Allometric Scaling Analysis of Metropolitan Areas of Iran

Rasol heidary soreshjani[1], Asghar zarabi[2]
[1.] (Corresponding Author), PhD Student, Department of urban planning, Geography faculty, University of Isfahan. Iran
[2.] Professor Department of urban planning, Geography faculty, University of Isfahan. Iran
Email: rasol_heidary@yahoo.com

Abstract: In this study the correlation of metropolitan areas growth is evaluated. Twelve cities of Iran with a population over 500,000 the total of which constitute more than half of the nation's population are the subject here. The growth rate in these metropolitan areas has been different in the last few decades. Their growth rates have not been balanced. The Allometric growth model is applied in this work through compares on of all elements with one another. An allometric scaling analysis method based on the idea from fractal theory, general system theory, and analytical hierarchy processes are proposed to make a comprehensive evaluation for the relative level of city development. Through applying Allometric model in the metropolitan areas different growth rates are determined. The recent areas like Karaj and Zahedan with very high growth rate kind of disturb the balanced growth rate of others, while Rasht and Ormieh are just the opposite.
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1. Introduction

City dwelling in on a rapid pace in Iran and the target cities are the metropolitans that have had a very high growth rate in the last fifty years in general. Of course some of them, according to national ranks: some have over grown, some had proportional low growth and yet some have been on the edge of extinction.

The Iran-Iraq imposed war during 1960 and 1968 almost destroyed two south-western metropolitan cities of Abadan and Khoramshahr which were close to the boarder, in a sense that they have not been able to recover their pre-ware status as far as population growth is concerned. In 1956 population of Abadan was 226083 heads, in 1986 the city was almost evacuated and in 2006 the population was 217988 heads. In general, population growth rate are evident on national bases; therefore, a study to evaluate this trend is necessary. In this article the Allometric growth model is adapted to evaluate 12 metropolitan areas of Iran (figure1). The senses used in this study are extracted from the records of the years 1956-66-76-86-96-2006.

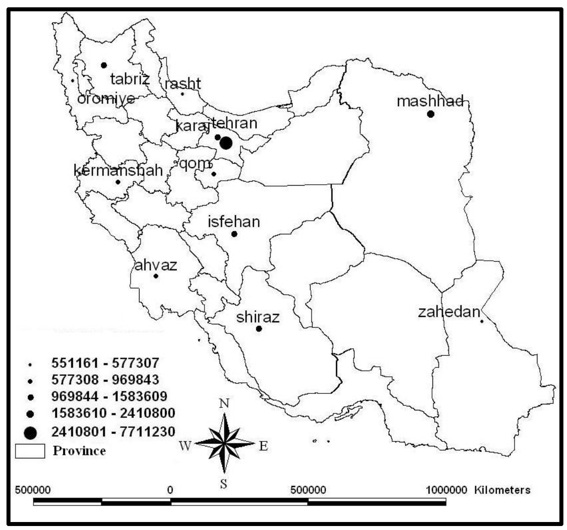

Figure 1. The twelve metropolitans of the republic Islamic Iran (2006)

2. Material and Methods

2-1. The Law of Allometric Growth

The law of allometric growth originated from biological sciences (gayon, 2000, pp 748-758 and lee, 1989, pp 463-476), it is well known that most rates in biology scale with the size of organism (peters, 1983), meaning that they are proportional to mass raised to some exponent (samanlego & e.moses, 2008, 25). and allometric analysis is introduced to social science by Naroll and Von Bertalanffy (Naroll & von Bertalanffy, 1956, pp76-89 and Naroll & von Bertalanffy, 1973, 244-252) From then on, the formulation gradually becomes a law of the urban geography, describing the relationship observed to be invariable between urban area and population for all cases in which the specified conditions are met (batty & Longley, 1994 and nordbeck, 1971, pp 54-67 and tobler, 1969, pp 30-34). In biology, the allometric rule was defined formally as follows (Beckman, 1958, pp247) "The rate of relative growth of an organ is a constant fraction of the rate of relative growth of the total organism." Actually it can also be restated in a broad sense as follows (chen & jiang, 2009, pp 54) "The rate of relative growth of a part of a system is a constant fraction of the rate of relative growth of the whole or another part of the system." Where urban systems are concerned, allometric growth can be divided into two types: longitudinal allometry and cross allometry. The cross allometry is also called transversal allometry, which can be divided into two types, too. One is based on the rank-size distribution of cities, and the other is based on hierarchy of cities or cascade structure of urban systems.

Then we can present our theory and method on spatio-temporal analysis of urban systems. An autonomous dynamic urban system S, which represents a city as a system or a system of cities, can be defined as a set of elements in interrelations. Conditioning that the number of elements is finite while denoting the measure of elements, pi(i=1,2, ., ., .), by Qi respectively, the spatio-temporal relationship of elements in a given urban system can be described mathematically in the form (von bertalanffy, 1968 and chen & wang, 1997) :

$$\frac{dQ_i}{dt} = f_i(Q_1, Q_2, \ldots Q_n)$$
(1)

in which i=1,2, ., .,, n. Eq. (1) can be developed in Taylor series. Considering the simplest case with an urban system consisting of only two elements, i.e., n = 2, and retaining only the first term, yields

$$\frac{dQ_i}{dt} = a_i Q_1, \ \frac{dQ_j}{dt} = a_j Q_{j,}$$
(2)

where a_i, a_j represent the intrinsic growth coefficients. Eq. (2) means that the increase in each element of an urban system depends only on the element itself. Obviously, allometric equation can be obtained such as

$$\frac{dQ_i}{dt} = a_{ij} \frac{Q_i}{Q_j} = \frac{dQ_j}{dt},$$
(3)

where Q_i, Q_j refer to two time series of some measurements of cities i and j, $\beta_j = e^c$ to the proportionality coefficient (C is an integral constant), α_{ij} to a allometric scaling exponent, and $d_{(i)}$ and $d_{(j)}$ to the generalized fractal dimension of Q_i and Q_j, respectively. Obviously we have

$$\alpha = \frac{d_i}{d_j}$$
(4)

This is the dimension equation based on the allometric growth of an urban system (Chen, 2008). If the relation between each pair of cities follows the allometric scaling law, equation (3), we can make an allometric scaling exponent (ASE) matrix such as

$$M = [\alpha_{ij}]_{n\times n} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \cdots & \alpha_{nn} \end{bmatrix} = [d_i/d_j]_{n\times n}$$
(5)

where M denotes the scaling exponent matrix, and n, the number of elements in the system. In theory, the relationships of ASEs are as below

$$\alpha_{ii} = \alpha_{jj} = 1, \alpha_{ij} = 1/\alpha_{ji}, \alpha_{ij} = \alpha_{is}/\alpha_{js}$$
(6)

where i, j, s =1, 2, …, n. Suppose that the fractal dimensions of cities form a vector in the form

$$D = [D_{(1)} \ D_{(2)} \ \ldots D_{(n)}]^T$$
(7)

Apparently, multiplying M on the right by D yields nD, namely

$$\begin{bmatrix} D_1/D_1 & D_1/D_2 & \cdots & D_1/D_n \\ D_2/D_1 & D_2/D_2 & \cdots & D_2/D_n \\ \vdots & \vdots & \cdots & \vdots \\ D_n/D_1 & D_n/D_2 & \cdots & D_n/D_n \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_n \end{bmatrix} = n \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_n \end{bmatrix} = nD,$$
(8)

which is a scaling relation of fractal dimension matrix. This means that D is just one of the eigenvectors of M, and n, the maximum eigenvalue corresponding to D. In theory, we can evaluate D and thus evaluate city development. However, in many case, it is hard to estimate values of $D_{(i)}$ or $D_{(j)}$. On the other hand, the ratio of $D_{(i)}$ to $D_{(j)}$ is easier to work out and more important in practice of urban analysis (Chen, 2010; Chen and Lin, 2009). After all, the value of a measurement, including fractal dimension, rests with comparison and relation. The allometric scaling exponent is just the ratio of fractal dimension. Therefore, we can calculate the α_{ij} values rather than $D_{(i)}$ and $D_{(j)}$ values for application. In order to evaluate the scaling exponent, we must solve the equation (M-nI)A=0. The equation has a nontrivial solution if and only if n is an eigenvalue of A, in that case the determinant of (M-nI) vanishes. It is clear that all the eigenvalues except one are zero. Here M has unit rank because every row is a constant multiple of the first row. As we known, the sum of the eigenvalues, n, equals to the trace of M. This implies that n is an eigenvalue of M and we have a nontrivial solution consisting of positive entries. In short, based on the second postulate given above, all the elements of matrix M are non-negative. According to the positive matrix theory, the matrix M has a non-negative largest eigenvalue λ_{max} =n since it conforms to the rule defined by equation (4). If the urban system follows the laws of allometric growth completely, the goodness of fit of the double logarithmic linear regression, R^2, is close to 1 in theory. For a perfect fit, we have

$$\lambda_{max} = \sum_{i=1}^{n} \lambda_i = \sum_{i=1}^{n} \propto_{ii} = \sum_{i=1}^{n} D_{(i)} / D_{(i)} = n \quad (9)$$

Where λ_i denotes the jth eigenvalue.

3.Results

The Allometric growth rate evaluation method: metropolitans areas of Iran

ASA can be used to evaluate city development and generate a rank of the relative growth potential for cities or elements of a city. The 12 Metropolitan areas with a population of more than 500,000 in Iran that have encountered different population changes in the last 50 years with a determined criteria of over-population (table 1).

ASA (Allometric scaling analysis) is based on the following postulate: each pair of elements in an urban system follow the allometric scaling law. Due to space-time translational asymmetry of geographical mathematical laws, we cannot guarantee that any pair of elements in a city or cities in a network always follows the allometric growth law. Therefore, it is necessary to make tests before implementing evaluation of city development. Through Allometric model the metropolitan areas are evaluated and compared two by two (table2 and figure 2).

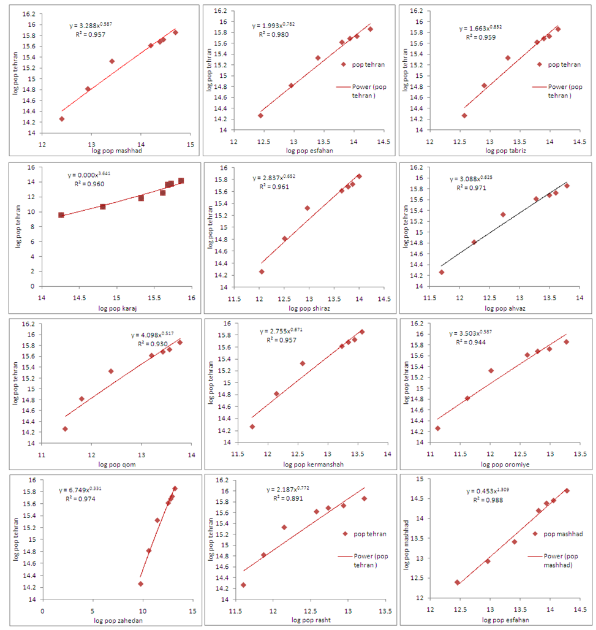

Figure 2: The allometric scaling relations between cities of Iran (1956-2006)

4.Discussions

Through applying Allometric model in the metropolitan areas different growth rates are determined. By analogy, ASA can be applied to intraurban development. Say, the same method can be use to evaluate the relative potential of different industrial sections in a city. For example, we can rank the development of the primary industry, secondary industry, and tertiary industry of Beijing. If we adopt different measures (say population, transport, greenbelt) to carry out ASA on cities as systems and systems of cities, we can develop a multilevel comprehensive evolution framework with cascade structure for city development.The recent areas like Karaj and Zahedan with very high growth rate kind of disturb the balanced growth rate of others, while Rasht and Ormieh are just the opposite. The most balanced growth rate is found in Isfahan, Shiraz and Tehran in the last five decades.

Table 1. The population cities with over 500000 in Iran (2006)

	1956	1966	1976	1986	1991	1996	2006
Tehran	1560934	2719730	4530223	6052584	6475527	6758845	7711230
Mashhad	241989	409616	667770	1463508	1759155	1887405	2410800
Isfahan	254708	424045	661510	986753	1127030	1266072	1583609
Tabriz	289996	403413	597976	971482	1088985	1191043	1378935
Karaj	14526	44243	137926	275100	799999	940968	1377450
shiraz	170659	269865	425813	848289	965117	1053025	1204882
Ahvaz	120098	206375	334399	579826	724652	804980	969843
qum	96499	134292	241219	543139	681253	777677	957496
kermanshah	125439	187930	290600	560514	624084	692986	784602
Ormieh	67605	110749	164419	300746	357399	435200	577307
zahedan	17495	39732	93740	281923	361623	419518	552706
rasht	109491	143557	188957	290897	340637	417748	551161

Source: Statistical Centre of Iran (years 1956-66-76-86-96-2006). General Census of Population and Settlement:http://www.amar.org.ir/

Table 2. The Twelve cities of Iran in Allometric scaling (2006)

	Tehran	Mashhad	Isfahan	Tabriz	Karaj	shiraz	Ahvaz	qum	kermsha	Ormieh	zahedan	rasht
Tehran	1	1.1721	1.0615	1.2211	0.6460	1.1286	1.1739	1.1606	1.2324	1.2667	0.4895	1.2722
Mashhad	0.8279	1	0.9179	0.9490	0.5101	0.9768	1.0148	0.9990	1.0695	1.1060	0.4438	1.2187
Isfahan	0.9385	1.0821	1	1.0189	0.5640	1.0552	1.1120	1.0739	1.1480	1.1824	0.5052	1.2843
Tabriz	0.7789	1.0510	0.9811	1	0.4529	0.9292	1.1129	0.9194	1.0079	1.0437	0.3843	1.1527
Karaj	1.3540	1.4899	1.4360	1.5471	1	1.3737	1.5402	1.3872	1.4575	1.4950	0.8801	1.5374
shiraz	0.8714	1.0232	0.9448	1.0708	0.6263	1	1.0774	1.0065	1.0761	1.1173	0.4778	1.2023
Ahvaz	0.8261	0.9852	0.8880	0.8871	0.4598	0.9226	1	0.9701	1.0585	1.0967	0.3951	1.1928
qum	0.8394	1.0010	0.9261	1.0806	0.6128	0.9935	1.0299	1	1.0481	1.1004	0.4608	1.2052
kermsha	0.7676	0.9305	0.8520	0.9921	0.5425	0.9239	0.9415	0.9519	1	1.0234	0.4680	1.1474
Ormieh	0.7333	0.8940	0.8176	0.9563	0.5050	0.8827	0.9033	0.8996	0.9766	1	0.3515	1.0871
zahedan	1.5105	1.5562	1.4948	1.6157	1.1199	1.5222	1.6049	1.5392	1.5320	1.6485	1	1.6823
rasht	0.7278	0.7813	0.7157	0.8473	0.4626	0.7977	0.8072	0.7948	0.8526	0.9129	0.3177	1

Corresponding Author:
Rasol heidary soreshjani, PhD Student, Department of urban planning, Geography faculty, University of Isfahan. Iran.
Email: rasol_heidary@yahoo.com

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