Location Selection of Gas Stations Using of Fuzzy GTMA and Fuzzy Prioritization Method (Case Study: Tehran Province Gas Company)

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Abstract: This paper presents an integrated fuzzy approach for Location Selection of Gas Stations. In the integrated approach, fuzzy concepts are used for decision-makers' subjective judgments to reflect the vague nature of the selection process. Fuzzy Prioritization Method (FPM) and Fuzzy GTMA are included in the integrated approach. FPM is used to determine the weights of criteria. Fuzzy GTMA aims to rank locations. We apply the integrated approach in Tehran Province Gas Company to demonstrate the application of the proposed method.
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Keywords: Fuzzy Prioritization Method (FPM), Graph theory and matrix approach (GTMA), location selection, Genetic algorithm, Fuzzy set.

1. Introduction

In order to minimize cost and maximize the use of resources, selecting a suitable location has become one of the most important issues for companies. Many potential criteria, such as investment cost, human resources, availability of acquirement material, climate etc., must be considered in selecting a particular plant location (Liang et al.1991).The facility location decision involves organizations seeking to locate, relocate or expand their operations (Ertuğrul et al, 2008). The facility location decision process encompasses the identification, analysis, evaluation and selection among alternatives (Yang and Lee, 1997). Selecting a plant location is a very important decision for firms because they are costly and difficult to reverse, and they entail a long term commitment. And also location decisions have an impact on operating costs and revenues. For instance, a poor choice of location might result in excessive transportation costs, a shortage of qualified labor, lost of competitive advantage, inadequate supplies of raw materials, or some similar condition that would be detrimental to operations (Stevenson, 1993). The conventional approaches for facility location problems like locational cost volume analysis, factor rating, and center of gravity method (Stevenson, 1993) tend to be less effective in dealing with the imprecise or vague nature of the linguistic assessment (Kahraman et al, 2003). In real life, the evaluation data of plant location suitability for various subjective criteria and the weights of the criteria are usually expressed in linguistic terms. And also, to efficiently resolve the ambiguity frequently arising in available information and do more justice to the essential fuzziness in human judgment and preference, the fuzzy set theory has been used to establish an ill defined multiple criteria decision-making problems (Liang, 1999). In this paper, FPM–Fuzzy GTMA integrated approach for location selection of Gas station will be introduced and the implementation process will be explained with a real case. We shall use the FPM method to analyze the structure of location selection problem and determine the weights of criteria and use Fuzzy GTAM method for final ranking. This paper is divided into five sections. In section "Introduction", the studied problem is introduced. Section "Principles of FPM and Fuzzy GTMA" briefly describes the proposed methodology. In section "Proposed FPM-FGTMA integrated approach", proposed FPM–FGTMA integrated approach for location selection is presented and the stages of the proposed approach and steps are determined in detail. A real case is explained in section "The application of proposed approach". In section "Conclusions and future research", conclusions and future research areas are discussed.

2. Principles of FPM and Fuzzy GTMA methods

Before explaining about fuzzy prioritization method, it has been described fuzzy sets and fuzzy numbers as follow:

2.1. Fuzzy sets and fuzzy numbers

Fuzzy set theory, which was introduced by Zadeh (1965) to deal with problems in which a source of vagueness is involved, has been utilized for incorporating imprecise data into the decision framework. A fuzzy set $\widetilde{A}$ can be defined mathematically by a membership function $\mu_{\widetilde{A}}(X)$, which assigns each element x in the universe of discourse X a real number in the interval [0,1]. A triangular fuzzy number $\widetilde{A}$ can be defined by a triplet (a, b, c) as illustrated in Fig 1.

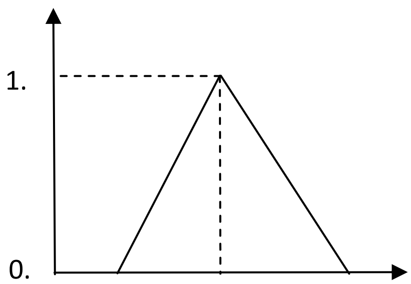

Fig1. A triangular fuzzy number $\widetilde{A}$.

The membership function $\mu_{\widetilde{A}}(X)$ is defined as

$$\mu_{\tilde{A}}(x)=\begin{cases} \frac{x-l}{m-l} & l \le x \le m \\ \frac{x-u}{m-u} & m \le x \le u \\ 0 & otherwise \end{cases} \quad (1)$$

Where l, m, and u are also considered as the lower bound, the mean bound, and the upper bound, respectively. The triangular fuzzy number $\tilde{N}$ is often represented as (l,m,u). According to Table 1, criteria compare with each other.After pairwise comparisons, are finished at a level, a fuzzy reciprocal judgment matrix $\tilde{A}$ can be established as

$$\tilde{A}=\{\tilde{a}_{ij}\}=\begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \tilde{a}_{nn} \end{bmatrix} \quad (2)$$

Table 1. Linguistic variables for important of each criteria

linguistic variables	triangular fuzzy numbers
very low	(0.00,0.00,0.00)
low	(0.10,0.20,0.30)
medium low	(0.20,0.35,0.50)
medium	(0.40,0.50,0.60)
medium high	(0.50,0.65,0.80)
high	(0.70,0.80,0.90)
very high	(0.80,1.00,1.00)

Where n is the number of the related elements at this level, and a_{ij} =1/ a_{ji}. Basic arithmetic operations on triangular fuzzy numbers $A_1 = (l_1,m_1,u_1)$, where $l_1 \le m_1 \le u_1$, and $A_2 = (l_2,m_2,u_2)$, where $l_2 \le m_2 \le u_2$, can be shown as follows:

Addition:

$$A_1 \oplus A_2 = (l_1 + l_2 , m_1 + m_2, u_1 + u_2) \quad (3)$$

Subtraction:

$$A_1 \ominus A_2 = (l_1 - u_2 , m_1 - m_2, u_1 - l_2) \quad (4)$$

Multiplication: if K is a scalar

$$K \otimes A_1 = \begin{cases} (kl_1 , km_1, ku_1), & k > 0 \\ (ku_1 , km_1, kl_1), & k < 0 \end{cases}$$

$$A_1 \otimes A_2 \approx (l_1 l_2 , m_1 m_2, u_1 u_2), \text{ if } l_1 \ge 0 , l_2 \ge 0 \quad (5)$$

$$\text{Division: } A_1 \varnothing A_2 \approx \left(\frac{l_1}{u_2}, \frac{m_1}{m_2}, \frac{u_1}{l_2}\right), \text{ if } l_1 \ge 0 , l_2 \ge 0 \quad (6)$$

Although multiplication and division operations on triangular fuzzy numbers do not necessarily yield a triangular fuzzy number, triangular fuzzy number approximations can be used for many practical applications (Kaufmann and Gupta, 1988). Triangular fuzzy numbers are appropriate for quantifying the vague information about most decision problems including Facility location selection. The primary reason for using triangular fuzzy numbers can be stated as their intuitive and computational-efficient representation (Karsak, 2002). A linguistic variable is defined as a variable whose values are not numbers, but words or sentences in natural or artificial language. The concept of a linguistic variable appears as a useful means for providing approximate characterization of phenomena that are too complex or ill defined to be described in conventional quantitative terms (Zadeh, 1975).

2.2. Fuzzy Prioritization Method (FPM)

Fuzzy prioritization method is describedby Wang et al (2007) as follow: suppose that a fuzzy judgment matrix is constructed as Eq. (2) in a prioritization problem, where n elements are taken into account. Among this judgment matrix A, the triangular fuzzy number a_{ij} is expressed as (l_{ij},m_{ij},u_{ij}), i and j=1,2,…,n, where l_{ij}, m_{ij}, and u_{ij} are the lower bound, the mean bound, and the upper bound of this fuzzy triangular set, respectively. Furthermore, we assume that $l_{ij}<m_{ij}<u_{ij}$ when i≠j. If i=j, then $a_{ij}= a_{ji}$ = (1, 1, 1). Therefore, an exact priority vector $w = (w_1, w_2,\ldots,w_n)^T$ derived from A must satisfy the fuzzy inequalities:

$$l_{ij} \lesssim \frac{w_i}{w_j} \lesssim m_{ij} \quad (7)$$

Where $w_i > 0$, $w_j > 0$, $i \neq j$, and the symbol $\lesseqgtr$ means "fuzzy less or equal to".To measure the degree of satisfaction for different crisp ratios w_i/ w_j with regard to the double side inequality (7), a function can be defined as:

$$\mu_{ij}\left(\frac{w_i}{w_j}\right)= \begin{cases} \frac{m_{ij}-(w_i/w_j)}{m_{ij}-l_{ij}} & 0 < \frac{w_i}{w_j} \leq m_{ij} \\ \frac{(w_i/w_j)-m_{ij}}{u_{ij}-m_{ij}} & , \frac{w_i}{w_j} > m_{ij} \end{cases} \quad (8)$$

Where $i \neq j$. Being different from the membership function (1) of triangular fuzzy numbers, the function value of μ_{ij} (w_i/ w_j) may be larger than one, and is linearly decreasing over the interval $(0,m_{ij}]$ and linearly increasing over the interval $[m_{ij},\infty)$, as shown in Fig. 2. The less value of μ_{ij} (w_i/ w_j) indicates that the exact ratio w_i/ w_j is more acceptable.

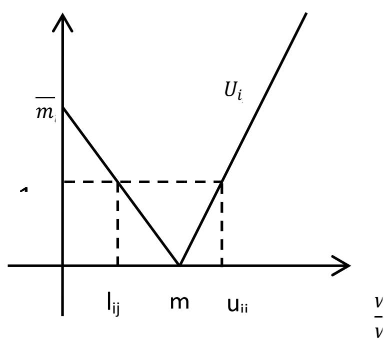

Fig 2. Function for measuring the satisfaction degree of w_i/ w_j

To find the solution of the priority vector $(w_1,w_2,\ldots,w_n)^T$, the idea is that all exact ratios w_i/ w_j should satisfy n(n-1) fuzzy comparison judgments (l_{ij},m_{ij},u_{ij}) as possible as they can, i and j=1,2,…,n, $i \neq j$. Therefore, in this study, the crisp priorities assessment is formulated as a constrained optimization problem:

Min J $(w_1 ,w_2 ,\ldots,w_n)$

$$= \min \sum_{i=1}^{n} \sum_{j=1}^{n} \left[m_{ij}\left(\frac{w_i}{w_j}\right)\right]$$

$$= \min \sum_{i=1}^{n} \sum_{j=1}^{n} \begin{bmatrix} \delta\left(m_{ij}-\frac{w_i}{w_j}\right)\left(\frac{m_{ij}-(w_i/w_j)}{m_{ij}-l_{ij}}\right)^P \\ +\delta\left(\frac{w_i}{w_j}-m_{ij}\right)\left(\frac{(w_i/w_j)-m_{ij}}{u_{ij}-m_{ij}}\right)^P \end{bmatrix}$$

Subject to

$\sum_{k=1}^{n} w_k = 1$, $w_k>0$, k=1,2,…,n.

Where $i \neq j$,P ϵ N , and

$$\delta(x) = \begin{cases} 0 & , x < 0 \\ 1 & , x \geq 0 \end{cases} \quad (9)$$

The power index P is fixed, and chosen by decision makers in a specific MCDM problem. A larger p is suggested, e.g. 10, as illustrated briefly in the next section. The function J $(w_1,w_2,\ldots,w_n)$ is non-differentiable. In some cases, decision-makers are unable or unwilling to give all pairwise comparison judgments of n elements. However, provided that the known fuzzy set of pairwise comparisons involves n elements, such as $F=\{a_{ij}\} =\{a_{12} ,a_{13} ,\ldots,a_{n1}\}$ or $\{a_{21} ,a_{31} ,\ldots,a_{n1}\}$, the solution of priority vector $(w_1,w_2,\ldots,w_n)^T$ will be still able to be derived based on the optimization problem above. In order to measure the consistency degree of the fuzzy comparison judgment matrix A as Eq. (2), an index γ can be defined after the optimal crisp priority vector $(w_1^*, w_2^*, \ldots, w_n^*)^T$is obtained:

$$\gamma = \exp\left\{-\max\left\{\mu_{ij}\left(\frac{w_i^*}{w_j^*}\right)\middle| i, j = 1,2,\ldots,n, i \neq j\right\}\right\}$$
(10)

Where $\mu_{ij}(w_i^*/w_j^*)$is the function of (8). The value of γ satisfies $0 < \gamma \leq 1$ always. If it is larger than $e^{-1}=0.3679$, all exact ratios satisfy the crisp inequalities $l_{ij} \leq w_i^*/w_j^* \leq m_{ij}$, i and j=1,2,…,n, $i \neq j$, and the corresponding fuzzy judgment matrix has good consistency. γ=1 indicates that the fuzzy judgment matrix is completely consistent. In conclusion, the fuzzy judgment matrix with a larger γ value is more consistent. For solving this optimization problem that has non-linear constraints, we used the genetic algorithm. The next section briefly describes the basics of the Genetic Algorithms (Rao, 2007).

2.3. Genetic Algorithms

Over the last decade, genetic algorithms (GAs) have been extensively used assearch and optimization tools in various problem domains, including the sciences, commerce, and engineering. The primary reasons for their success are their broad applicability, ease of use, robustness and global perspective (Goldberg 1989; Mitchell, 1996; Gen and Cheng, 1997; Vose, 1999; Deb, 2002). The genetic algorithms are inspired by Darwin's theory evolution. The algorithm is started with a set of solution (represented by chromosomes) called a population. Solutions from one population are used to form a new population. This is motivated by that thenew population will be better than the old one. Solutions to forming new solutions (offsprings) are selected according to their fitness. The more suitable they are, the more chances they have of reproducing. The iteration is stopped after the completion of maximal number of iterations (generations) or on the attainment ofthe best result.The decision variables of multiple objective, multiple variable, constrained or unconstrained optimization problems solved by GAs may be represented by either binary coding or real coding. GAs employ three important genetic operators for solving optimization problems, and these operators are briefly described below.

Reproduction or selection operator: GA begins with a set of solutions calledpopulation (represented by chromosomes or strings). The primary objective of their production operator is to make duplicates of good solutions, and eliminate bad solutions in a population, while keeping the population size constant. This is achieved by identifying good solutions in a population, making multiple copies of good solutions, and eliminating bad solutions from the population so that multiple copies of good solutions can be placed in the population.

Crossover operator: This operator is applied to the strings of the mating poolafter the reproduction operator has been applied. The latter cannot create any new solutions in the population, and it only makes more copies of good solutions at the expense of not-so-good solutions. The creation of new solutions is performed by the crossover operator. In crossover operation, two strings are randomly selected from the mating pool, and some portions of the strings are exchanged between strings to create new strings.

Mutation operator: The crossover operator is mainly responsible for the search aspect of genetic algorithms, even though the mutation operator is also used for this purpose. Mutation is intended to prevent all solutions in the population being concentrated into a local optimum of the solved problem. The bit wise mutation operator changes a 1 into 0, and vice versa, with a small mutation probability. The need for mutation is to maintain diversity in the population.

The three GA operators reproduction or selection, crossover, and mutation, aresimple and straight-forward. The reproduction operator selects good strings, while the crossover operator recombines good substrings from two good strings to hopefully form a better spring. The mutation operator alters a string locally to hopefully create a better string. The basic genetic algorithm is outlined below:

1. [Start] Choose a coding to represent problem decision variables, a reproduction or selection operator, a crossover operator, and a mutation operator. Choose population size n, crossover probability p_c, and mutation probability p_m. Initialize a random population of strings of size‘s’. Choose a maximum allowable generation (*i.e.*, iteration) number t_{max}. Set t=0
2. [Fitness] Evaluate the fitness function of each string in the population
3. [New population] Create a new population by repeating the following steps until the new population is complete

[Reproduction or selection] Select two parent strings from a population according to their fitness (the better fitness, the bigger the chance of being selected)

[Crossover] Crossover the parents to form new offspring (children). If no crossover is performed, then the offspring are the exact copy of parents.

[Mutation] Mutate the new offspring at each locus (position in string).

[Accepting] Place the new offspring in the new population

4. [Replace] Use the newly generated population for a further run of the algorithm
5. [Test] If $t > t_{max}$, or other termination criteria, are satisfied, then terminate and return the best solution in current population
6. [Loop] Go to step 2

The above procedure is repeated until an optimum solution is reached. More details on the genetic algorithms and their applications can be found in literature (Goldberg 1989, Mitchell 1996, Gen and Cheng 1997, Vose 1999, Deb 2002).

2.4. The GTMA method

Graph theory is a logical and systematic approach. The advanced theory of graphs and its applications are very well documented. Rao (2007) in his book presents this methodology and shows some of its applications. Graph/digraph model representations have proved to be useful for modeling and analyzing various kinds of systems and problems in numerous fields of science and technology (Darvish et al, 2009). The matrix approach is useful in analyzing the graph/digraph models expeditiously to derive the system function and index to meet the objectives (Rao, 2007). The graph theory and matrix methods consist of the digraph representation, the matrix representation and the permanent function representation. The digraph is the visual representation of the variables and their inter dependencies. The matrix converts the digraph into mathematical form and the permanent function is a mathematical representation that helps to determine the numerical index (Faisal, 2007).

The step by step explanation of the methodology is as follows:

Step 1. Identifying equipment selection attributes. In this step all the criteria which affect the decision is determined. This can be done by using relevant criteria available in the literature or getting information from the decision maker.

Step 2. Determine equipment alternatives. All potential alternatives are identified.

Step 3. Graph representation of the criteria and their inter dependencies. Equipment selection criterion is defined as a factor that influences the selection of an alternative. The equipment selection criteria digraph models the alternative selection criteria and their inter relationship. This digraph consists of a set of nodes N = {n_i}, with i = 1, 2,...,M and a set of directed edges E = {e_{ij}}. A node n_i represents i-th alternative selection criterion and edges represent the relative importance among the criteria. The number of nodes M considered is equal to the number of alternative selection criteria considered. If a node 'i' has relative importance over another node 'j' in the alternative selection, then a directed edge or arrow is drawn from node i to node j (i.e. e_{ij}). If 'j' has relative importance over 'i' directed edge or arrow is drawn from node j to node i (e_{ji}) (Rao, 2007).

Step 4. Develop equipment selection criteria matrix of the graph. Matrix representation of the alternative selection criteria digraph gives one-to-one representation. A matrix called the equipment selection criteria matrix. This is an M in M matrix and considers all of the criteria (i.e. A_i) and their relative importance (i.e. a_{ij}). Where A_i is the value of the i-th criteria represented by node n_i and a_{ij} is the relative importance of the i-th criteria over the j-th represented by the edge e_{ij} (Rao, 2007 & Faisal et al, 2007).

The value of A_i should preferably be obtained from available or estimated data. When quantitative values of the criteria are available, normalized values of a criterion assigned to the alternatives are calculated by v_i/v_j, where v_i is the measure of the criterion for the i-th alternative and v_j is the measure of the criterion for the j-th alternative which has a higher measure of the criterion among the considered alternatives. This ratio is valid for beneficial criteria only. A beneficial criteria means its higher measures are more desirable for the given application. Whereas, the non-beneficial criterion is the one whose lower measures are desirable and the normalized values assigned to the alternatives are calculated by v_j/v_i.

Step 5. Obtaining alternative selection criteria function for the matrix. The permanent of this matrix, is defined as the alternative selection criteria function. The permanent of a matrix was introduced by Cauchy in 1812. At that time, while developing the theory of determinants, he also defined a certain subclass of symmetric functions which later Muir named permanents (Nourani, 1999). The permanent is a standard matrix function and is used in combinatorial mathematics (Faisal, 2007 & Rao, 2006). The permanent function is obtained in a similar manner as the determinant but unlike in a determinant where a negative sign appears in the calculation, in a variable permanent function positive signs replace these negative signs (Faisal, 2007 & Rao, 2006). Application of the permanent concept will lead to a better appreciation of selection attributes. Moreover, using this no negative sign will appear in the expression (unlike determinant of a matrix in which a negative sign can appear) and hence no information will be lost (Rao, 2006).

$$\text{CS Matrix} = \begin{bmatrix} A_1 & a_{12} & a_{13} & a & a & a_{1.m} \\ a_{21} & A_2 & a_{23} & \cdots & \cdots & a_{2.m} \\ a_{31} & a_{32} & A_3 & \cdots & \cdots & a_{3.m} \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \ddots & \cdots & \cdots & \cdots & \vdots \\ a_1 & a_1 & a_1 & \cdots & \cdots & A_m \end{bmatrix} \quad 11)$$

The per (CS) contains terms arranged in (M + 1) groups, and these groups represent the measures of criteria and the relative importance loops. The first group represents the measures of M criteria. The second group is absent as there is no self-loop in the digraph. The third group contains 2- criterion relative importance loops and measures of (M-2) criteria. Each term of the fourth group represents a set of a 3-criterion relative importance loop, or its pair, and measures of (M-3) criteria. The fifth group contains two sub-groups. The terms of the first sub-group is a set of two 2-criterion relative importance loops and the measures of (M-4) criteria. Each term of second sub-group is a set of a 4-attribute relative importance loop, or its pair, and the measures of (M-4) criteria. The sixth group contains two subgroups. The terms of the first sub-group are a set of a 3-criterion relative importance loop, or its pair, and 2-criterion importance loop and the measures of (M-5) criteria. Each term of the second sub-group is a set of a 5-criterion relative importance loop, or its pair, and the measures of (M-5) criteria. Similarly other terms of the equation are defined. Thus, the CS fully characterizes the considered alternative selection evaluation problem, as it contains all possible structural components of the criteria and their relative importance. It may be mentioned that this equation is nothing but the determinant of an M-M matrix but considering all the terms as positive.

Step 6. Evaluation and ranking of the alternatives, in this step all alternatives are ranked according to their permanent values calculated in the previous step.

$$\begin{aligned}
\mathrm{per}\,(Cs) = {} & \prod_{i=1}^{M} A_i + \sum_{i=1}^{M-1}\sum_{j=i+1}^{M} \dots \sum_{M=t+1}^{M} (a_{ij}a_{ji})\, A_k A_l A_m A_n A_o \dots A_t A_M \\
& + \sum_{i=1}^{M-2}\sum_{j=i+1}^{M-1}\sum_{k=i+1}^{M} \dots \sum_{M=t+!}^{M} (a_{ij}a_{jk}a_{ki} + a_{ik}a_{kj}a_{ji}) A_l A_m A_n A_o \dots A_t A_M \\
& + \sum_{i=1}^{M-3}\sum_{j=i+1}^{M}\sum_{k=i+1}^{M-1}\sum_{l=i+2}^{M} \dots \sum_{M=t+1}^{M} (a_{ij}a_{ji} + a_{kl}a_{lk}) A_m A_n A_o \dots A_t A_M \\
& + \sum_{i=1}^{M-3}\sum_{j=i+1}^{M}\sum_{k=i+1}^{M-1}\sum_{l=i+2}^{M} \dots \sum_{M=t+1}^{M} (a_{ij}a_{jk}a_{kl}a_{li} + a_{il}a_{lk}a_{kj}a_{ji}) A_m A_n A_o \dots A_t A_M + \\
& \dots\dots \\
& \sum_{i=1}^{M-2}\sum_{j=1}^{M-1}\sum_{j=i+1}^{M}\sum_{l=1}^{M-1}\sum_{m=l+1}^{M-2}\sum_{m=t+1}^{M} (a_{ij}a_{jk}a_{ki} + a_{ik}a_{kj}a_{ji})(a_{lm}a_{m}) A_n A_o \dots A_t A_m + \\
& \sum_{i=1}^{M-4}\sum_{j=i+1}^{M-1}\sum_{k=j+1}^{M}\sum_{l=1}^{M}\sum_{m=l+1}^{M} \dots\dots \sum_{M=t+1}^{M} (a_{ij}a_{jk}a_{kl}a_{lm}a_{mi} + a_{im}a_{mj}a_{lk}a_{kj}a_{ji}) A_n A_o \dots\dots A_t A_m + \\
& \sum_{i=1}^{M-3}\sum_{j=i+1}^{M-1}\sum_{k=j+1}^{M}\sum_{l=1}^{M}\sum_{m=l+1}^{M-1}\sum_{n=m+1}^{M} \dots\dots \sum_{M=t+1}^{M} (a_{ij}a_{jk}a_{ki} + a_{ik}a_{kj}a_{ji})(a_{lm}a_{mn}a_{nl} \\
& \qquad + a_{ln}a_{nm}a_{ml}) A_o \dots\dots A_t A_m \\
& + \sum_{i=1}^{M-5}\sum_{j=i+1}^{M-1}\sum_{k=j+1}^{M}\sum_{l=1}^{M-2}\sum_{m=l+1}^{M-1}\sum_{n=m+1}^{M} \dots\dots \sum_{M=t+1}^{M} (a_{ij}a_{jk}a_{ki} + a_{ik}a_{kj}a_{ji}) + \\
& (a_{lm}a_{mn}a_{nl} + a_{ln}a_{nm}a_{ml}) A_o \dots A_t A_M \\
& + \sum_{i=1}^{M-5}\sum_{j=i+1}^{M-1}\sum_{k=j+1}^{M}\sum_{l=1}^{M}\sum_{m=l+1}^{M}\sum_{n=m+1}^{M} \dots\dots \sum_{M=t+1}^{M} (a_{ij} + a_{jk}a_{kl}a_{lm}a_{mn}a_{nj} + a_{in}a_{nm}a_{ml}a_{lk}a_{kj}a_{ji}) A_o \dots A_t A_M
\end{aligned}$$

(12)

3. Proposed FPM-FGTMA integrated approach

The integrated approach, composed of FPM and Fuzzy GTMA methods, for location selection problem consists of 3 basic stages: (1) Data gathering, (2) FPM computations,(3) fuzzy GTMA computations. In the first stage, alternative equipment and the criteria which will be used in their evaluation are determinedand the decision hierarchy is formed. After determining the decision hierarchy, criteria used in location selection are assigned weights using FPM in thesecond stage. In this phase, pairwise comparison matrices are formed to determine the criteria weights. The experts make individual evaluations using the scale, provided in Table 1, to determine the values ofthe elements of pairwise comparison matrices. Computing the geometric mean of the values obtained from individual evaluations, a final pairwise comparison matrix on which there is a consensus is found. The weights of the criteria are calculated based on this final comparison matrix. In the next step of this phase, according to final comparison matrix, optimization problem is formed and this optimization problem will solve using of Genetic algorithm and the weights of criteria are determined. Location priorities are found by using fuzzy GTMA computations in the third stage.

4. The application of proposed approach

In this section, we demonstrate the application of the proposed method in Tehran Province Company. This Company desires to find a new location and it has ten alternatives (from A_1 to A_{10}). First of all, a committee of decision-makers is formed. There are ten decision-makers in the committee. Then evaluation criteria are determined as Gain the ground for establishment of station (C_1), The distance of stations from each other (C_2), Proximity to the high pressure supply lines (C_3), Establishment of station in wide streets(C_4), Availability in crisis times (C_5) and Distance from residential areas due to noise pollution (C_6).

4.2. FPM calculations

After forming the decision hierarchy for location selection problem, the criteria to be used in evaluation process are assigned weights by using FPM method. In this phase, the experts are given the task of forming individual pairwise comparison matrix by using the scale given in Table 1. Geometric means of these values are found to obtain the pairwise compassion matrix on which there is a consensus (Table 2).

Table 2. Fuzzy comparison matrix

	C_1	C_2	C_3	…	C_5	C_6
C_1	(1.00,1.00,1.00)	(0.67,0.82,0.90)	(0.60,0.77,0.87)	…	(0.37,0.50,0.63)	(0.77,0.93,0.97)
C_2	(0.87,1.26,1.56)	(1.00,1.00,1.00)	(0.70,0.88,0.93)	…	(0.57,0.72,0.80)	(0.73,0.87,0.93)
C_3	(1.17,1.36,1.75)	(1.08,1.18,1.50)	(1.00,1.00,1.00)	…	(0.63,0.77,0.83)	(0.50,0.65,0.80)
C_4	(1.07,1.17,1.37)	(1.04,1.08,1.31)	(1.17,1.36,1.75)	…	(0.80,1.00,1.00)	(0.73,0.87,0.93)
C_5	(1.04,1.08,1.31)	(1.17,1.36,1.75)	(1.26,1.42,1.73)	…	(1.00,1.00,1.00)	(0.63,0.77,0.83)
C_6	(1.04,1.08,1.31)	(1.07,1.17,1.37)	(1.25,1.54,2.00)	…	(1.26,1.42,1.73)	(1.00,1.00,1.00)

After that we formulate the fuzzy comparison matrix as a constrained optimization problem and we solve this optimization problem using of Genetic algorithm. In order to employ Genetic algorithm, we use the MATLAB toolbox.Some settings that are used: Population Size equal to 100, the number of direct transfer to the next generation (Elite count) equal to 2, crossover fraction equal to 0.8 and the stopping conditions are described as follow: transfer from 100 generation and a lack of improvement in 50 generation. The results obtained from solving optimization problem using of Genetic algorithm are presented in Table 3.

Table 3.The weight of criteria

W_1	W_2	W_3	W_4	W_5	W_6
0.102316	0.05036	0.178527	0.077652	0.474809	0.11565

The establishments of station in wide streets, proximity to the high pressure supply lines and distance from residential areas due to noise pollution are determined as the three most important criteria in location selection processby FPM.

4.3. Fuzzy GTMA calculations

The weights of the criteria are calculated by FPM up to now, and then these values can be used in Fuzzy GTMA. After calculating the weights, we formed the fuzzy decision matrix of GTMA and after that we normalized the Fuzzy decision matrix of GTMA that shows in Table 4.

Table 4.Decision matrix of Fuzzy GTMA

	C_1	C_2	C_3	…	C_5	C_6
A_1	(0.28,0.43,0.55)	(0.87,0.80,0.90)	(0.25,0.35,0.50)	…	(0.87,0.80,0.90)	(0.50,0.50,0.60)
A_2	(0.00,0.00,0.22)	(0.50,0.50,0.50)	(1.00,1.00,1.00)	…	(0.12,0.20,0.30)	(0.50,0.50,0.60)
A_3	(0.57,0.62,0.66)	(0.25,0.35,0.50)	(0.62,0.65,0.80)	…	(0.62,0.65,0.80)	(0.00,0.00,0.20)
A_4	(0.28,0.43,0.55)	(0.62,0.65,0.80)	(0.12,0.20,0.30)	…	(0.25,0.35,0.50)	(0.62,0.65,0.80)
A_5	(0.14,0.25,0.33)	(1.00,1.00,1.00)	(0.50,0.50,0.60)	…	(0.50,0.50,0.60)	(0.00,0.00,0.20)
A_6	(0.71,0.81,0.88)	(0.50,0.50,0.60)	(0.87,0.80,0.90)	…	(0.12,0.20,0.30)	(0.87,0.80,0.90)
A_7	(1.00,1.00,1.00)	(0.25,0.35,0.50)	(0.62,0.65,0.80)	…	(0.25,0.35,0.50)	(1.00,1.00,1.00)
A_8	(0.28,0.43,0.55)	(0.12,0.20,0.30)	(0.00,0.00,0.20)	…	(0.62,0.65,0.80)	(0.50,0.50,0.60)
A_9	(0.57,0.62,0.66)	(0.62,0.65,0.80)	(0.87,0.80,0.90)	…	(1.00,1.00,1.00)	(0.00,0.00,0.20)
A_{10}	(1.00,1.00,1.00)	(0.12,0.20,0.30)	(0.87,0.80,0.90)	…	(0.87,0.80,0.90)	(0.25,0.35,0.50)

In Fuzzy GTMA method, we carry out pair-wise comparison with respect to their weight that shows in Table 5.

Table 5.Pair-wise comparison of criteria with respect to each other

	C_1	C_2	C_3	C_4	C_5	C_6
C_1		0.670	0.364	0.569	0.177	0.469
C_2	0.330		0.220	0.393	0.096	0.303
C_3	0.636	0.780		0.697	0.273	0.607
C_4	0.431	0.607	0.303		0.141	0.402
C_5	0.823	0.904	0.727	0.859		0.804
C_6	0.531	0.697	0.393	0.598	0.196	
w_j	0.102	0.05	0.179	0.078	0.475	0.116

Because in Fuzzy GTMA method our decision matrix is fuzzy, we should obtain the fuzzy permanent matrix for each criterion. For example, for calculating fuzzy permanent matrix for A_1, first we should obtain the permanent matrix with the lower bound of fuzzy decision matrix as well as we should obtain the permanent matrix with the mean bound and the upper bound that show from Table 6 to Table 8.

Table 6.Pair-wise comparison of criteria with respect to A_1 with the lower bound of fuzzy decision matrix

A_1-l	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0.286	0.670	0.364	0.569	0.177	0.469
C_2	0.330	0.875	0.220	0.393	0.096	0.303
C_3	0.636	0.780	0.250	0.697	0.273	0.607
C_4	0.431	0.607	0.303	0.875	0.141	0.402
C_5	0.823	0.904	0.727	0.859	0.875	0.804
C_6	0.531	0.697	0.393	0.598	0.196	0.500

Table 7. Pair-wise comparison of criteria with respect to A_1 with the mean bound of fuzzy decision matrix

A_1-m	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0.438	0.670	0.364	0.569	0.177	0.469
C_2	0.330	0.800	0.220	0.393	0.096	0.303
C_3	0.636	0.780	0.350	0.697	0.273	0.607
C_4	0.431	0.607	0.303	0.800	0.141	0.402
C_5	0.823	0.904	0.727	0.859	0.800	0.804
C_6	0.531	0.697	0.393	0.598	0.196	0.500

Table 8.Pair-wise comparison of criteria with respect to A_1 with the upper bound of fuzzy decision matrix

A_1-u	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0.556	0.670	0.364	0.569	0.177	0.469
C_2	0.330	0.900	0.220	0.393	0.096	0.303
C_3	0.636	0.780	0.500	0.697	0.273	0.607
C_4	0.431	0.607	0.303	0.900	0.141	0.402
C_5	0.823	0.904	0.727	0.859	0.900	0.804
C_6	0.531	0.697	0.393	0.598	0.196	0.600

The permanent matrix for Table 6, Table 7 and Table 8 are 7.7899, 7.9039 and 10.1868. According to this method the fuzzy permanent matrix for A_1 is (7.7899, 7.9039, 10.1868)

After that we obtain the fuzzy permanent matrix of all alternatives that shows in Table 9.

Table 9. The fuzzy permanent matrix

Alternative	Fuzzy permanent matrix
A_1	(7.7899, 7.9039, 10.1868)
A_2	(4.65560, 4.8762, 6.2865)
A_3	(4.18790, 4.5214, 6.4050)
A_4	(4.7910, 5.64320, 7.7352)
A_5	(6.0805, 6.3377, 7.73520)
A_6	(5.32620, 5.5082, 7.22910)
A_7	(6.59020, 7.4056, 9.4466)
A_8	(4.63020, 4.9934, 6.7364)
A_9	(6.8177, 7.1140, 9.10550)
A_{10}	(7.24110, 7.3763, 9.1772)

In the next step, by using of extent analysis method, we obtain the crisp permanent matrix and we rank locations based on crisp permanent matrix. Finally, we rank all locations with respect to their permanent matrix that shows in Table 10.

Table 10. Ranking of locations

Alternative	Crisp Permanent matrix	rank
A_1	0.293045	1
A_2	0.036202	5
A_3	0.020108	6
A_4	0.007266	9
A_5	0.009106	8
A_6	0.001895	10
A_7	0.225284	2
A_8	0.011683	7
A_9	0.183106	4
A_{10}	0.212304	3

According to Table 10, A_1 is the best location among other locations and other locations of Gas stations ranked as follow:
$A_1 > A_7 > A_{10} > A_9 > A_2 > A_3 > A_8 > A_5 > A_4 > A_6$.

5. Conclusions

Location selection is the determination of a geographic site for a firm's operations. The facility location decision involves organizations seeking to locate, relocate or expand their operations. The facility location decision process encompasses the identification, analysis, evaluation and selection among alternatives. In this paper, fuzzy prioritization method and fuzzy GTMA are combined that fuzzy GTMA uses FPM result weights as input weights. Then a real case study is presented to show applicability and performance of the method. It can be said that using linguistic variables makes the evaluation process more realistic. Because evaluation is not an exact process and has fuzziness in its body. Here, the usage of FPM weights in fuzzy GTMA makes the application more realistic and reliable. As a future direction, other decision-making methods such as fuzzy ELECTRE, fuzzy similarity-based approach can be used in this area.

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