Differential Transformation Approach to A SIR Epidemic Model with Constant Vaccination

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Abstract: In this article SIR model that monitors the temporal dynamics of a childhood disease in the presence of preventive vaccine is developed. The qualitative analysis reveals the vaccination reproductive number R_v for disease control and eradication. The aim of this paper is to apply the differential transformation method (DTM) which is used to compute an approximation to the solution of the non-linear system of differential equations governing the problem. Graphical results are presented and discussed quantitatively to illustrate the solutions.
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1. Introduction

The purpose of this paper is to employ the differential transformation method (DTM) to systems of differential equations which describes SIR epidemic model. The DTM is a semi-analytical numerical technique depending on Taylor series that promises to be useful in various fields of mathematics. The DTM derives from the differential equation system with initial conditions a system of recurrence equations that finally leads to a system of algebraic equations whose solutions are the Coefficients of a power series solution.

Over the years, diligent vaccination campaigns have resulted in high levels of permanent immunity against the childhood disease among the population. Childhood diseases are the most common form of infectious diseases. These are diseases such as measles, mumps, chicken pox, etc. to which children are born susceptible, and usually contract within five years. Since young children are in particularly close contact with their peers, at school and play, such diseases can spread quickly. Meanwhile, the development of vaccines against infectious childhood diseases has been a boon to mankind and protecting children from diseases that can be prevented by vaccination is a primary goal of health administrators. Since vaccination is considered to be the most effective strategy against childhood diseases, the development of a framework that would predict the optimal vaccine coverage level needed to prevent the spread of these diseases is crucial.

The SIR model is a standard compartmental model that has been used to describe many epidemiological diseases ([7], [9],[11]),[13] The way several childhood diseases spread through a population fits into this framework. At time t Suppose the population consist of:

S (t)-susceptible population: those so far uninfected and therefore liable to infection.

I (t)-infective population: those who have the disease and are still at large.

R (t)-isolated population: or who have recovered and are therefore immune.

This model assumes that the efficacy of the vaccine is 100 per cent and the natural death rates μ in the classes remain unequal to births, so that the population size N is realistically not constant. Citizens are born into the population at a constant birth rate π with extremely very low-childhood disease mortality rate. We denote the fraction of citizens vaccinated at birth each year as p (with $0 < p < 1$) and assume the rest are susceptible. A susceptible individual will move into the infected group through contact with an infected individual, approximated by an average contact rate β. An infected individual recovers at a rate γ, and enters removed group. The removed group also contains people who are vaccinated. The differential equations for the *SIR* model are:

$$\frac{dS}{dt} = (1-p)\pi N - \beta\frac{SI}{N} - \mu S \quad , \qquad (1.1)$$

$$\frac{dI}{dt} = \beta\frac{SI}{N} - (\gamma+\mu)I \quad , \qquad (1.2)$$

$$\frac{dR}{dt} = p\pi N + \gamma I - \mu R \quad . \qquad (1.3)$$

We also have the relationship $N = S+I+R$ and assume μ, π, β, γ are all positive constant parameters.

Adding equations (1.1)-(1.3) we obtain:

$$\frac{dN}{dt} = (\pi-\mu)N \quad , \qquad (1.4)$$

So that we are now dealing with a varying total population [4]. A summary of the process is drawn in a flow chart shown in Figure 1.

The groups can be scaled by population N using the new variables, $s = \frac{S}{N}$, $i = \frac{I}{N}$, and $r = \frac{R}{N}$. The population is now normalized, meaning

$s + i + r = 1$; and we have the new system

$$\frac{di}{dt} = \beta si - (\gamma + \pi)i \ , \qquad (1.5)$$

$$\frac{dr}{dt} = p\pi + \gamma i - \pi r \ , \qquad (1.6)$$

This problem was solved by Makinde[10] using Adomian decomposition method (ADM) and Yildirim [15] using homotopy perturbation method.

2. Qualitative analysis

We can analyses the system qualitatively by studying the subsystem in the closed set $\Gamma = \{(s,i) \in \Re_+ : 0 \le s + i \le 1\}$, because r does not appear in equations (1.5) and (1.6). A qualitative investigation of the subsystem described by equations (1.5) and (1.6) reveals that the long- term behavior falls into two categories: endemic or die out.

When the disease dies out naturally, the solution asymptotically approaches a disease free equilibrium E_0 of the form:

$$E_0 = (1 - p, 0) \qquad (2.1)$$

The threshold that determines the stability of this equilibrium is the vaccination reproduction number:

$$R_v = \frac{\beta(1-p)}{\gamma+\pi} \qquad (2.2)$$

The disease free equilibrium is locally stable if $R_v < 1$ Global asymptotic stability for disease free equilibrium is also achieved using a Bendixson-Dulac argument for $R_v < 1$, i.e. there are no periodic solutions ([2][3]). Equation (2.2) also reveals that there is a critical vaccination proportion, $p_c = (\beta - \gamma - \pi)/\beta$ Above which the disease free equilibrium is stable, i.e. $p > p_c$ Thus, in order to successfully prevent disease, the vaccination proportion should be large enough. When the disease free equilibrium is unstable, there exists an endemic equilibrium E_u of the form:

$$E_u = \left(\frac{1-p}{R_v}, \frac{\pi}{\beta}(R_v - 1)\right) \qquad (2.3)$$

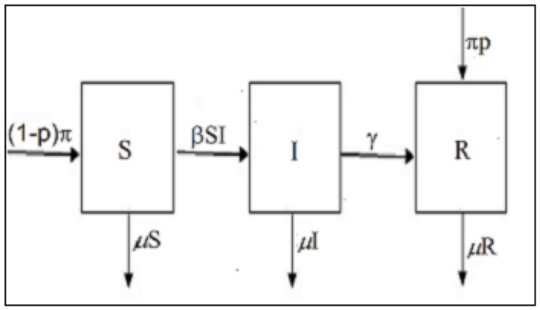

Figure 1: Flow chart for the SIR model

From equation (2.3) it Very obvious that E_u will only exist provided $R_v > 1$. The eigenvalues $(\delta_{1,2})$ of the Jacobian matrix evaluated at the endemic equilibrium E_u is given as:

$$\delta_{1,2} = \left(-\frac{\pi}{2}R_v \pm \frac{1}{2}\sqrt{\pi^2 R_v^2 - 4R_v\pi(\gamma + \pi)}\right) \qquad (2.4)$$

The endemic equilibrium E_u is locally asymptotically stable provided:

$$1 < R_v \le \frac{4(\gamma+\pi)}{\pi} \ , \qquad (2.5)$$

i.e. the eigenvalues are complex with negative real part and E_u can be classified as a spiral sink. This behavior can be interpreted as follows; for initial low levels of infectives, the numbers of susceptible build. Then, the number of infectives begins to increase until that process is faster than the number of susceptible being added to the population. Eventually, there are too few people to infect, the outbreak ends, and the number of susceptibles being to increase again.

3. Basic definitions of differential transformation method

Pukhov [12] proposed the concept of differential transformation, where the image of a transformed function is computed by differential operations, which is different from the traditional integral transforms as are Laplace and Fourier. Thus, this method becomes a numerical-analytic technique that formalizes the Taylor series in a totally different manner. Differential transformation is a computational method that can be used to solve linear (or non-linear) ordinary (or partial) differential equations with their corresponding boundary conditions. A pioneer using this method to solve initial value problems is Zhou [16] , who introduced it in a study of electrical circuits. Additionally, differential transformation has been applied to solve a variety of problems that are modeled with differential equations [6], [14], [1], [8]:

The method consists of, given system of differential equations and related initial conditions; these are transformed into a system of recurrence equations that finally leads to a system of algebraic equations whose solutions are the coefficients of a power series solution.

For the sake of clarity in the presentation of the DTM and in order to help to the reader we summarize

the main issues of the method that may be found in [16].

Definition 3.1 A differential transformation $Y(k)$ of function y (x) is defined as follows [5]

$$Y(k) = \frac{1}{k!}\left[\frac{d^k y(x)}{dx^k}\right]_{x=0} \quad (3.1)$$

In (3.1), $y(x)$ is the Original function and $Y(k)$ is the transformed function. Differential inverse transform of $Y(k)$ is defined as follows

$$y(x) = \sum_{k=0}^{\infty} x^k Y(k) \quad (3.2)$$

In fact. From (3.1) and (3.2), we obtain

$$y(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}\left[\frac{d^k y(x)}{dx^k}\right]_{x=0} \quad (3.3)$$

Equation (3.3) implies that the concept of differential transformation is derived from the Taylor series expansion.

From Equation (3.1) and (3.2), it is easy to obtain the following mathematical operations:

1- If $y(x) = g(x) \pm h(x)$ then
$Y(k) = G(k) \pm H(k)$.

2- If $y(x) = cg(x)$ then $Y(k) = cG(k), c$ is a constant.

3- If $y(x) = \frac{d^n g(x)}{dx^n}$,then
$Y(x) = \frac{(k+n)}{k!} G(x+n)$.

4- If $y(x) = g(x)h(x)$ then
$Y(k) = \sum_{l=0}^{k} G(l)H(k-l)$.

5- If $y(x) = x^n$ then

$Y(k) = \delta(k-n) = \begin{cases} 1, & k = n \\ 0, & k \neq n \end{cases}$, δ is the Kronecker delta .

6- If $y(x) = u(x)v(x)w(x)$ then

$$Y(k) = \sum_{s=0}^{k}\sum_{m=0}^{k-s} U(s)V(m)W(k-s-m)$$

.

4. The operation properties of differential transformation

If x (t) and y (t) are two uncorrelated functions with time t where X (k) and $Y(K)$ are the transformed functions corresponding to $x(t)$ and $y(t)$ then we can easily proof the fundamental mathematics operations performed by differential

Transformation and are listed as follows [1]:

(1) **Linearity**. If $X(k) = D[x(t)]$, $Y(k) = D[y(t)]$ and c_1 and c_2 are independent of t and k then

$$D[c_1 x(t) \pm c_2 y(t)] = c_1 X(k) \pm c_2 Y(k) \quad (4.1)$$

Thus, if c is a constant, then

$$D[c] = c\delta(k)$$

where δ is the kronecer delta function.

(2) **Convolution**. If

$$z(t) = x(t)y(t),\ x(t) = D^{-1}[X(t)],\ y(t) = D^{-1}[Y(t)]$$

And ⊗ denote the convolution and Symbol D denoting the differential transformation process. Then

$$D[z(t)] = D[x(t)y(t)] = X(k) \otimes Y(k) = \sum_{l=0}^{k} x(l)Y(k-l) \quad (4.2)$$

If $y(x) = y_1(x)\,y_2(x) \ldots y_{n-1}(x)\,y_n(x)$ then

$$Y(k) = \sum_{k_{n-1}}^{k}\sum_{k_{n-2}}^{k_{n-1}} \ldots \sum_{k_2}^{k_3}\sum_{k_1=0}^{k_2} Y_1(k_1)\,Y_2(k_2-k_1) \ldots Y_{n-1}(k_{n-1}-k_{n-2})Y_n(k-k_{n-1}) \quad (4.3)$$

The proof of above properties is deduced from the definition of the differential transformation.

5. Application

By using the fundamental operations of differential transformation method. We obtained the following recurrence relation to the system (1.5) - (1.7) with respect to time t one gets

$$S(k+1) = \frac{1}{k+1}\left\{(1-p)\pi\delta(k) - \beta\sum_{l=0}^{k} S(l)I(k-l) - \pi S(k)\right\} \quad (5.1)$$

$$I(k+1) = \frac{1}{k+1}\left\{\beta\sum_{l=0}^{k} S(l)I(k-l) - (\gamma+\pi)I(k)\right\} \quad (5.2)$$

$$R(k+1) = \frac{1}{k+1}\{p\pi\delta(k) + \gamma I(k) - \pi R(k)\} \quad (5.3)$$

We consider the following values for parameters into four cases:

Case 1

$s_0 = 1$ Initial population of $s(t)$. who are Susceptible.

$i_0 = 0$ Initial population of $i(t)$. who are infective.

$r_0 = 0$ Initial population of $r(t)$. who are Immune.

$\beta = 0.8$ Rate of change of susceptible population to infective population.

$\gamma = 0.03$ Rate of change of infective population to immune population.

$\pi = 0.4$ Constant birth rate.

$p = 0.9$ The fraction of citizens vaccinated at birth each year.

From the initial condition

$s(0) = 1,\ i(0) = 0, r(0) = 0$ we have $S(0) = 1$, $I(0) = 0$, $R(0) = 0$ and from equations (5.1) – (5.3) we have

$S(1) = -0.36,\ S(2) = 0.072,\ S(3) = -0.96 \times 10^{-2},\ S(4) = 0.96 \times 10^{-3},\ S(5) = -0.768 \times 10^{-4},\ S(6) = -0.512 \times 10^{-6}, \ldots$

$I(1) = 0, I(2) = 0, I(3) = 0, I(4) = 0, I(5) = 0, I(6) = 0, \ldots$

$R(1) = 0.36,\ R(2) = -0.072, R(3) = 0.96 \times 10^{-2},\ R(4) = -0.96 \times 10^{-3},\ R(5) = 0.768 \times 10^{-4},\ R(6) = -0.512 \times 10^{-5}, \ldots$

Therefore, the closed form of the solution can be easily written as:

$s(t) = \sum_{k=0}^{\infty} S(k)\, t^k = 1 - 0.36t + 0.072 \times t^2 - 0.96 \times 10^{-2}t^3 + 0.96 \times 10^{-3}t^4 - 0.768 \times 10^{-4}t^5 - 0.569 \times 10^{-6}t^6$,

$i(t) = \sum_{k=0}^{\infty} I(k)\, t^k = 0$

$r(t) = \sum_{k=0}^{\infty} R(k)t^k = 0.36t - 0.072 \times t^2 + 0.96 \times 10^{-2}t^3 - 0.96 \times 10^{-3}t^4 + 0.768 \times 10^{-4}t^5$

Case 2

s_0 =0.8 Initial population of $s(t)$. who are susceptible.

$i_0 = 0.2$ Initial population of $i(t)$. who are infective.

$r_0 = 0$ Initial population of $r(t)$. who are immune.

$\beta = 0.8$ Rate of change of susceptible population to infective population.

$\gamma = 0.03$ Rate of change of infective population to immune population.

$\pi = 0.4$ Constant birth rate.

$p = 0.9$ The fraction of citizens vaccinated at birth each year. From the initial condition $s(0) = 0.8,\ i(0) = 0.2,\ r(0) = 0$ we have $S(0) = 0.8,\ I(0) = 0.2,\ R(0) = 0$ and from equations (5.1) – (5.3) we have $S(1) = -0.408,\ S(2) = 0.1008,\ S(3) = -0.8224 \times 10^{-2},\ S(4) = -0.1812 \times 10^{-2}, S(5) = 0.2839 \times 10^{-3},\ S(6) = -0.487 \times 10^{-4}, S(7) = -0.197 \times 10^{-5},\ S(8) = 0.1567 \times 10^{-7},\ S(9) = 0.456 \times 10^{-9},\ S(10) = -0.175 \times 10^{-11}, \ldots$

$I(1) = 0.042, I(2) = -0.02823, I(3) = -0.117 \times 10^{-2},\ I(4) = 0.276 \times 10^{-2}, I(5) = -0.376 \times 10^{-3},\ I(6) = 0.474 \times 10^{-4},\ I(7) = 0.199 \times 10^{-5},\ I(8) = -0.154 \times 10^{-7},\ I(9) = -0.457 \times 10^{-9}, I(10) = 0.175 \times 10^{-11}, \ldots$

$R(1) = 0.366, R(2) = -0.07257, R(3) = 0.9394 \times 10^{-2},\ R(4) = -0.948 \times 10^{-3},\ R(5) = 0.924 \times 10^{-4},\ R(6) = 0.445 \times 10^{-5},\ R(7) = -0.1697 \times 10^{-7}, R(8) = -0.269 \times 10^{-9},\ R(9) = 0.182 \times 10^{-11}, \ldots$

Therefore, the closed form of the solution can be easily written as:

$s(t) = \sum_{k=0}^{\infty} S(k)t^k = 0.8 - 0.408t + 0.1008 \times t^2 - 0.8224 \times 10^{-2}t^3 - 0.1812 \times 10^{-2}t^4 + 0.2839 \times 10^{-3}t^5 - 0.487 \times 10^{-4}t^6 - 0.197 \times 10^{-5}t^7 + 0.1567 \times 10^{-7}t^8 + 0.456 \times 10^{-9}t^9 - 0.175 \times 10^{-11}t^{10}$,

$i(t) = \sum_{k=0}^{\infty} I(k)\, t^k = 0.2 + 0.042t - 0.02823 \times t^2 - 0.117 \times 10^{-2}t^3 + 0.276 \times 10^{-2}t^4 - 0.376 \times 10^{-3}t^5 + 0.474 \times 10^{-4}t^6 + 0.199 \times 10^{-5}t^7 - 0.154 \times 10^{-7}t^8 - 0.457 \times 10^{-9}t^9 + 0.175 \times 10^{-11}t^{10}$

,

$r(t) = \sum_{k=0}^{\infty} R(k)t^k = 0.366t - 0.07257 \times t^2 + 0.9394 \times 10^{-2}t^3 - 0.948 \times 10^{-3}t^4 + 0.924 \times 10^{-4}t^5 + 0.445 \times 10^{-5}t^6 - 0.1697 \times 10^{-7}t^7 - 0.269 \times 10^{-9}t^8 + 0.182 \times 10^{-11}t^9$

.

Case 3

s_0 =0.8 Initial population of $s(t)$. who are susceptible.

$i_0 = 0.2$ Initial population of $i(t)$. who are infective.

$r_0 = 0$ Initial population of $r(t)$. who are immune.

$\beta = 0.8$ Rate of change of susceptible population to infective population.

$\gamma = 0.03$ Rate of change of infective population to immune population.

$\pi = 0.4$ Constant birth rate.

$p = 0.3$ The fraction of citizens vaccinated at birth each year. From the initial condition $s(0) = 0.8,\ i(0) = 0.2,\ r(0) = 0$ we have $S(0) = 0.8,\ I(0) = 0.2,\ R(0) = 0$ and from equations (5.1) – (5.3) we have $S(1) = -0.168,\ S(2) = 0.0336,\ S(3) = -0.2464 \times 10^{-2},\ S(4) = -0.1252 \times 10^{-3}, S(5) = 0.223 \times 10^{-5},\ S(6) = -0.193 \times 10^{-3}, S(7) = -0.7804 \times 10^{-4}, S(8) = 0.425 \times 10^{-5}, S(9) = 0.1094 \times 10^{-5}, S(10) = -0.294 \times 10^{-7}, \ldots$

$I(1) = 0.042, I(2) = -0.903 \times 10^{-2}, I(3) = -0.7217 \times 10^{-3},\ I(4) = 0.4492 \times 10^{-3},\ I(5) = -0.3084 \times 10^{-4},\ I(6) = 0.186 \times 10^{-3},\ I(7) = 0.787 \times 10^{-4},\ I(8) = -0.416 \times 10^{-5},\ I(9) = -0.1099 \times 10^{-5}, I(10) = 0.294 \times 10^{-7}, \ldots$

$R(1) = 0.126, R(2) = -0.02457, R(3) = 0.3186 \times 10^{-2},\ R(4) = -0.324 \times 10^{-3},\ R(5) = 0.286 \times 10^{-8},\ R(6) = 0.1535 \times 10^{-5},\ R(7) = -0.6596 \times 10^{-6}, R(8) = -0.924 \times 10^{-7},\ R(9) = 0.437 \times 10^{-8}, \ldots$

Therefore, the closed form of the solution can be easily written as:

$s(t) = \sum_{k=0}^{\infty} S(k)t^k = 0.8 - 0.168t + 0.0336 \times t^2 - 0.2464 \times 10^{-2}t^3 - 0.1252 \times 10^{-3}t^4 + 0.223 \times 10^{-5}t^5 - 0.193 \times 10^{-3}t^6 - 0.7804 \times 10^{-4}t^7 + 0.425 \times 10^{-5}t^8 + 0.1094 \times 10^{-5}t^9 - 0.294 \times 10^{-7}t^{10}$,

$i(t) = \sum_{k=0}^{\infty} I(k)t^k = 0.2 + 0.042t - 0.903 \times 10^{-2}t^2 - 0.7217 \times 10^{-3}t^3 + 0.4492 \times 10^{-3}t^4 - 0.3084 \times 10^{-4}t^5 + 0.186 \times 10^{-3}t^6 + 0.787 \times 10^{-4}t^7 - 0.416 \times 10^{-5}t^8 - 0.1099 \times 10^{-5}t^9 + 0.294 \times 10^{-7}t^{10}$

,

$r(t) = \sum_{k=0}^{\infty} R(k)t^k = 0.126t - 0.02457 \times t^2 + 0.3186 \times 10^{-2}t^3 - 0.324 \times 10^{-3}t^4 + 0.286 \times 10^{-8}t^5 + 0.1535 \times 10^{-5}t^6 - 0.6596 \times 10^{-6}t^7 - 0.924 \times 10^{-7}t^8 + 0.437 \times 10^{-8}t^9$

.

Case 4

s_0 =0.8 Initial population of $s(t)$. who are susceptible.

$i_0 = 0.2$ Initial population of $i(t)$. who are infective.

$r_0 = 0$ Initial population of $r(t)$. who are immune.

$\beta = 0.8$ Rate of change of susceptible population to infective population.

$\gamma = 0.03$ Rate of change of infective population To immune population.

$\pi = 0.4$ constant birth rate.

$p = 0.0$ The fraction of citizens vaccinated at birth each year. From the initial condition $s(0) = 0.8,\ i(0) = 0.2,\ r(0) = 0$ we have $S(0) = 0.8,\ I(0) = 0.2,\ R(0) = 0$ and from equations (5.1) – (5.3) we have $S(1) = -0.048,\ S(2) = 0,\ S(3) = 0.416 \times 10^{-3},\ S(4) = 0.269 \times 10^{-4}, S(5) = -0.771 \times 10^{-5},\ S(6) = -0.208 \times 10^{-3}, S(7) = -0.140 \times 10^{-3}, S(8) = 0.108 \times 10^{-4}, S(9) = 0.456 \times 10^{-5}, S(10) = -0.175 \times 10^{-6}, \ldots$

$I(1) = 0.042, I(2) = 0.57 \times 10^{-3}, I(3) = -0.4977 \times 10^{-3},\ I(4) = -0.1496 \times 10^{-4},\ I(5) = 0.685 \times 10^{-5},\ I(6) = 0.198 \times 10^{-3},\ I(7) = 0.141 \times 10^{-3},\ I(8) = -0.105 \times 10^{-4},\ I(9) = -0.458 \times 10^{-5}, I(10) = 0.175 \times 10^{-6},$

$R(1) = 0.6 \times 10^{-2}, R(2) = -0.57 \times 10^{-3},\ R(3) = 0.817 \times 10^{-4},\ R(4) = -0.119 \times 10^{-4},\ R(5) = 0.862 \times 10^{-6},\ R(6) = 0.357 \times 10^{-5},\ R(7) = -0.117 \times 10^{-5}, R(8) = -0.269 \times 10^{-6},\ R(9) = 0.182 \times 10^{-7},$

Therefore, the closed form of the solution can be easily written as:

$s(t) = \sum_{k=0}^{\infty} S(k)t^k = 0.8 - 0.048t + 0.416 \times 10^{-3}t^3 + 0.269 \times 10^{-4}t^4 - 0.771 \times 10^{-5}t^5 - 0.208 \times 10^{-3}t^6 - 0.140 \times 10^{-3}t^7 + 0.108 \times 10^{-4}t^8 + 0.456 \times 10^{-5}t^9 - 0.175 \times 10^{-6}t^{10},$

$i(t) = \sum_{k=0}^{\infty} I(k)t^k = 0.2 + 0.042t + 0.57 \times 10^{-3}t^2 - 0.4977 \times 10^{-3}t^3 - 0.1496 \times 10^{-4}t^4 + 0.685 \times 10^{-5}t^5 + 0.198 \times 10^{-3}t^6 + 0.141 \times 10^{-3}t^7 - 0.105 \times 10^{-4}t^8 - 0.458 \times 10^{-5}t^9 + 0.175 \times 10^{-6}t^{10}$

,

$r(t) = \sum_{k=0}^{\infty} R(k)t^k = 0.6 \times 10^{-2}t - 0.57 \times 10^{-3}t^2 + 0.817 \times 10^{-4}t^3 - 0.119 \times 10^{-4}t^4 + 0.862 \times 10^{-6}t^5 + 0.357 \times 10^{-5}t^6 - 0.117 \times 10^{-5}t^7 - 0.269 \times 10^{-6}t^8 + 0.182 \times 10^{-7}t^9.$

The approximate solutions $S(t)$, $I(t)$ and $R(t)$ are displayed in Figs. 2-5, respectively. In each figure four different values of $s_0, i_0, r_0, \beta, \gamma, \pi$ and p are considered.

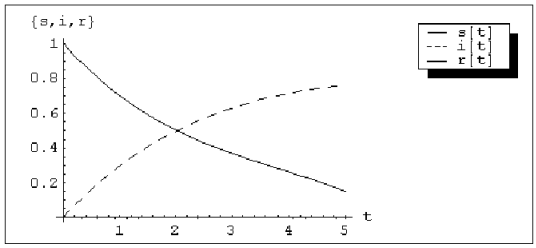

Figure 2

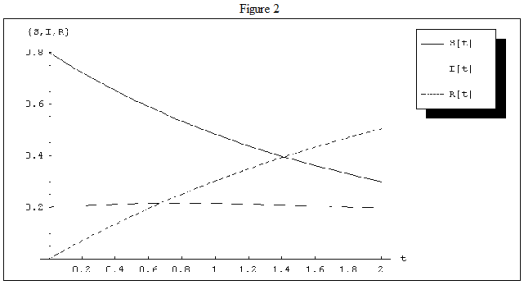

Figure. 3

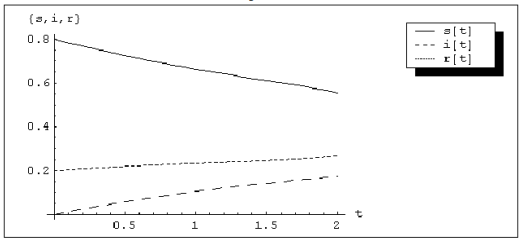

Figure. 4

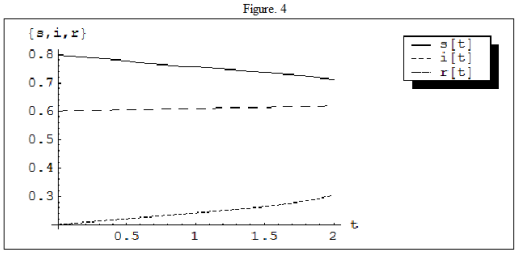

Figure. 5

6. Conclusions

In this paper, SIR epidemic model with constant vaccination strategy are solved numerically using the DTM for approximating the solutions. This method is a powerful tool which enables to find analytical solution in case of linear and non- linear systems of differential

equations. This method is better than numerical methods, since it is free from rounding off error. In the present paper, the method yields a series solution which converges faster than the series obtained by another method (see Refs. [10], [15]) The basic reproductive rate,R_v, is derived. If$R_v < 1$, the disease-free equilibrium is globally stable so that the disease always dies out, and if $R_v > 1$, the disease-free equilibrium becomes unstable.

Figure 2 describes case 1 and shows the impact of high-vaccination coverage on the disease free initial population groups. As expected, the population of the susceptible group decreases with time while that of the removed group gradually increases due to inclusion of vaccinated susceptible group.

figure 3 describes Case 2 and illustrates the impact of high- vaccination coverage on the initial population groups with low level of infective group. The population of the susceptible and infective group decrease with time while that of the removed group increases due to inclusion of vaccinated and recovered people with permanent immunity and the disease outbreak ends.

Figure 4 describes Case 3 and illustrates the effect of low- vaccination coverage on the initial population groups with low level of infective group. The population of the susceptible group decrease with time A small increase in the population of removed group is also noticed

Fiure5 describes Case 4 and illustrates the impact of initial low levels of infective group on the vaccination free population. As expected, the population of susceptible group decreases while that of infective group temporally increases. The disease rapidly spread to the entire population. The only contribution to removed group is the very small proportion of recovered people with permanent immunity.

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