Flexural Behavior of Reinforced Concrete Flat Slabs with Depression in Shear and Flexure Zones

A.M. Elbehairy and M.Rabie

Structural engineering Department, faculty of Eng. Cairo University, Egypt.
rabie_eng@yahoo.com

Abstract: Depressions in flat slabs are a common design problem whenever sanitary needs are required. Presence of depression affects the overall behavior of the slab. This effect is dependent on depression size and location. In this research the effect of depression in shear & flexure zones of both column and field strips on the behavior of flat slabs is investigated. A finite element model of a single floor of a flat slab building is used for investigation. Linear analysis is conducted for the study. A slab composed of three square panels is analyzed for one hundred eight different study cases of depression sizes and locations. Bending moments and deflections of the different study cases were investigated and compared with recommendations of different building codes. Design aids in chart form are concluded. Finally recommendations for the design of flat slabs with depression are given.

[A.M. Elbehairy, M.Rabie . **Flexural Behavior of Reinforced Concrete Flat Slabs with Depression in Shear and Flexure Zones**. *J Am Sci* 2012;8(7):675-686] **(**ISSN: 1545-1003) http://www.americanscience.org. 101

Keywords: Flat slabs; Depression; Field strip; Column strip; Shear zone; Flexural zone

1. Introduction

Reinforced concrete flat slabs are widely spread in the last few decades. In fact, it is being considered ideal from both the construction and architectural point of view.

The main objective of this research is to study the behavior of the flat slab with a depression near column, taking into consideration variable parameters as depression size and location. Finally; a presentation of a number of design aids (charts) from which the values of the affected bending moments & maximum deflection can be easily achieved. The study is applying linear analysis using finite elements method (Staad III program) for different study cases to investigate the following:

- The zone affected with the presence of depression for different slabs having different depression size & location.
- The effect of depression size and location on:
  - I) Distribution of moments at the different slab locations.
  - II) The value of maximum deflection and deflection profile of the slab.

Finally, the summary and conclusions of this research in addition to suggested recommendations for design are presented.

Various researches were conducted to study the behavior of flat slabs under flexure, yet few of them considered the effect of the presence of an opening or depression in the slab. Provisions of different codes of design are also discussed.

There are many methods for calculating flexure in flat slab panels, of which the Strip method and the Equivalent Frame method are the most used. In the Strip method the slab panels are assumed to be divided into column and field strips.

In the equivalent frame analysis the structure is considered to be divided longitudinally and transversely into frames consisting of a row of columns and strips of slab. Drops and possible column line beams are considered in the determination of the effective cross section of the beams. The columns are assumed fixed at the floors above and below for gravity loading, as shown in Figure (1).

For the distribution of the negative moment across the width of the slab. **Whittle** (1985) [1], indicated that the distribution of the negative moments across the width of the slab varies with the aspect ratio of the panel as shown in the figure (2.3). **Regan** (1981)[2], suggested in case of uniformly distributed loads, dividing the slab width into three bands as shown in figure (2):

The Egyptian Code (2007) [3], allows using Yield Line method if the ratio of maximum negative to maximum positive moments lies between 1.0 and 1.5.

ACI-318 (1995), [4], **BS8110** (1985), [5], and **Egyptian Code** (1995) [3] allow the design of flat slabs which have columns on straight lines with difference not more than 10% panel length and perpendicular to the other direction according to the either the Empirical method or the Equivalent frame method. Although ratios for division of moments between column and field strips may vary slightly from one code to another, the concept is always the same.

Elkafrawy A. and Elkfafrawy M. (1989), [6], also studied the effects of small openings (as specified by codes) on the elastic behavior of a flat plate floor. It was found that high local bending moments occur in the area around the opening in a direction parallel to the opening edge. These moments result in high principal moments near the sharp corner., so additional reinforcement should be added to control the crack width.

Ibrahim (1994), [7]**,** studied the effects of openings near columns on the behavior of the flat plates.. He found that the Opening at more than three times slab thickness from column face have a slight effect on the behavior of the flat slab. and A wide opening located at the column face with a width greater than that of the column should be avoided.

The American Code ACI-318 (1995), [4], **Canadian Code CAN3-A23.3-M84** (1984), [8], and the **Australian Code AS3600-88** (1988), [9], required no special analysis when dealing with openings in flat slabs in the following cases:

- Opening of any size may be located in the area common to intersecting middle strips, provided total amount of reinforcement required for the panel without the opening is maintained.
- In the area common to intersecting column strips, not more than one-eighth the width of column strip in either span shall be interrupted by an opening. An amount of reinforcement equivalent to that interrupted by an opening shall be added on the sides of the opening.
- In the area common to one column strip and one middle strip, not more than one-quarter the reinforcement in either strip shall be interrupted by openings. An amount of reinforcement equivalent to that interrupted by opening shall be added on the sides of the opening.

Ahmed[10] **S. Sanger (1997),** studied the effect of opening size , opening location with respect to column and opening aspect ratio (length/width) on the flexural behavior of the flat slab and deflection . he found that the maximum effect of the presence of the opening on the slab flexural behavior is in the region close to the opening and it decreases as we get far from it. and the opening longitudinal dimension has the major effect while its transversal dimension has a minor effect.

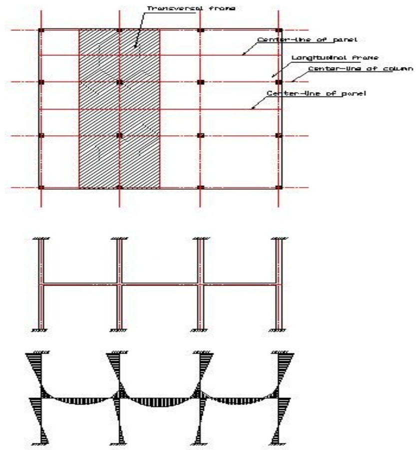


Figure (1) Equivalent Frame Analysis

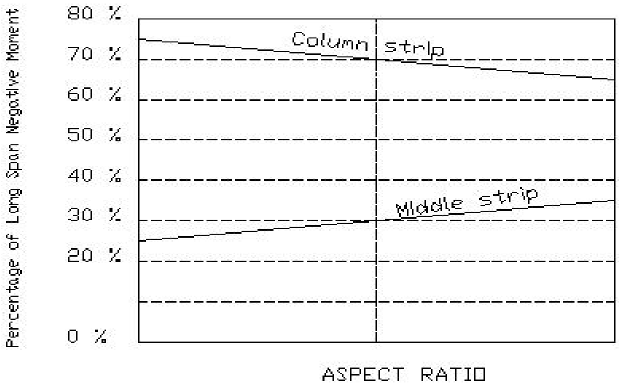


Figure (2) DISTRIBUTION OF LONG-SPAN NEGATIVE MOMENT IN INTERNAL PANEL OF FLAT SLAB , WHITTLE (1985)

Finite Element Analysis of Flat Slabs

Numerical Modeling

In this work , Finite Element analysis is being used to study the effect of depression in contact with column in flat slabs (with different sizes & different locations) on the distribution of the positive & negative moments in X & Y directions .

A commercial linear analysis software package – "STAAD-III (VER.22) "is used for the entire linear analysis.

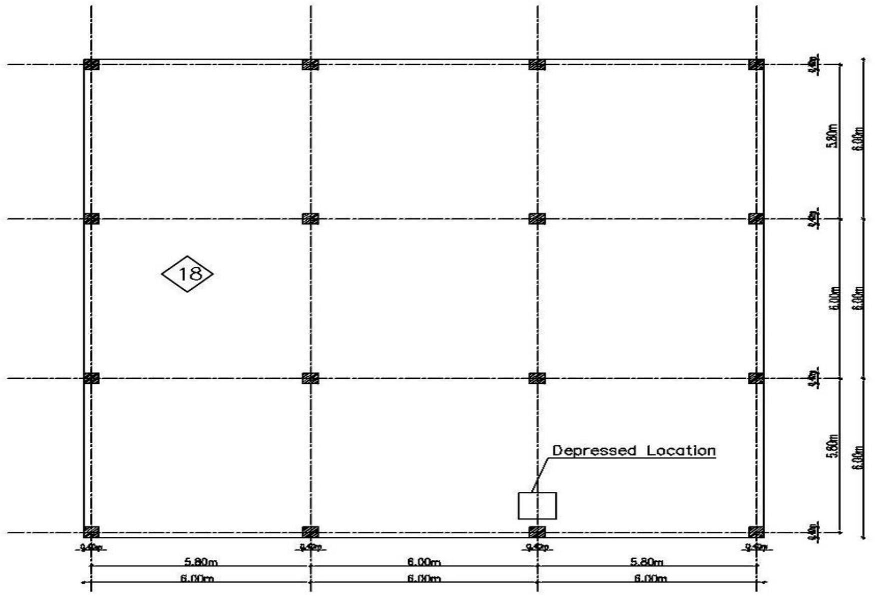
NLREG is an extremely powerful program for performing statistical regression analysis.. NLREG can perform simple linear regressions of the form: Y = A*X+ B, but it can also perform non linear regression, fitting virtually any type of function to a set of data values .

It was decided to choose the equally spanned panels (3X3) mentioned implicitly by the "ECP " to be the ideal case to apply the empirical method of design.

A (3x3) panels with variable depression sizes & locations were analyzed under a distributed working load of 900 kg/m^2 & the bending moments were determined & compared with those of slab without depression (REFERENCE CASE).

In all cases the used panel was square of length 6.0 m , column 0.40x0.40 m , and slab thickness 0.18 m . Figs (3,4) show the chosen model & the different notations given to the depression dimensions, for both exterior panel depression & interior panel depression respectively. The reinforced concrete used in the study has a characteristic strength (Fcu)=250 kg/cm^2 & Young's modulus (Ec)=221000kg/cm^2.

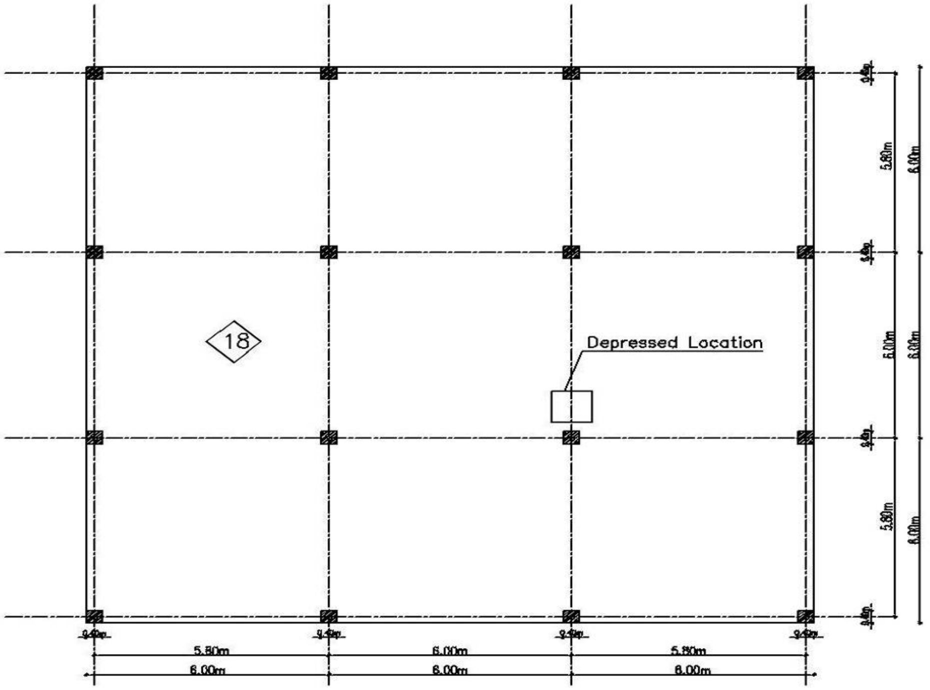
Plate bending shell elements were used . For the best numerical conditions of stiffness equations; it was decided that the (length / width) ratio of individual elements is not to exceed 1 to 3 .



DEPRESSION VARIABLES :

A : Distance between depression and column interior face .
B : Depression transversal dimension [width] .
C : Depression longitudinal dimension [length] .
D : Depression depth .

FIG.(3) EXTERIOR PANEL DEPRESSION NOTATIONS.



DEPRESSION VARIABLES :

A : Distance between depression and column interior face .
B : Depression transversal dimension [width] .
C : Depression longitudinal dimension [length] .
D : Depression depth .

FIG.(4) INTERIOR PANEL DEPRESSION NOTATIONS.

Study Cases:
A number of 108 cases are studied as follows:
a) Cases of exterior panel depression (54 cases):

Depressions lying at 0 cm from column edge (i.e. in contact with the col. face) with dimensions varying from (1.00 x 1.00) m to (3.00 x 3.00)m & depths varying from 5 cm to 10 cm as shown in table (4-1).

b) Cases of interior panel depression (54 cases):

Depressions lying at 0 cm from column edge (i.e. in contact with the col. face) with dimensions varying from (1.00 x 1.00)m to (3.00 x 3.00)m & depths varying from 5 cm to 10 cm as shown in table (4-1).

Table (1): Study Cases For The Analysis (For Either Exterior Or Interior Panel Depression):

CASE #	DIMENSIONS				RELATIVE DIMENSIONS			
	A (1) (m)	B (1) (m)	C (1) (m)	DEPTH(D)	A/L (2)	B/L (2)	C/L (2)	D/t (3)
1	0	1.00	1.00	5 cm	0	1/6	1/6	5/18
2	0	1.00	2.00		0	1/6	1/3	
3	0	1.00	3.00		0	1/6	1/2	
4	0	2.00	1.00		0	1/3	1/6	
5	0	2.00	2.00		0	1/3	1/3	
6	0	2.00	3.00		0	1/3	1/2	
7	0	3.00	1.00		0	1/2	1/6	
8	0	3.00	2.00		0	1/2	1/3	
9	0	3.00	3.00		0	1/2	1/2	
10	0	1.00	1.00	6 cm	0	1/6	1/6	6/18
11	0	1.00	2.00		0	1/6	1/3	
12	0	1.00	3.00		0	1/6	1/2	
13	0	2.00	1.00		0	1/3	1/6	
14	0	2.00	2.00		0	1/3	1/3	
15	0	2.00	3.00		0	1/3	1/2	
16	0	3.00	1.00		0	1/2	1/6	
17	0	3.00	2.00		0	1/2	1/3	
18	0	3.00	3.00		0	1/2	1/2	
19	0	1.00	1.00	7 cm	0	1/6	1/6	7/18
20	0	1.00	2.00		0	1/6	1/3	
21	0	1.00	3.00		0	1/6	1/2	
22	0	2.00	1.00		0	1/3	1/6	
23	0	2.00	2.00		0	1/3	1/3	
24	0	2.00	3.00		0	1/3	1/2	
25	0	3.00	1.00		0	1/2	1/6	
26	0	3.00	2.00		0	1/2	1/3	
27	0	3.00	3.00		0	1/2	1/2	
28	0	1.00	1.00	8 cm	0	1/6	1/6	8/18
29	0	1.00	2.00		0	1/6	1/3	
30	0	1.00	3.00		0	1/6	1/2	
31	0	2.00	1.00		0	1/3	1/6	
32	0	2.00	2.00		0	1/3	1/3	
33	0	2.00	3.00		0	1/3	1/2	
34	0	3.00	1.00		0	1/2	1/6	
35	0	3.00	2.00		0	1/2	1/3	
36	0	3.00	3.00		0	1/2	1/2	
37	0	1.00	1.00	9 cm	0	1/6	1/6	9/18
38	0	1.00	2.00		0	1/6	1/3	
39	0	1.00	3.00		0	1/6	1/2	
40	0	2.00	1.00		0	1/3	1/6	
41	0	2.00	2.00		0	1/3	1/3	
42	0	2.00	3.00		0	1/3	1/2	
43	0	3.00	1.00		0	1/2	1/6	
44	0	3.00	2.00		0	1/2	1/3	
45	0	3.00	3.00		0	1/2	1/2	
46	0	1.00	1.00	10 cm	0	1/6	1/6	10/18
47	0	1.00	2.00		0	1/6	1/3	
48	0	1.00	3.00		0	1/6	1/2	
49	0	2.00	1.00		0	1/3	1/6	
50	0	2.00	2.00		0	1/3	1/3	
51	0	2.00	3.00		0	1/3	1/2	
52	0	3.00	1.00		0	1/2	1/6	
53	0	3.00	2.00		0	1/2	1/3	
54	0	3.00	3.00		0	1/2	1/2	

(1) See fig.(3) & (4).

(2) L=Flat slab span length (6m).

(3) t =Flat slab thickness(18cm)

2. Discussions of the analysis results

Bending Moments:

The suggested finite element model was used for all depression sizes and locations. The slab was divided into 7 strips in X-direction & 7 strips in Y-direction.

The percentage change in bending moment stated through the analysis represents the increase (for positive %) & the decrease (for negative %) in moments, due to the presence of the depression, with respect to the reference case (without depression).

Cases of Exterior Panel Depression:

B.M. about X-axis (MX) :

Only the 5 cm depression depth cases & the 10 cm depth ones are presented in this research as being the lower & upper study cases limits.

A complete study of the different percentage changes is investigated using The NLREG computer program & it was noted that each local percentage change is linearly varying with the depression studied parameters i.e. varying according to a linear equation in which:

- The dependent variable = The percentage change (increase or decrease) in B.M.
- The independent variables = The depression width (B), The depression length (C), The depression depth (D) & The depression volume (B X C X D)…in general.

Consequently correction factors are deduced as shown as in table (2) & fig. (5) show the different correction factors (F) of the STAAD III B.M. (MX) due to an exterior panel depression adjacent to column in case of Y-direction strips as shown in appendix.

Similarly, correction factors are deduced as shown as in Table (3) & fig. (6) Show the different Correction factors (F) of the STAADIII B.M. (MY) due to an external panel depression adjacent to column in case of X-direction strips as shown in appendix.

Cases of Interior Panel Depression:

B.M. about X-axis(MX):

Correction factors are deduced as shown as in table (4) & fig. (7) Show the different correction factors (F) of the STAADIII B.M. (MX) due to an external panel depression adjacent to column in case of Y-direction strips as shown in appendix.

Similarly, correction factors are deduced as shown as in table (5) & fig. (8) Show the different correction factors (F) of the STAADIII B.M. (MY) due to an internal panel depression adjacent to column in case of X-direction strips as shown in appendix.

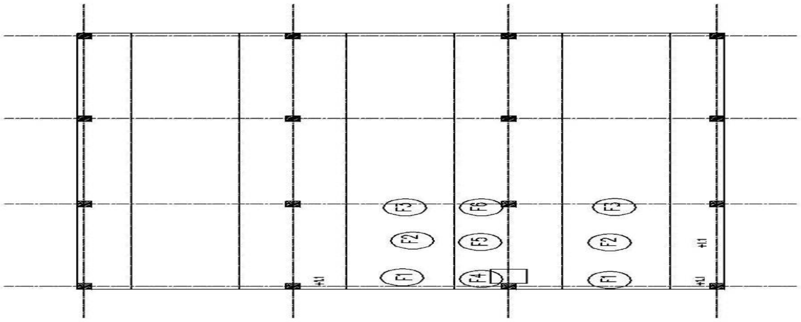


FIGURE (5) : CORRECTION FACTORS (F *) OF THE STAADIII B.M.(MX) DUE TO AN EXTERIOR PANEL DEPRESSION ADJACENT TO COL.

CASE OF Y–DIR. STRIPS

$* \; F_i = \dfrac{M_{i\ corr.}}{M_{i\ r}}$

WHERE :

- $M_{i\ corr.}$ = CORRECTED B.M. IN FLAT SLAB WITH DEPRESSION.
- $M_{i\ r}$ = CALCULATED B.M. IN FLAT SLAB WITHOUT DEPRESSION (REFERENCE CASE).

TABLE (3) : CORRECTION FACTORS (F) OF THE STAADIII B.M.(MY) DUE TO AN EXTERIOR PANEL DEPRESSION ADJACENT TO COL.

CASE OF X–DIR. STRIPS

STRIPS	INTERIOR PANEL			EXTERIOR PANEL ADJACENT TO DEPRESSION	
	– VE	+ VE	– VE	FIELD B.M.	COL.B.M.
INTERIOR COL. STRIP NOT DEPRESSED	1	1	+1.1	1	1
ADJACENT & DEPRESSED FIELD STRIP	+1.2	1	F7 [SEE CHARTS (12)& (13)]	1	1
THE DEPRESSED EDGE COL. STRIP	+1.2	+1.2	F8 ** [SEE CHARTS (14)& (15)]	+1.2	F9 [SEE CHARTS (16)& (17)]
OTHERWISE	1				

** DESIGN –VE B.M AT THE COLUMN IN CONTACT TO DEPRESSION.
SEE FIG.(6)

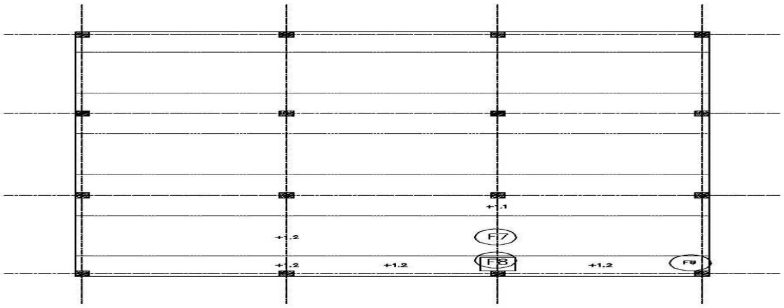


FIGURE (6) : CORRECTION FACTORS (F *) OF THE STAADIII B.M.(MY) DUE TO AN EXTERIOR PANEL DEPRESSION ADJACENT TO COL.

CASE OF X−DIR. STRIPS

$* \; F_i = \frac{M_{i\ corr.}}{M_{i\ r}}$

WHERE :

TABLE (5) : CORRECTION FACTORS (F) OF THE STAADIII B.M.(M) DUE TO AN INTERIOR PANEL DEPRESSION ADJACENT TO COL.

CASE OF X−DIR. STRIPS

STRIPS	INTERIOR PANEL			EXTERIOR PANEL ADJACENT TO DEPRESSION	
	− VE	+ VE	− VE	FIELD B.M.	COL.B.M.
INTERIOR COL. STRIP NOT DEPRESSED	1	1	+1.1	1	1
ADJACENT & DEPRESSED FIELD STRIP	+1.2	+1.1	F14 [SEE CHARTS (26)& (27)]	1	1
THE DEPRESSED INTERIOR COL. STRIP	+1.1	+1.15	1 **	+1.1	+1.15
ADJACENT FIELD STRIP	1	1	+1.15	1	1
OTHERWISE	1				

** DESIGN −VE B.M AT THE COLUMN IN CONTACT TO DEPRESSION.

SEE FIG.(8)

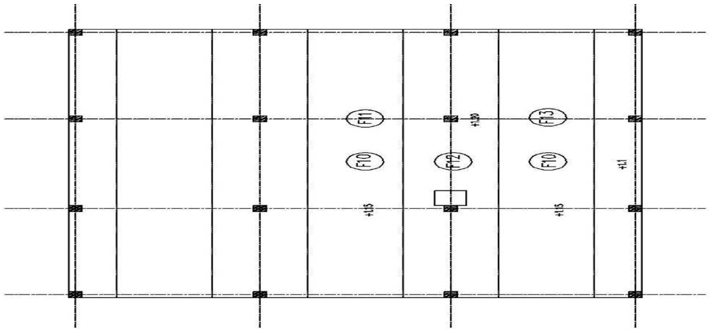


FIGURE (7) : CORRECTION FACTORS (F *) OF THE STAADIII B.M.(MX) DUE TO AN INTERIOR PANEL DEPRESSION ADJACENT TO COL.

CASE OF Y−DIR. STRIPS

$* \; F_i = \frac{M_{i\ corr.}}{M_{i\ r}}$

WHERE :

- $M_{i\ corr.}$ = CORRECTED B.M. IN FLAT SLAB WITH DEPRESSION.
- $M_{i\ r}$ = CALCULATED B.M. IN FLAT SLAB WITHOUT DEPRESSION (REFERENCE CASE).

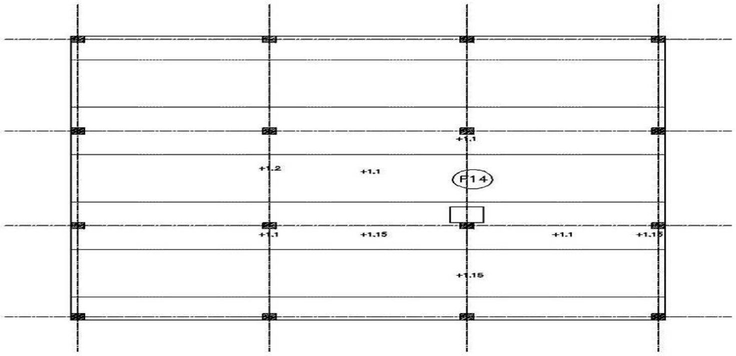


FIGURE (8) : CORRECTION FACTORS (F)* OF THE STAADIII B.M.(MY) DUE TO AN INTERIOR PANEL DEPRESSION ADJACENT TO COL.

CASE OF X–DIR. STRIPS

* $F_i = \frac{M_{i\ corr.}}{M_{i\ r}}$

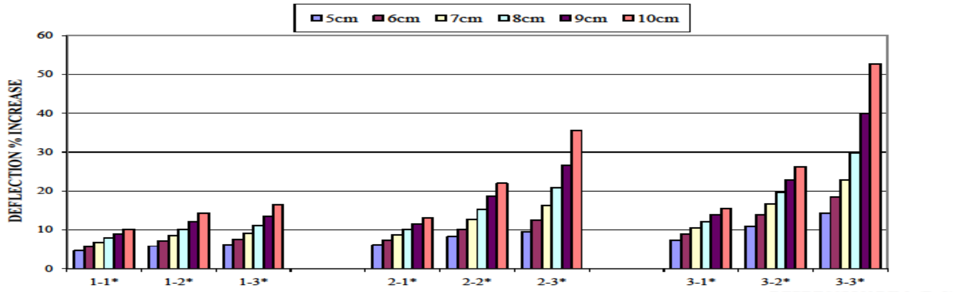
WHERE :

- $M_{i\ corr.}$ = CORRECTED B.M. IN FLAT SLAB WITH DEPRESSION.
- $M_{i\ r}$ = CALCULATED B.M. IN FLAT SLAB WITHOUT DEPRESSION (REFERENCE CASE).

Deflection

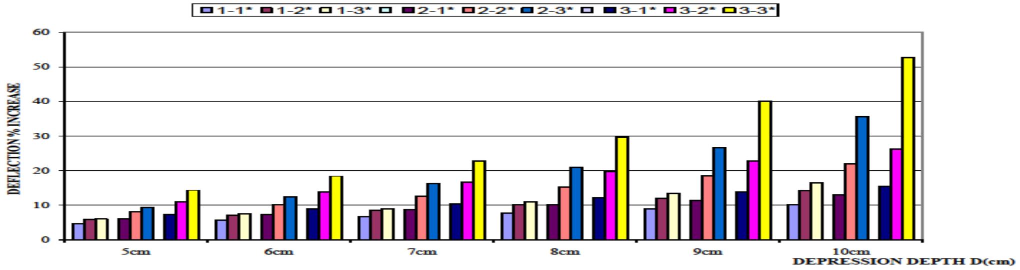
The effect of depression width, length and depth on the percentage of increase in the slab maximum deflection are shown in figures 9, 10, 11, 12, 13, 14

FIG (9) EFFECT OF DEPRESSION DEPTH (D) ON THE PERCENTAGE INCREASE OF THE SLAB MAXIMUM DEFLECTION (EXTERIOR PANEL DEPRESSION).



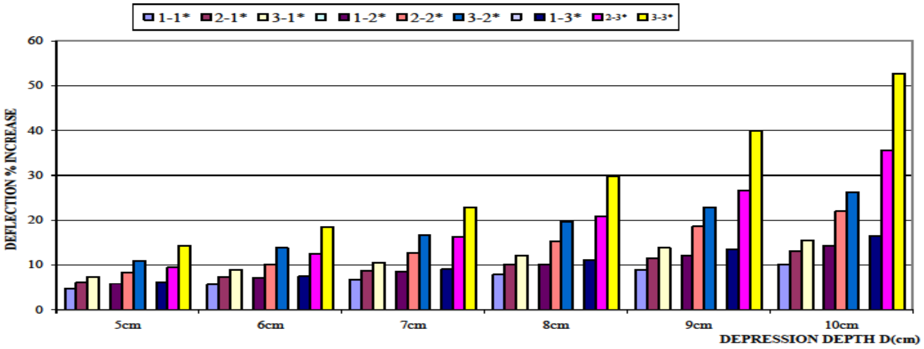
* The first number denotes the depression width B in m & the second number denotes the depression length C in cm.

FIG (10) EFFECT OF DEPRESSION LENGTH (C) ON THE PERCENTAGE INCREASE OF THE SLAB MAXIMUM DEFLECTION (EXTERIOR DEPRESSION).



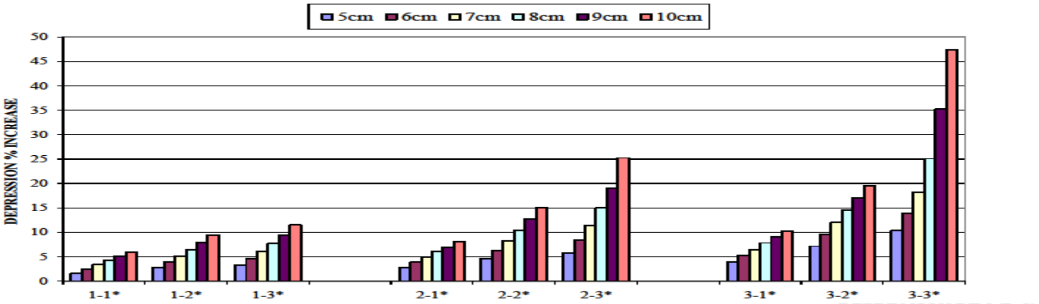
* The first number denotes the depression width B in m & the second number denotes the depression length C in m.

FIG (11) EFFECT OF DEPRESSION WIDTH (B) ON THE PERCENTAGE INCREASE OF THE SLAB MAXIMUM DEFLECTION (EXTERIOR PANEL DEPRESSION).



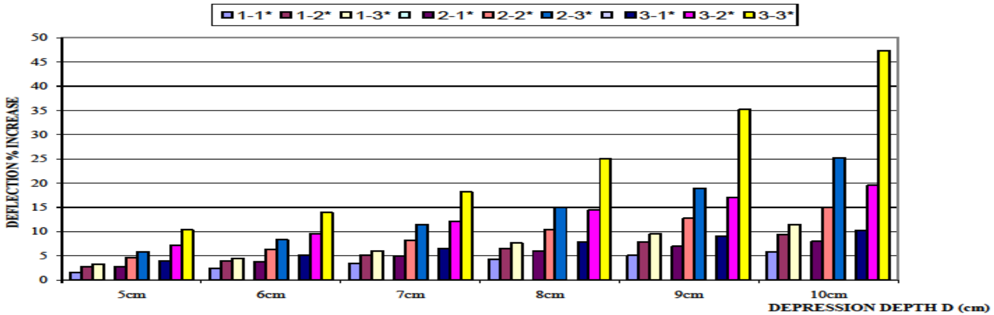
* The first number denotes the depression width B in m & the second number denotes the depression length C in m.

FIG (12) EFFECT OF DEPRESSION DEPTH (D) ON THE PERCENTAGE INCREASE OF THE SLAB MAXIMUM DEFLECTION (INTERIOR PANEL DEPRESSION).



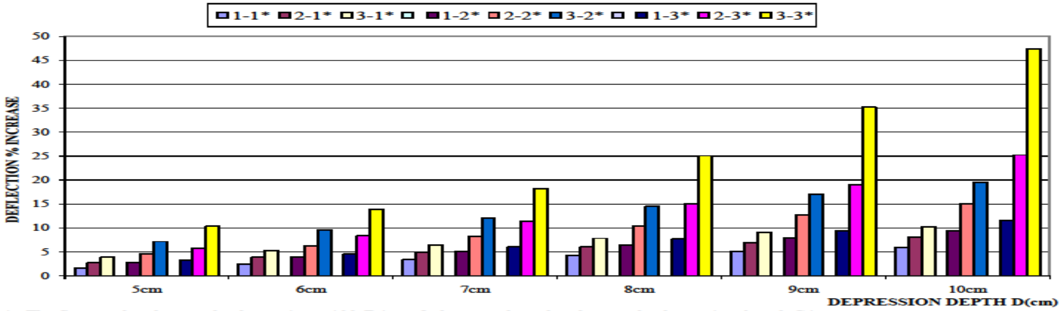
* The first number denotes the depression width B in m & the second number denotes the depression length C in cm.

FIG (13) EFFECT OF DEPRESSION LENGTH (C) ON THE PERCENTAGE INCREASE OF THE SLAB MAXIMUM DEFLECTION (INTERIOR PANEL DEPRESSION).



* The first number denotes the depression width B in m & the second number denotes the depression length C in m.

FIG (14) EFFECT OF DEPRESSION WIDTH (B) ON THE PERCENTAGE INCREASE OF THE SLAB MAXIMUM DEFLECTION (INTERIOR PANEL DEPRESSION).



* The first number denotes the depression width B in m & the second number denotes the depression length C in m.

A complete study of the different deflection factors is investigated using The NLREG computer program. It was noted that the deflection factor is linearly varying with the depression studied parameters i.e. varying according to a linear equation in which:

- The dependent variable = deflection factor (FD).
- The independent variables = The depression width (B), The depression length (C), The depression depth (D) & The depression volume (B X C X D).

Consequently deflection factors can be easily deduced. The previously mentioned equations are presented in a chart form for simplicity & ease of design solution. One can deduce the FD12 deflection factor in case of an exterior panel depression from charts (D1) & (D2) as shown in appendix.

Similarly, Deflection factor in case of an interior panel depression from charts (D3) & (D4) as shown in appendix.

3. Conclusion

- The maximum effect of the presence of the depression on the slab flexural behavior is concentrated at the region close to depression & it decreases as we get far from it.
- The common axis between depression & column in contact with depression represents an axis of symmetry of the % increase in B.M.(MX or MY) to the right & to the left from the depression for both the depressed col. strip & the adjacent field strip(s).
- For all depression sizes; only bending moments at sections intersecting depression showed a decrease in their values & on the other hand, other sections showed an increase or decrease.
- Increase of both B.M. and maximum deflection in case of interior panel depression are relatively lesser than those recorded in case of exterior

panel depression. This means that locating a depression area in an interior panel represents a more economic solution compared to exterior panel.

- For any chosen depression depth, there is no significant change in the maximum deflection recorded value by exchanging values between depression width & depression length, which means that rotating the depression area does not significantly alter the maximum deflection recorded value.
- Correction factors for the field & col. –ve & +ve design B.M.s located at the zones affected by the depression can be deduced from a number of design aids (charts)- sample of them are shown in appendix.
- Maximum immediate deflection values based on the gross moment of inertia can be deduced from a number of design aids (charts).

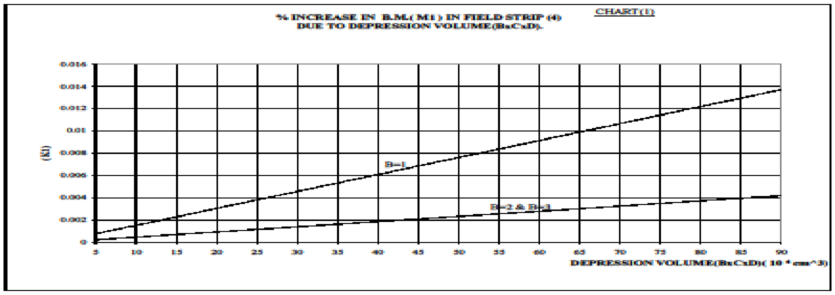
References

1. **Whittle, R.T. (1985):** “Design of Reinforced Concrete Flat Slabs to BS8110”. Ciria , Report, 110:1985 .
2. **Regan, P.E.,(1981) :** “Behaviour of Reinforced Concrete Flat Slabs” . Ciria Report, 89:1981.
3. **Egyptian Code (**1995): “Design and Construction of Reinforced Concrete Structures” . Permanent Committee for The Egyptian Code for Design and Construction of Reinforced Concrete Structures , .
4. **ACI Committee 318-1995 (**1995): “Building Code Requirements for Reinforced Concrete” . ACI 318-83 , American Concrete Institute , Detroit , .
5. **BS 8110 (**1985): “Structural Use of Concrete” . British Standard Institute, London , .
6. **El Kafrawy, A.F. and El kafrawy , M.F. (**1989): “Slab Opening in Reinforced Concrete Flat Plates” . Engineering Bulletin, Helwan University vol., 1, Dec..
7. **Ibrahim, W.W. (**1994): “Behavior of Flat Plates with Opening “ , Ph.D. Thesis , Zagazig University, .
8. **Can 3-A23.3-M84 (**1984): “Design of Concrete for Buildings” . Canadian Standards Association , Rexdale, .
9. **AS3600-88,(1988) :** “Australian Standard for Concrete Structures” .Standard Association of Australia, North Sidney, 1988 .
10. **Sanger , Ahmed. S.(**1997): “Behaviour of R.C. Flat Slabs with Openings near Columns under Flexure” , MS.D. Thesis , Cairo University .

6/15/2012

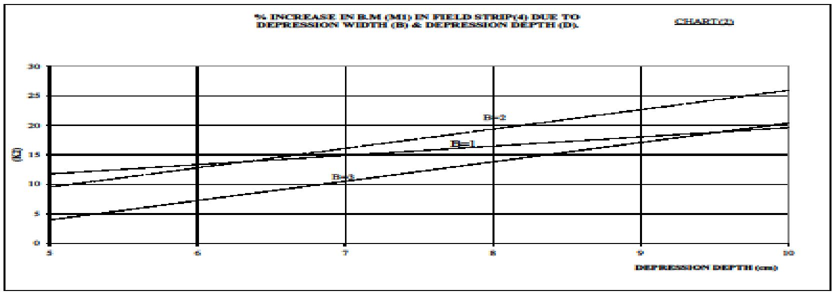
Appencdi

F1 CHARTS



CHART(1)

% INCREASE IN B.M.(M1) IN FIELD STRIP (4) DUE TO DEPRESSION VOLUME(BxCxD).



CHART(2)

% INCREASE IN B.M (M1) IN FIELD STRIP(4) DUE TO DEPRESSION WIDTH (B) & DEPRESSION DEPTH (D).

I - Get the % increase portion due to the depression volume (K1) from chart (1).

FD12 CHARTS

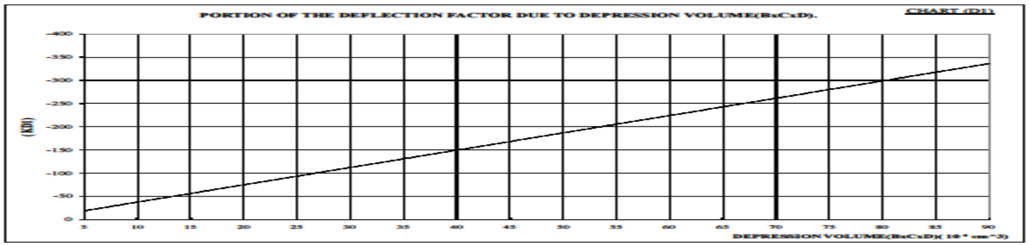
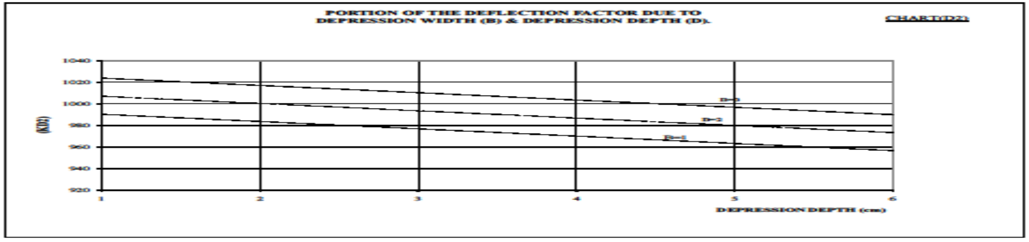


CHART (D1)

PORTION OF THE DEFLECTION FACTOR DUE TO DEPRESSION VOLUME(BxCxD).



CHART(D2)

PORTION OF THE DEFLECTION FACTOR DUE TO DEPRESSION WIDTH (B) & DEPRESSION DEPTH (D).

I - Get the portion of the deflection factor due to depression volume (K_{D1}) from chart (D1) .
ii - Get the portion of the deflection factor due to depression width & depth (K_{D2}) from chart (D2).
iii - The final deflection factor FD12 = KD1 + KD2 + 17.2 C .
iv - IMMEDIATE DEFLECTION (δ) = L / FD12 …Based upon the gross moment of inertia Ig.