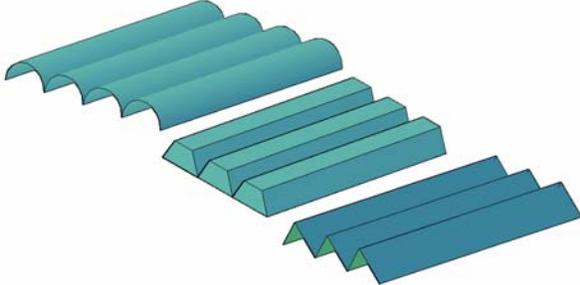


Figure 1. Considered corrugated bed shapes

To study the effect of different corrugated bed shapes, a rectangular flume was used. The flume is 60 cm wide, 60 cm deep and 20 m long. The flume is constructed of bricks sealed with smooth cement mortar. The flume consists of head and tail tanks, main and by-pass channels. A centrifugal pump is used to supply water to the head tank from the ground sump; water is controlled using a control valve installed on the pipe connected to the feeding pump. The head tank has a gravel box which is used to provide an even flow distribution across the flume, and also, the head tank contains a calibrated weir model which is made of wood. The water head over the calibrated weir was measured with a precise vertical scale. The flow enters main channel through an inlet screen to absorb any water eddies. The main channel contains a streamlined lip sluice gate. The downstream bay is made of smooth cement-sand mortar overlaid on a 0.5 m layer of sand to prevent leakage, this bay is represented the smooth bed also, prepared for the corrugated sheets fixation. The rear reach of the channel is filled with a 0.30 m deep layer of a uniform PVC material with $D_{50} = 2.50\text{ mm}$, and a specific weight of 1080 kg/m^3 in order to represent the movable bed. Two Precise point gauges are installed, one of them monitors the bed level and the other is to measure the initial and sequent depth of jump. These gauges are mounted on carriages moving in the flow and the perpendicular directions. Downstream water depth is controlled using a flap gate. The length of floor and the dimensions of the corrugated sheets are kept constant for all runs. Corrugated bed shapes are changed to reach the most suitable one.

3. EXPERIMENTAL PROCEDURE

To achieve the main purpose of this research, Three discharges are considered ($Q = 20, 26$

and 30 Lit/sec). For each discharge, four values of initial depth of hydraulic jumps are used so, three types of hydraulic jumps (weak, oscillating and steady) were investigated. 48 runs were conducted including 12 runs with smooth bed. These twelve smoothed bed runs were considered as a reference case.

For each run, the backwater feeding is started first until its depth reaches higher than the required downstream water depth, and then, the upstream feeding is pumped. To adjust the tail water depth, the tail gate is screwed gradually until the considered depth is adjusted. For each run, the discharge is measured using ultrasonic-flow meter, and calibrated weir equation, a streamlined lip gate diminishes the turbulence. The gate avoids vena contracta and provides a uniform flow. A precise point gauge is used to adjust the initial depth of jump, (gate opening) and another point gauge is used to measure the sequent depth. The length and the roller length of hydraulic jump are measured by a precise scale. After several trials, half and two hours period was selected as a constant time for all runs, after the running time, the run was stopped and the flume was drained and the expecting scouring area was recorded with a precise point gauge to monitor the bed topography on a grid $10\text{ cm} \times 10\text{ cm}$.

4. RESULTS AND DISCUSSION

4.1 Conjugated Depth

For hydraulic jumps over corrugated beds with a supercritical depth y_1 , the sequent depth of the jump y_2 can be written to be function of:

$$f(y_1, u_1, g, \rho, \nu, t, S) = 0 \quad (1)$$

In which, g is the gravitational acceleration ρ , ν are the density and viscosity of water, respectively, and other variables have been previously defined. By the application of Buckingham's theory, the dimensionless relationship thus, may be written as:

$$\frac{y_2}{y_1} = f\left(F_{r1} = \frac{u_1}{\sqrt{gy_1}}, R_1 = \frac{u_1 y_1}{\nu}, \frac{t}{y_1}, \frac{S}{y_1}\right) \quad (2)$$

For all experimental work, value of Reynolds's number $R_1 > 25000$, so the effect of viscosity is considered to be neglected, then the values of Reynolds's number can be eliminated from analysis.

Farhad Izadjoo and Shafai-Bajestan M. (2005) and Ead S. Ali (2007) are concluded that, the relative roughness have small influence on the sequent depth ratio, so the relation between initial Froude number Fr_1 and y_2/y_1 for the considered corrugation shapes is plotted, as shown in Figure 3.

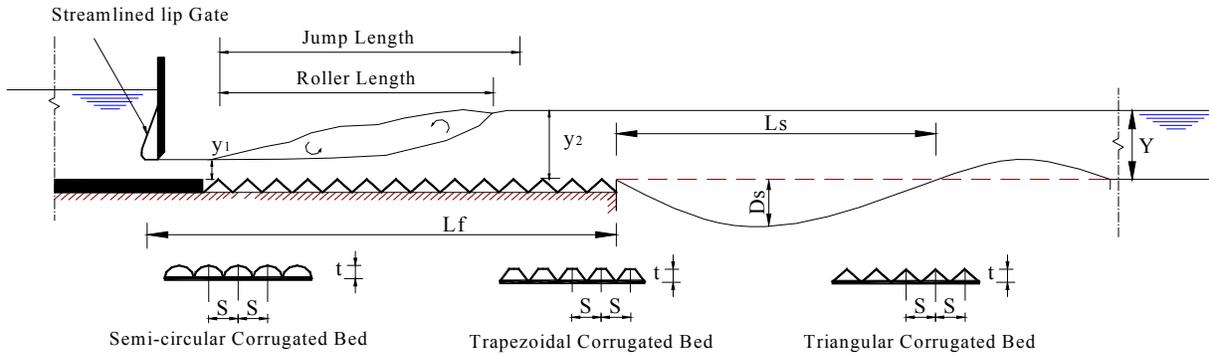


Figure 2. Layout of a hydraulic Jump on corrugated beds

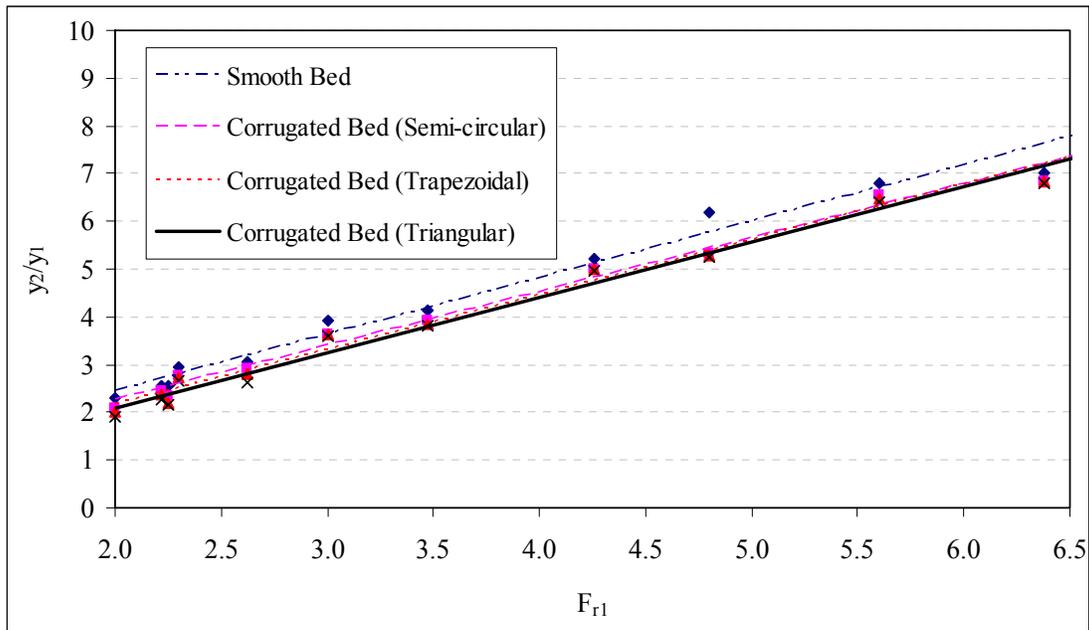


Figure 3. Relation between the ratio of y_2/y_1 and Fr_1

One can see that, for considered shapes of corrugations beds, the relative sequent depth is smaller than of smooth bed. It is apparent that, there is a small difference in effect on the sequent depth of hydraulic jump y_2 by semi-circular, trapezoidal and triangular corrugated beds. Statistical analysis of results for the proportional of y_2/y_1 and Fr_1 provides us with the following equation:

$$\frac{y_2}{y_1} = F_{r1}^{1.078} \quad (3)$$

Figure 4 shows that, the developed equation are closer to Farhad Izadjoo and Shafai-Bajestan M. (2005) than Blanger equation and Ead S. Ali (2007). The Blanger equation is for classical jumps only. The equation of Ead S. Ali (2007) was developed based on the investigation of two relative roughness t/y_1 , but the present equation is developed based on the

investigation of five different values of relative roughness.

In order to show the amount of difference between sequent water depth y_2 and sequent depth of classical jump y_2^* , a dimensionless index D which is defined as:

$$D = \frac{y_2^* - y_2}{y_2^*} \quad (4)$$

Dimensionless index D is computed for all experimental runs and plotted against Fr_1 , as shown in Figure 5. The results indicate that, D is variable with flow discharge and has a maximum value of 0.14, 0.145 and 0.174, for semi-circular, trapezoidal, and triangular corrugated beds, respectively. This mean that, the required tail water depths for jumps over different corrugated beds is respectively, 86%, 85.5%

and 82.6%, of the same variable for jump over smooth bed.

Comparing of the value of D obtained in this study with the same value for type II and III USBR stilling basins which are, respectively 0.17 and 0.21, Peterka (1958). The results obtained by Ead and Rajaratnam (2002) in which the D was equal to 0.25. The results obtained by Farhad Izadjoo and Shafai-Bajestan M. (2005) in which the D was equal to 0.20. One can see that, Triangular shape of corrugation has highly decreased the required tail water depth.

4.2 Jump Length

Figure 6 shows that, the relation between the initial Froude number and the dimensionless length of the jump L_j/y_2 . From this figure, it is noticed that, the length of hydraulic jump over the corrugated beds is smaller than the length of jump over smooth bed. The semi-circular, trapezoidal and triangular corrugated beds reduce the length of jump by 10%, 11% and 14%, respectively. The length of roller jump is found to be depended largely on corrugated beds. For jumps of $F_{r1} \leq 3$, the corrugated bed shapes have small influence on the jump length. Statistical analysis of the results for the L_j and F_{r1} provides us with the following equation:

$$L_j = 3.86 + 6.18 (y_2 - 0.944y_1) \quad (5)$$

Figure 7 shows the comparison between the developed equation and the equations of Hager (1992) and Elevator ski. (1959). The present equation is in good agreement with Elevator ski. (1959).

4.3 Bed Shear Stress

One of the main objectives of installed corrugated bed sheets is to increase the bed shear stress, the sequent water depth and the length of the hydraulic jump thus is reduced. In the present section, the bed shear stress is calculated using the momentum equation as following:

$$F_\tau = (P_1 - P_2) + (M_1 - M_2) \quad (6)$$

where: P_1 , P_2 , M_1 and M_2 are the integrated pressures and momentum fluxes at sections prior and after the hydraulic jump, respectively. Also the shear force index ε can be written as:

$$\varepsilon = \frac{F_\tau}{0.5 \gamma y_1^2} \quad (7)$$

The relation between shear force index ε and F_{r1} is shown in Figure 8, it is apparent that, Shear stresses over the semi-circular, trapezoidal and triangular corrugated beds are almost 8, 9, and 11 times the corresponding stresses at smooth bed. One can see that, the triangular corrugated bed is the best of all considered models.

Statistical analysis of the results data for the relationship between ε and F_{r1} provides us with the following equation:

$$\varepsilon = 4.95 - 3.97 F_{r1} + 0.99 F_{r1}^2 \quad (8)$$

Figure 9 shows that, the developed equation are close to Farhad Izadjoo and Shafai-Bajestan M. (2005).

To illustrate the effect of increasing bed shear stress on the ratio of $\Delta E/E_1$, Figure 10 is plotted in order to show the relation between $\Delta E/E_1$ and F_{r1} . It is noticed that, the relative loss for smooth bed ranges from 10% to 62%, the relative loss for semi-circular, trapezoidal and triangular corrugated beds ranges from 14% to 64%, 15% to 65% and 16% to 66%, respectively. It is worth to mention that, the values of energy dissipation of steady jump in this research agree well with those that determined by Chow (1959). It is noticed that, increasing the bed shear stresses leads to increase the relative loss.

4.4 Maximum Scour Depth

The downstream maximum scour hole depth D_s is dependent on the following independent variables.

$$\frac{D_s}{y_1} = f(F_{r1}, \frac{y_2}{y_1}, \frac{L_j}{y_1}, \frac{t}{y_1}) \quad (9)$$

Figure 11 illustrates the relation between D_s/y_1 , and F_{r1} with respect to the considered corrugated bed sheets. One can see that, for the considered flow conditions, using the semi-circular, trapezoidal and triangular corrugated beds, the scour depth decreases by a range from 22% to 31%, 25% to 34%, and 30% to 36% respectively, in comparison with the maximum scour hole depth of the smooth bed. The considered corrugated beds have small influence on the maximum scour hole depth.

It is important to predict the relative scour depth for the different cases being under investigation. Based on the experimental data and using the statistical methods at presence of the different corrugated beds, several models were proposed and their coefficients were estimated. Out of all trials, the best equation predicting the relative scour depth can be put in the following form.

Figure 12 shows the comparison between the measured relative scour depth (D_s/y_1) and the calculated one using equation 10. It can be noticed that, the predicted data agrees well with the measured one. The developed model of the relative depth equation has been validated through the tested condition. It is worth to mention that, there is an acceptable agreement between the measured data and the calculated ones. The regression statistics have been listed in Table (1).

$$\frac{D_s}{y_1} = -1.51F_{r1} + 0.793\frac{y_2}{y_1} + 0.115\frac{L_j}{y_1} + 2.571\frac{t}{y_1} - 0.330 \quad (10)$$

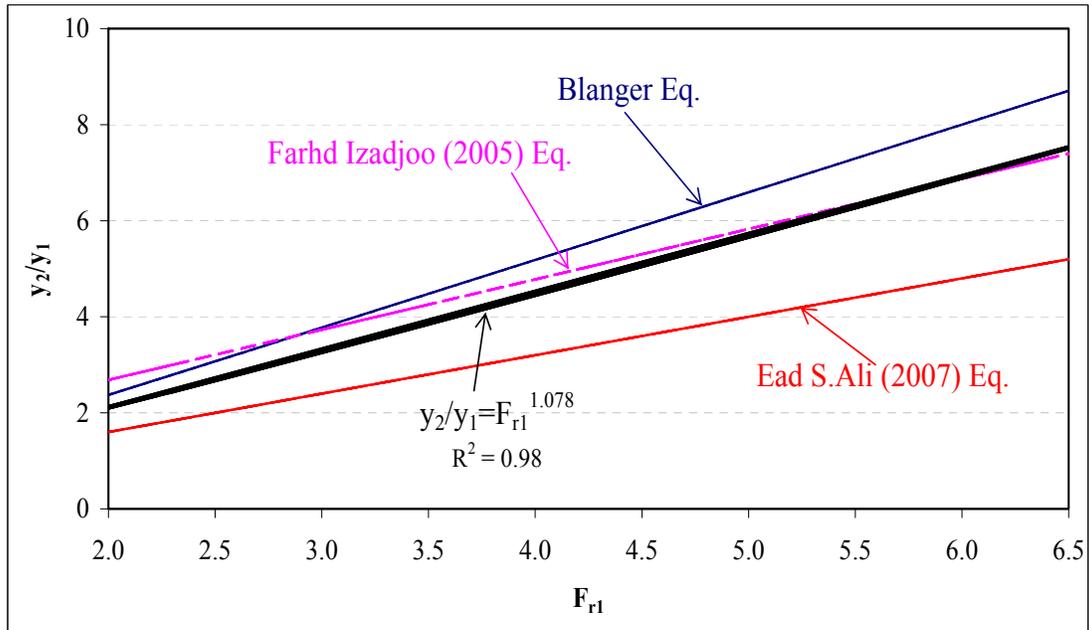


Figure 4. Relation between y_2/y_1 and F_{r1} for classical jumps and jumps over corrugated beds

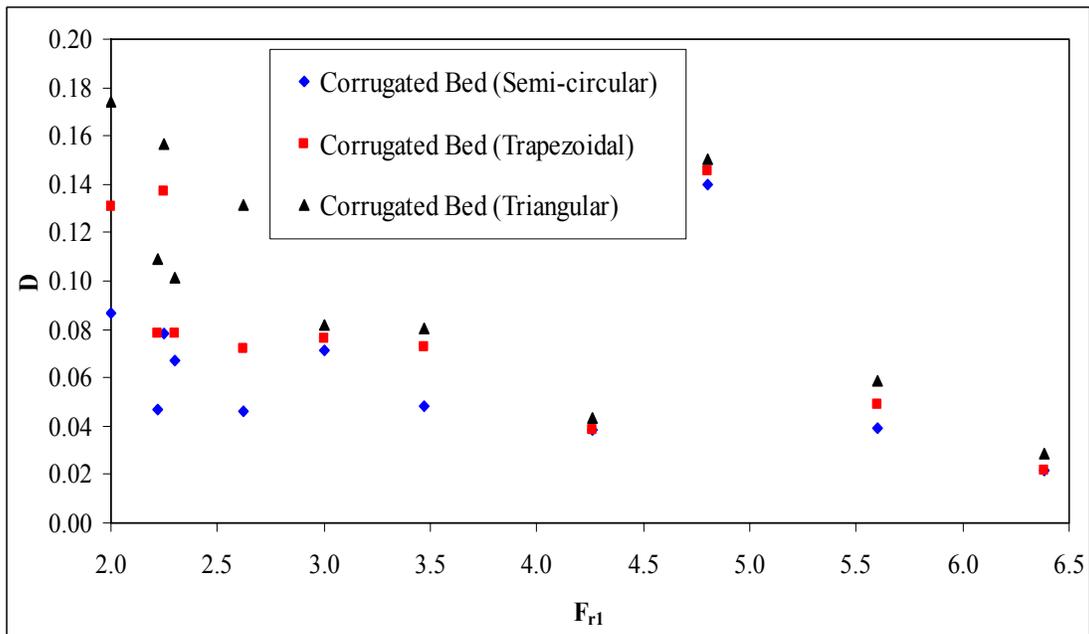


Figure 5. Relation between depth dimensionless index, D and F_{r1}

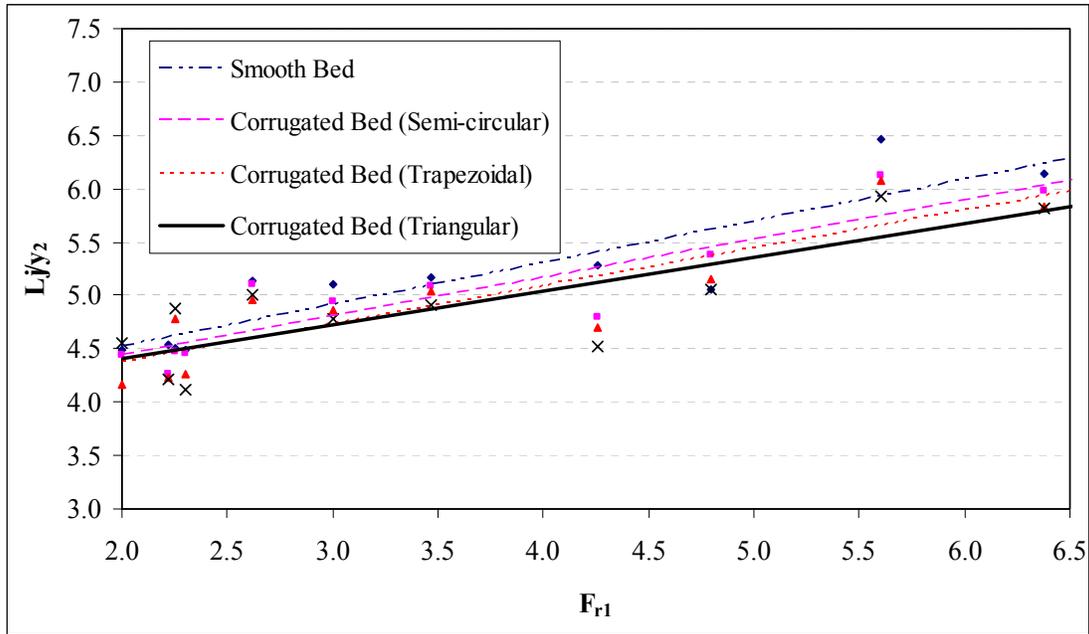


Figure 6. Relation between the ratio of L_j/y_2 and F_{r1}

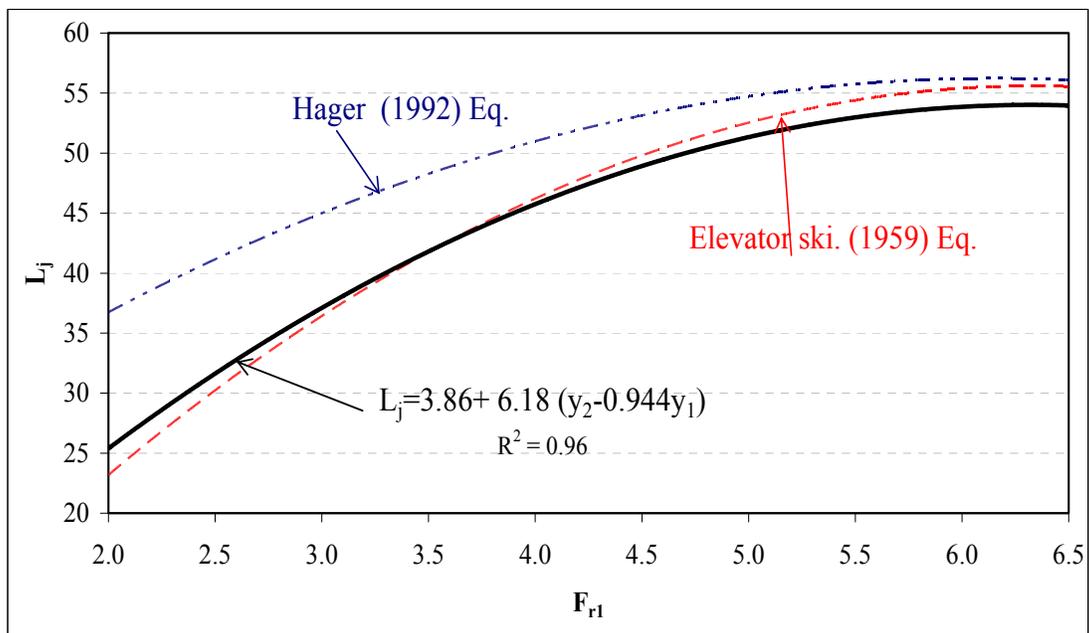


Figure 7. Length of jumps over smoothed beds and over corrugated beds

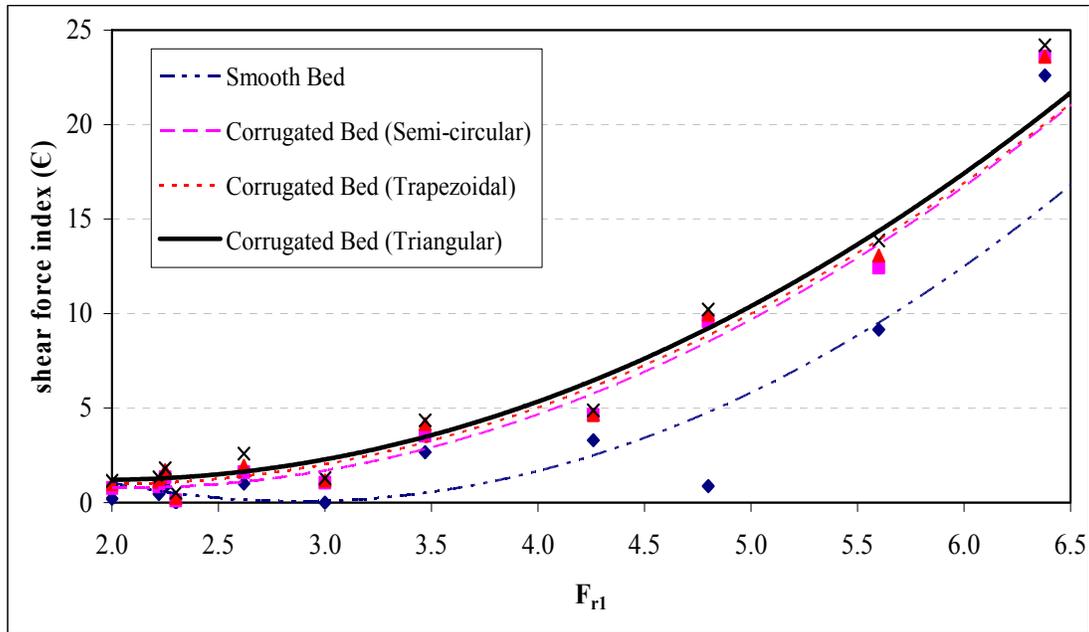


Figure 8. Relation between shear force index and initial Froude number

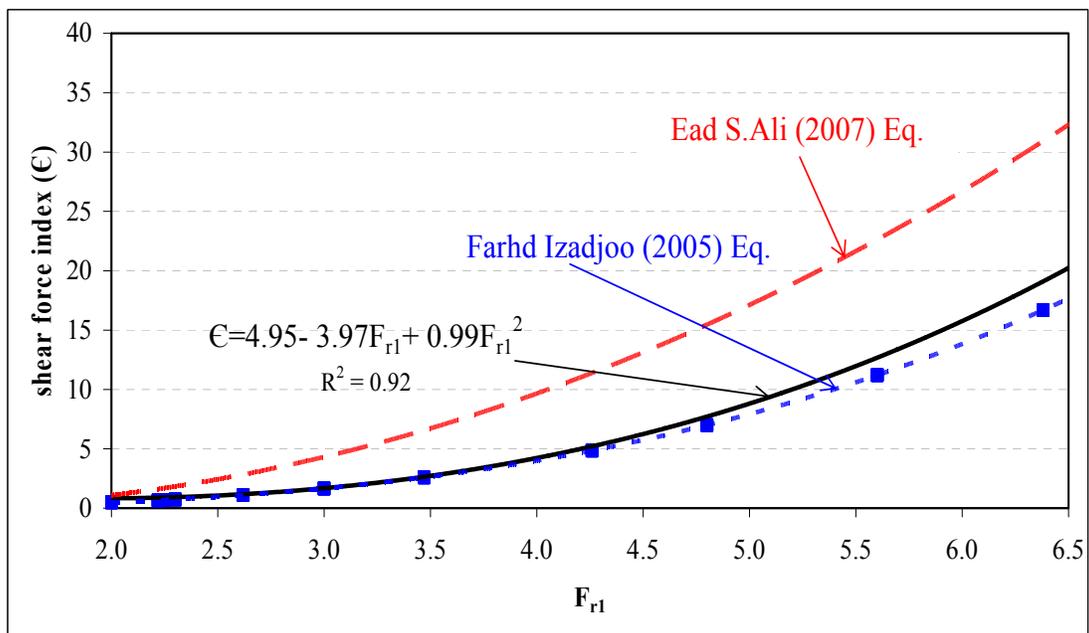


Figure 9. Shear force index for hydraulic jump over corrugated beds

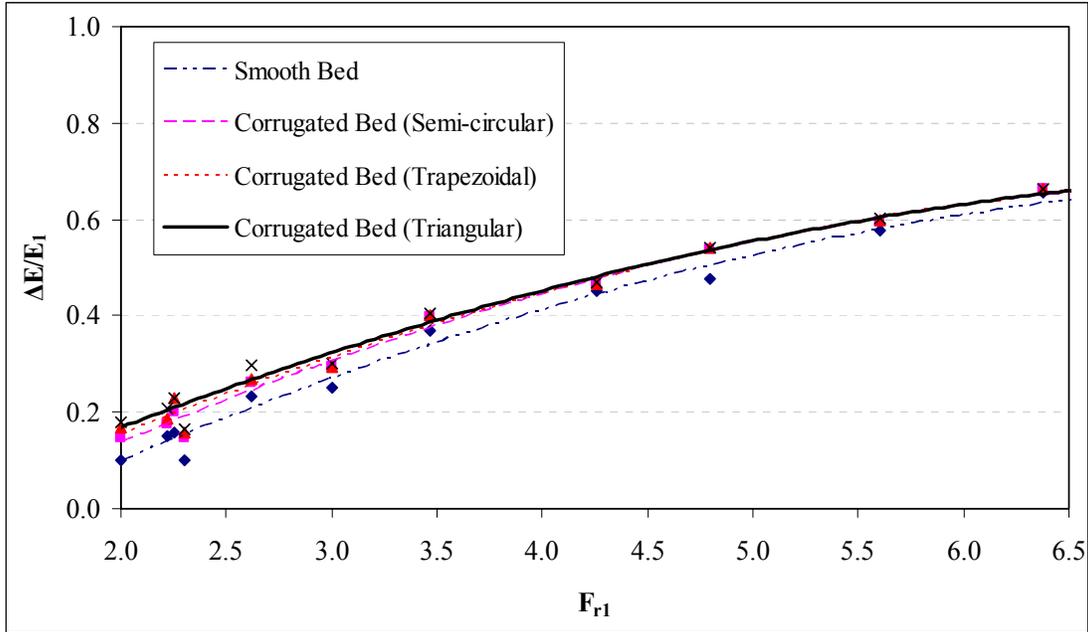


Figure 10. Relation between $\Delta E/E_1$ and F_{r1} for considered corrugated beds

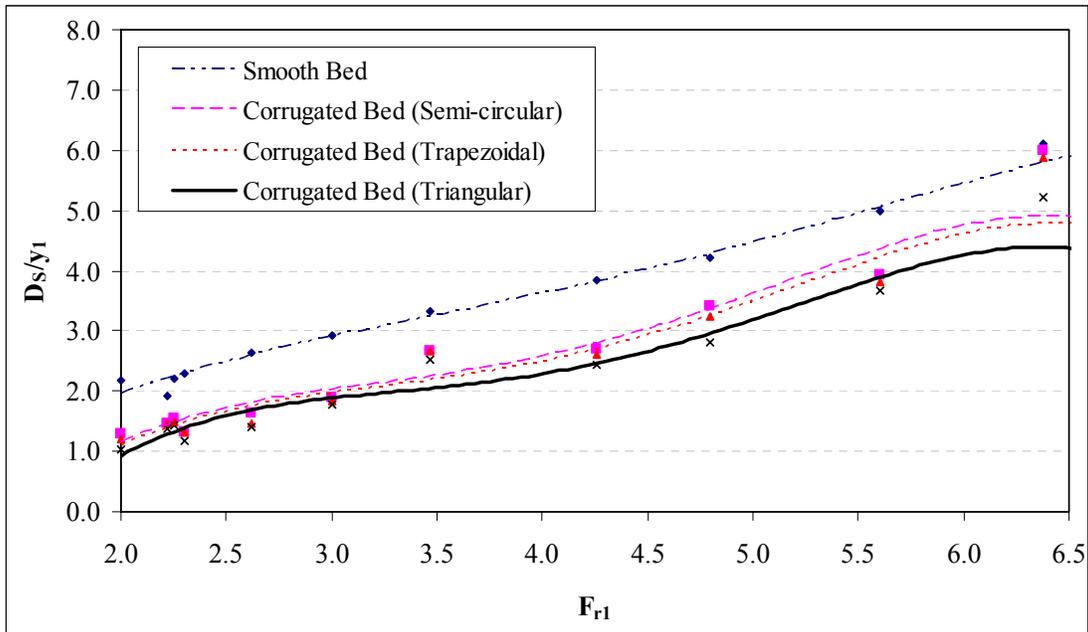


Figure 11. Relation between relative scour depth D_s/y_1 and F_{r1} for the considered corrugated beds

4.5 Maximum Scour Length

The maximum scour length L_s is dependent on the following independent variables:

$$\frac{L_s}{y_1} = f\left(F_{r1}, \frac{y_2}{y_1}, \frac{L_j}{y_1}, \frac{t}{y_1}\right) \quad (11)$$

Figure 13 describes the relation between L_s/y_1 , and F_{r1} with respect to the considered corrugated bed sheets. One can see that, for the considered flow

conditions, using semi-circular, trapezoidal and triangular corrugated beds, the maximum scour length decreases by a range from 17% to 24, 23% to 25% and 24% to 30% respectively, in comparison with the scour length of smooth bed.

As mentioned before, it is important to predict the relative scour length. Based on the experimental data and using the statistical methods, several

models were proposed and their coefficients were estimated. Out of all trials, the best equation predicting the relative scour length can be put in the following form.

$$\frac{L_s}{y_1} = -17.55F_{r1} + 8.05\frac{y_2}{y_1} + 1.19\frac{L_j}{y_1} + 51.79\frac{t}{y_1} \quad (12)$$

Figure 14 shows the comparison between the measured relative scour length (L_s/y_1) and the calculated one using equation 12. It can be noticed that, the predicted data agrees well with the measured one. The developed model of the relative length equation has been validated through the tested condition. It is worth to mention that, there is an acceptable agreement between the measured data and the calculated ones. The regression statistics have been listed in Table (1).

Table (1). The Regression Statistics of Equations 10 and 12

Regression Statistics	Equation (10)	Equation (12)
Coefficient of Multiple Determination (R^2)	0.937418	0.965448
Adjusted coefficient of multiple (Ra^2)	0.931596	0.962234
Standard Error	0.3746	4.011
No. of Observation	36	36

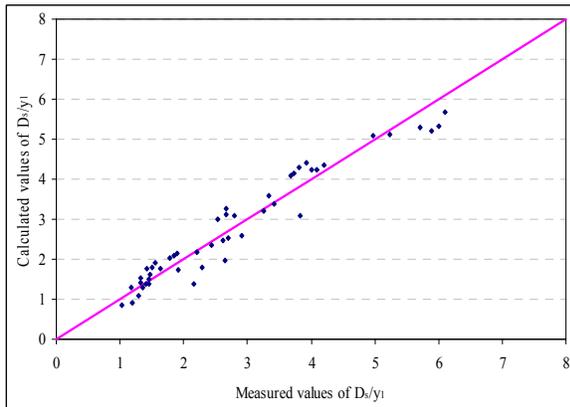


Figure 12. Comparison between calculated and measured values of D_s/y_1

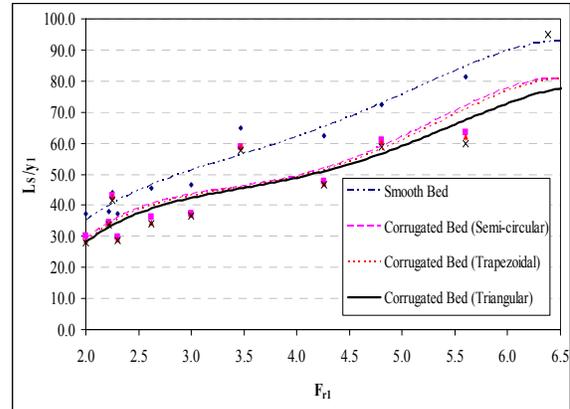


Figure 13. Relation between relative scour length, L_s/y_1 and F_{r1} for the considered corrugated beds

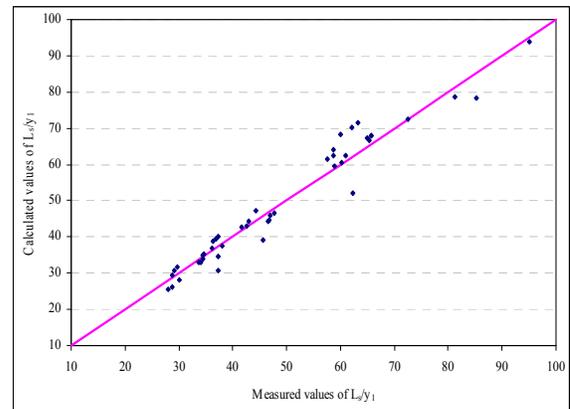


Figure 14. Comparison between calculated and measured values of L_s/y_1

5. CONCLUSIONS

The results of the experimental and statistical study for the hydraulic jump over corrugated beds and downstream local scour have been presented; the discussion and analysis of the results highlighted the following conclusions:

- Corrugated bed sheets increase these eddies and consequently increase the bed shear stress,
- Corrugated bed sheets reduce the sequent depth of hydraulic jump and the results agree well with Farhad Izadjoo and Shafai-Bajestan M. (2005),
- The required tail water depths for jumps over different semi-circular, trapezoidal and triangular corrugated beds is respectively, 86%, 85.5% and 82.6% of the same variable for jump over smooth bed. So, triangular shape of corrugation has highly decreased the required tail water depth,
- The semi-circular, trapezoidal and triangular corrugated beds reduce the length of jump by 10%, 11% and 14%, respectively,

- The length of roller jump is found to be depended largely on corrugated beds,
- For jumps of $F_{r1} \leq 3$, the corrugated bed shapes have small influence on the length of jump,
- Shear stresses over the semi-circular, trapezoidal and triangular corrugated beds are almost 8, 9, and 11 times the corresponding stresses of smooth bed,
- The relative energy loss for smooth bed ranges from 10% to 62%, while, for semi-circular, trapezoidal and triangular corrugated beds ranges from 14% to 64%, 15% to 65% and 16% to 66%, respectively,
- Many parameters affect the scour properties but only, some of these parameters such as Froude number, ratio between conjugate depths and the relative roughness are used to express the scour properties,
- Corrugation beds reduce the maximum scour length by a range from 17% to 30% and reduce the maximum scour depth by a range from 22% to 36% in comparison with the smooth bed,
- Triangular shape is the best corrugated shape among the investigated shapes,
- The results of the present study confirm the effectiveness of corrugated beds for energy dissipation downstream the hydraulic structures.

6. LIST OF ABBREVIATIONS

The following symbols were used through this research:

B	: Channel width
b	: Basin width
D	: Dimensionless index
D_{50}	: Mean size of bed material
D_s	: Maximum scour depth
E_1	: Specific energy at the initial water depth of a hydraulic jump
E_2	: Specific energy at the sequent water depth of a hydraulic jump
ΔE	: Energy losses through a hydraulic jump
F_r	: Tail Froude number
F_{r1}	: Froude number at the initial water depth of a hydraulic jump
F_τ	: Total shear forces
g	: Gravitational acceleration
L_f	: Floor length
L_j	: Length of hydraulic jump
L_s	: Maximum scour length
M_1	: Momentum forces before the jump
M_2	: Momentum forces after the jump

P_1	: Hydrostatic pressure force before the jump
P_2	: Hydrostatic pressure force after the jump
Q	: Flow discharge
R_l	: Reynolds's number
S	: Corrugations wave length
S_o	: Bed slope of the channel
t	: Height of the corrugations
u_1	: Flow velocity before the jump
Y	: Tail water depth
y_1	: Initial water depth of a hydraulic jump
y_2	: Sequent water depth of a hydraulic jump
y_2^*	: Subsequent sub-critical flow for classical jump
ρ	: Density of the fluid
ρ_s	: Density of bed material
C	: Shear force index; and
ν	: Kinematics viscosity of water

7. ACKNOWLEDGEMENTS

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