Multi-criteria Group Purchasing Decision-making Process Based on Marketing Mix (4Ps) Using Fuzzy TOPSIS

Mirzaei.Afshin[1], Sheikh.Reza[2], Moodi.Najmeh[1]

[1]Department of Industrial Engineering and Management, Shahrood University of Technology ,SHAHROOD, IRAN, Mirzaei.Afshin@gmail.com; najmehmoodi@gmail.com
[2]Faculty of Industrial Engineering and Management Shahrood University of Technology, SHAHROOD, IRAN, resheikh@Shahroodut.ac.ir

Abstract: In this paper, we discussed two cases. First, we challenged decision making with multi-criteria. Secondly, group decision making has been discussed. Specifically, we introduced Marketing Mix (4Ps) as Multi-Criteria Decision-making (MCDM). Then, we discussed Fuzzy TOPSIS methodology; which can smooth group decision making while we have Multi-Criteria (Marketing Mix (4Ps)). To clarify our proposed procedure, a numerical example is discussed.

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1. Introduction

With regards to purchasing, whether it is the traditional process or online process, we face the problem of comparison and decision-making. The criteria are different from buyer and seller's point of view, and each of them try to maximize their profit rate, The most widely known marketing criteria are the 4 P's, (Price, Product, Place, Promotion), which expressed by Kotler and Borden in 1964[1]. Following Kotler and Borden's research, many other people explored in this area[2]. Other people examined the criteria of marketing mix (4Ps) and considered those criteria for different times[3].Historically, with regards to all studies and research pertaining to this subject, it can be observed that the only criteria, which is expressed and estimated in all studies and research, is marketing mix (4Ps). Buyers and consumers consider this criterion most, and based on them, they make their decision about purchasing. These factors are price, quality characteristics or product, purchasing place and promotion. All of these contribute to a consumer's buying practices. Finally, there are different prices, different qualities and features, various supply places, numerous services and side advantages for certain or special goods. This pushes most consumer's to be decisive when presented with so many variables. By considering the high level of uncertainty and fuzziness of the criteria, the problem of decision-making is doubled. Our suggested method for solving this problem is to use Fuzzy TOPSIS technique, which based on the decision-makers, can have the best selection. In classical MCDM methods, including classical TOPSIS, the ratings and the weights of the criteria are known precisely. However, under many conditions, crisp data is inadequate to model real-life situations, since human judgments, including preferences, are often vague and cannot estimate one's preference with an exact numerical value. A more realistic approach may be to use linguistic assessments, instead of numerical values, that is, to suppose that the ratings and weights of the criteria in the problem are assessed by means of linguistic variables. Lingual expressions, for example, low, medium, high, etc. are regarded as the natural representation of the judgment. These characteristics indicate the applicability of fuzzy set theory in capturing the decision makers' preference structure. Fuzzy set theory aids in measuring the ambiguity of concepts that are associated with human being's subjective judgment. Moreover, since in the group decision making, evaluation is resulted from different evaluator's view of linguistic variables, its evaluation must be conducted in an uncertain, fuzzy environment. The following chapter, the suggested model and one operational case of purchasing decision-making with multi-decision makers are presented.

A survey of the MCDM methods has been presented by Hwang and Yoon[4]. Technique for Order Performance by Similarity to Ideal Solution (TOPSIS), one of the known classical MCDM methods, also was first developed by Hwang and Yoon[4]. It is based upon the concept that the chosen alternative should have the shortest distance from the Positive Ideal Solution (PIS), i.e., the solution that maximizes the benefit criteria and minimizes the cost criteria; and the farthest from the Negative Ideal

Solution (NIS), i.e., the solution that maximizes the cost criteria and minimizes the benefit criteria.

There are many examples of applications of fuzzy TOPSIS in literature (For instance: The evaluation of service quality[5].Intercompany comparison[6]; the applications in aggregate production planning[7], Facility location selection [8]and large scale nonlinear programming[9]). The modifications proposed in this paper can be implemented in all real world applications of Fuzzy TOPSIS.

2. Methodology

Defining the criteria for purchase selection (Marketing Mix (4Ps))

The criteria considered here, in selection of the best purchase in a dynamic environment are:

1-Price (Product cost, Transportation cost, Development & tooling cost)

2-Product (Quality, Installation ease, Life cycle, Characteristics)

3-Place (Lead time, Distance)

4-Promotion (Guarantee, Flexibility of service)

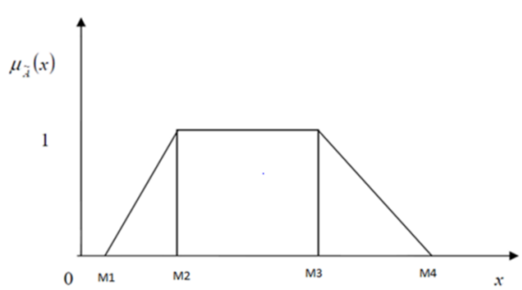
Definitions and Formulations

In this section we will cover some basic definitions and formulas that are used in our paper.

1. A fuzzy set A in a universe of discourse X is characterized by a membership function $\mu_{A(x)}$ which associates with each element x in X a real number in the interval [0, 1]. The function value is termed the grade of membership of x in A (defined by Zadeh[10]).

$$\tilde{A}= \{(x, \mu_{\tilde{A}(x)})|\ x\in X\} \quad (1)$$

2. A trapezoidal fuzzy number $\tilde{M}$ can be defined by (M_1,M_2,M_3,M_4) shown in Fig.1. The membership function $\mu_{\tilde{M}(x)}$ is defined as:



$$\mu_{\tilde{M}(x)} = \begin{cases} \dfrac{x-M_1}{M_2-M_1} & M_1 \le x < M_2 \\ 1 & M_2 \le x < M_3 \\ \dfrac{x-M_4}{M_3-M_4} & M_3 \le x < M_4 \\ 0 & \text{otherwise} \end{cases}$$

Figure - 1. Trapezoidal fuzzy number

If in a trapezoidal fuzzy number $M_2=M_3$ then $\tilde{M}$ is a triangular fuzzy number.

3. According to the expressed principles, in fuzzy set addition or subtraction of each two trapezoidal fuzzy number is a trapezoidal fuzzy number while multiplication of each two trapezoidal fuzzy number only is estimated of trapezoidal fuzzy number.

Let $\tilde{M} = (M_1,M_2,M_3,M_4)$ and $\tilde{N}= (N_1,N_2,N_3,N_4)$ be two trapezoidal fuzzy numbers and consider r as a real number, then four main definitions calculate as follow:

- $-\tilde{N}=(-N_4,-N_3,-N_2,-N_1)$
- $\tilde{M}+\tilde{N}=[M_1+N_1, M_2+N_2, M_3+N_3, M_4+N_4]$
- $\tilde{M}-\tilde{N}==[M_1-N_4, M_2-N_3, M_3-N_2, M_4-N_1]$
- $\tilde{M}\times r= [M_1r,M_2r,M_3r,M_4r]$
- If $\tilde{M}>0$ and $\tilde{N}>0$ then $\tilde{M}\times\tilde{N}= [M_1N_1,M_2N_2,M_3N_3,M_4N_4]$

4. The α -cut of fuzzy number $\tilde{M}$ is defined: $[\tilde{M}]_\alpha=\{x\in|\ R\ \ \mu_{\tilde{M}(x)} \ge \alpha\}$ Where $\alpha \in[0, 1]$ (2)

5. Let $\tilde{A}$ and $\tilde{B}$ be two trapezoidal fuzzy numbers. The method is usually defined to calculate the distance between fuzzy numbers as:

$$d(\tilde{A},\tilde{B})=\frac{1}{2}\int_0^1([\tilde{A}]_\alpha^L+[\tilde{A}]_\alpha^U-[\tilde{B}]_\alpha^L-[\tilde{B}]_\alpha^U)\,d\alpha \quad (3)$$

Where $[\tilde{A}]_\alpha^L, [\tilde{A}]_\alpha^U, [\tilde{B}]_\alpha^L, [\tilde{B}]_\alpha^U$ Are the lower and upper limits of fuzzy numbers $\tilde{A}$ and $\tilde{B}$.

This kind of distance measurement first expressed with Wu and Yao, and with respect to (3) they defined the following:

$$\tilde{B}<\tilde{A} \text{ If } d(\tilde{A},\tilde{B})>0,\ \tilde{B}>\tilde{A} \text{ If } d(\tilde{A},\tilde{B})<0,$$
$$\tilde{B}\approx\tilde{A} \text{ If } d(\tilde{A},\tilde{B})=0$$

3. Calculations

The importance weight of each criterion can be obtained by either directly assign or indirectly using pairwise comparisons[11]. Here, it is suggested that the decision makers use the linguistic variables (shown in Tables 1 to 4) to evaluate the importance of the criteria, and the ratings of alternatives with respect to various criteria. Assume that a decision group has K persons, and for each alternative of $A_1, A_2, \ldots, A_m$ there is $C_1, C_2, \ldots, C_n$ criteria to consider. Each of decision makers has a trapezoidal fuzzy number for each alternative of $A_1, A_2, \ldots, A_m$ respect to each of $C_1, C_2, \ldots, C_n$ criteria.it means there is k matrix respect to each decision maker as follow:

$$\begin{cases} \tilde{x}_{ijk} = (\tilde{x}_{ijk}^a, \tilde{x}_{ijk}^b, \tilde{x}_{ijk}^c, \tilde{x}_{ijk}^d) \\ \tilde{w}_{jk} = (\tilde{w}_{jk}^a, \tilde{w}_{jk}^b, \tilde{w}_{jk}^c, \tilde{w}_{jk}^d) \end{cases}$$

j =1, 2,..., n and i = 1, 2,...,m (4)

To calculate the aggregated importance of the criteria and the rating of alternatives, with respect to each criterion, we should consider each k matrix which calculated above as follow:

$$\begin{cases} \tilde{x}_{ij} = \left(x_{ij}^a, x_{ij}^b, x_{ij}^c, x_{ij}^d\right) \\ \tilde{w}_j = \left(w_j^a, w_j^b, w_j^c, w_j^d\right) \end{cases}$$
j =1, 2,..., n and i = 1, 2,...,m (5)

Where $\begin{cases} x_{ij}^a = \min\{x_{ijk}^a\}, x_{ij}^b = \frac{1}{K}\sum_{k=1}^{K} x_{ijk}^b \\ x_{ij}^c = \frac{1}{K}\sum_{k=1}^{K} x_{ijk}^c, x_{ij}^d = \max\{x_{ijk}^d\} \end{cases}$

And $\begin{cases} W_j^a = \min\{w_{jk}^a\}, W_j^b = \frac{1}{K}\sum_{k=1}^{K} w_{jk}^b \\ W_j^c = \frac{1}{K}\sum_{k=1}^{K} w_{jk}^c, x_{ij}^d = \max\{w_{jk}^d\} \end{cases}$

As stated previously, a fuzzy multi-criteria group decision making problem can be concisely expressed in matrix format as:

$$\widetilde{D} = \begin{bmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \cdots & \tilde{x}_{mn} \end{bmatrix}, \widetilde{W} = [\tilde{w}_1, \quad \tilde{w}_2, \cdots \quad \tilde{w}_n] \quad (6)$$

To avoid the complicated normalization formula used in classical TOPSIS, in some papers (see for example[12]) the linear scale transformation is used to transform the various criteria scales into a comparable scale. Therefore, it is possible to obtain the normalized fuzzy decision matrix

Denoted by $\widetilde{R}$

$$\widetilde{R} = [\tilde{r}_{ij}]_{m\times n} \quad (7)$$

$$\tilde{r}_{ij}\left(\frac{x_{ij}^a}{\theta_j^*}, \frac{x_{ij}^b}{\theta_j^*}, \frac{x_{ij}^c}{\theta_j^*}, \frac{x_{ij}^d}{\theta_j^*}\right) \quad (j \epsilon\, B) \quad (8)$$

$$\tilde{r}_{ij}\left(\frac{\rho_j^-}{x_{ij}^d}, \frac{\rho_j^-}{x_{ij}^c}, \frac{\rho_j^-}{x_{ij}^b}, \frac{\rho_j^-}{x_{ij}^a}\right) \quad (j \epsilon\, C) \quad (9)$$

Where B and C are the set of benefit criteria and cost criteria, respectively and $\rho_j^-(j\epsilon\, C)$, $\theta_j^*(j\epsilon\, B)$ are min $\{x_{ij}^a\}$ and max $\{x_{ij}^d\}$.

The normalization method mentioned above is to preserve the property that the ranges of normalized trapezoidal fuzzy numbers belong to [0, 1].In this paper, to avoid these computations and make a more easy and practical procedure, we simply define all of fuzzy numbers in this interval to omit the need of normalization method. Constructing the fuzzy numbers scalable and in [0, 1], we avoid calculations (8) through (9) and therefore we have: $\tilde{r}_{ij} = \tilde{x}_{ij}$ and $\widetilde{R} = \widetilde{D}$.

Considering the different importance of each criterion, one can now construct the weighted normalized fuzzy decision matrix as:

$$\widetilde{Q} = [\tilde{q}_{ij}]_{m\times n}, i = 1,2,\dots,m \text{ and } j = 1,2,\dots,n$$

Where

$$\tilde{q}_{ij} = \tilde{r}_{ij}(\cdot)\tilde{w}_j$$

According to the weighted normalized fuzzy decision matrix, we know that the elements $\tilde{q}_{ij}$ are normalized positive trapezoidal fuzzy numbers and their ranges belong to the closed interval [0, 1]. Then, we can define the Fuzzy Positive- Ideal Solution (FPIS,F^*) and Fuzzy Negative Ideal Solution (FNIS,F^-) as:

$$F^* = \left(\widetilde{V}_1^*, \widetilde{V}_2^*, \dots, \widetilde{V}_n^*\right), F^- = \left(\widetilde{V}_1^-, \widetilde{V}_2^-, \dots, \widetilde{V}_n^-\right) \quad (10)$$

Where

$$\widetilde{V}_j^* = (1,1,1,1) \text{ and } \widetilde{V}_j^- = (0,0,0,0).$$

The distance of each alternative A_i (i=1, 2,…, m) from $F^*$and F^- can be calculated as:

$$d_i^*(\alpha) = \sum_{j=1}^{n} d(\widetilde{V}_j^*, \tilde{q}_{ij}) = \sum_{j=1}^{n}(2v_j^* - q_{ij}^a - q_{ij}^d) + \left(\frac{\alpha}{2}\right)\sum_{j=1}^{n}(q_{ij}^a + q_{ij}^d - q_{ij}^b - q_{ij}^c) \quad \forall\, i, \quad (11)$$

$$d_i^-(\alpha) = \sum_{j=1}^{n} d(\tilde{q}_{ij}, \widetilde{V}_j^-) = \sum_{j=1}^{n}(q_{ij}^a + q_{ij}^d - 2v_j^-) + \left(\frac{\alpha}{2}\right)\sum_{j=1}^{n}(q_{ij}^b + q_{ij}^c - q_{ij}^a - q_{ij}^d) \quad \forall\, i, \quad (12)$$

Where d(*,*) is the distance measurements between two fuzzy numbers and $\alpha \in [0, 1]$.Moreover, a closeness coefficient is usually defined to determine the ranking order of all alternatives once d_i^* and d_i^- of each alternative A_i(i =1,2,…,m) has been calculated. The closeness coefficient of each alternative is calculated as (12):

$$CC_i(\alpha) = \frac{d_i^-(\alpha)}{d_i^-(\alpha) + d_i^*(\alpha)} \qquad i=1, 2,\dots, m \qquad (13)$$

Obviously, according to Eq. (13), an alternative A_i would be closer to FPIS (i.e. F^* defined in Eq. (10)) and farther from FNIS (i.e. F^- defined in Eq. (10)) as CC_i approaches 1. In other words, the closeness coefficient calculated by Eq. (13), can determine the ranking order of all alternatives and indicate the best one among a set of given feasible alternatives.

4. Numerical Example

Hereby, to illustrate our proposed approach of this paper we will discuss a numerical example.

Assume there is accompany ('Z') which wants to purchase a PLC (Programmable Logic Controller) which control their supply chain and it has high level of importance to their supply chain. This company has three expert decision makers (engineers)D_1, D_2 and D_3.

Assume there are three models of this PLC's A_1, A_2 and A_3 , while each of them has four different criteria C_1, C_2 , C_3, C_4 . As our paper based on Marketing Mix (4Ps), it has been used these criteria as follow:

- Product qualities and Characteristics (c_1).
- Price and transportation costs (c_2).
- Lead time (c_3).
- Promotion like guarantee and flexibility of services (c_4).
- The proposed method is applied to solve this problem and the computational procedure is summarized as follows:

Step 1. The decision makers use the linguistic weighting variables (shown in Fig 2) to assess the importance of the criteria.

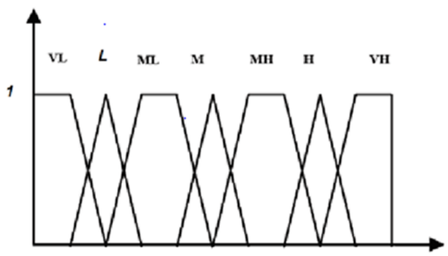


Figure - 2 Linguistic Variables for the importance weight of each criterion

Step 2. The decision makers use the linguistic rating variables (shown in Fig 3) to evaluate the rating of alternatives with respect to each criterion. Final aggregated results are calculated and as the linguistic fuzzy decision matrix.

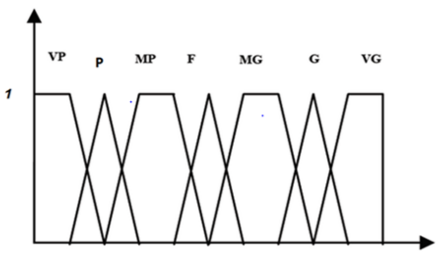


Figure - 3 Linguistic Variables for the ratings

Step3. The linguistic evaluations (shown in Tables 1 to 4) are converted into symmetric trapezoidal fuzzy numbers in order to construct the fuzzy decision matrix.

Table 1. Importance of each criterion

D \ C	D_1	D_2	D_3
C_1	H (.5,.8,.9,1)	VH (.8,.9,1,1)	MH (.3,.55,.7,.85)
C_2	L (0,0,.1,.25)	ML (.1,.15,.3,.45)	M (.2,.35,.5,.65)
C_3	M (.2,.35,.5,.65)	MH (.3,.55,.7,.85)	H (.5,.8,.9,1)
C_4	VH (.8,.9,1,1)	MH (.3,.55,.7,.85)	VL (0,0,0,.1)

Table 2. Importance of each PLC's obtained from D1

D_1	C_1	C_2	C_3	C_4
A_1	MG (.3,.55,.7,.9)	VG (.6,.9,1,1)	F (.2,.35,.5,.65)	VP (0,0,0,.1)
A_2	G (.4,.65,.8,.95)	F (.2,.35,.5,.65)	G (.4,.65,.8,.95)	F (.2,.35,.5,.65)
A_3	F (.2,.35,.5,.65)	VP (0,0,0,.1)	VG (.6,.9,1,1)	F (.2,.35,.5,.65)

Table 3. Importance of each PLC's obtained from D2

D_2	C_1	C_2	C_3	C_4
A_1	F (.2,.35,.5,.65)	G (.4,.65,.8,.95)	P (0,.1,.1,.2)	Mp (.1,.2,.3,.5)
A_2	MG (.3,.55,.7,.9)	P (0,.1,.1,.2)	VG (.6,.9,1,1)	P (0,.1,.1,.2)
A_3	P (0,.1,.1,.2)	VP (0,0,0,.1)	F (.2,.35,.5,.65)	Mp (.1,.2,.3,.5)

Table 4. Importance of each PLC's obtained from D3

D_3	C_1	C_2	C_3	C_4
A_1	G (.4,.65,.8,.95)	G (.4,.65,.8,.95)	VG (.6,.9,1,1)	P (0,.1,.1,.2)
A_2	G (.4,.65,.8,.95)	MG (.3,.55,.7,.9)	F (.2,.35,.5,.65)	F (.2,.35,.5,.65)
A_3	MG (.3,.55,.7,.9)	F (.2,.35,.5,.65)	MG (.3,.55,.7,.9)	Mp (.1,.2,.3,.5)

Step4. The (normalized) fuzzy decision matrix is constructed using Eq. (7) or simply in this paper Eq. (6).

Table – 5. The normalized fuzzy decision matrix

$\tilde{x}_{ij}$	A_1	A_2	A_3	weight
C_1	(.2,.51,.67,1)	(.3,.61,.76.95)	(0,.33,.43,.9)	(.3,.75,.86,1)
C_2	(.4,.73,.87,1)	(0,.33,.43,.9)	(0,.11,.16,.65)	(0,.17,.3,.65)
C_3	(0,.45,.53,1)	(.2,.63,.76,1)	(.2,.53,.73,1)	(.2,.57,.7,1)
C_4	(0,.1,.13.5)	(0,.26,.37.65)	(.1,.25,.36,.65)	(0,.48,.57,1)

Step 5. The weighted normalized fuzzy decision matrix is constructed using Eq. (6).

Table 6. The weighted normalized fuzzy decision matrix

$\tilde{q}_{ij}$	A_1	A_2	A_3
C_1	(.06,.382,.576,1)	(.09,.457,.653,.95)	(0,.247,.369,.9)
C_2	(0,.124,.261,.65)	(0,.056,.129,.585)	(0,.018,.048,.422)
C_3	(0,.256,.371,1)	(.04,.359,.532,1)	(.04,.302,.511,1)
C_4	(0,.048,.074,.5)	(0,.124,.273,.65)	(0,.12,.205,.65)

Step 6. FPIS and FNIS are defined as:

$$F^* = [(1,1,1,1),(1,1,1,1),(1,1,1,1),(1,1,1,1)]$$

and

$$F^- = [(0,0,0,0),(0,0,0,0),(0,0,0,0),(0,0,0,0)]$$

Step7. The distance of each candidate from FPIS and FNIS are calculated, respectively using Eq. (11),(12).

Table 7. Distance of each candidate from FPIS and FNIS

$\alpha = 0$	C_1	C_2	C_3	C_4
$d(\widetilde{A_1}, F^*)$	.94	1.35	1	1.5
$d(\widetilde{A_1}, F^-)$	1.06	.65	1	.5
$d(\widetilde{A_2}, F^*)$	.96	1.415	.96	1.35
$d(\widetilde{A_2}, F^-)$	1.04	.585	1.04	.65
$d(\widetilde{A_3}, F^*)$	1.1	1.578	.96	1.35
$d(\widetilde{A_3}, F^-)$	.9	.422	1.04	.65

Step 8. The closeness coefficient is calculated for each candidate.

Table 8. The closeness coefficient

A \ CC	CC_i
A_1	$\frac{3.21}{3.21+4.79}$=.401
A_2	$\frac{3.315}{3.315+4.68}$=.414
A_3	$\frac{3.012}{3.012+4.988}$=.376

5. Results and Discussion

According to these closeness coefficients, the ranking order of the three candidates will be A2, A1 and A3, respectively. Obviously, the best selection is candidate A2 having a greater closeness coefficient.

In this paper we considered the multi-criteria decision making problem when there is a group of decision makers based on Marketing Mix (4Ps). While crisp data are inadequate to model the real life situations in MCDM.

We used FUZZY TOPSIS with trapezoidal fuzzy numbers and we presented Fuzzy Positive-Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS).This method can be useful for companies or managers whom have decision makers' team in management to get the best choices.

Corresponding Author:
Afshin Mirzaei
Department of Industrial Engineering and Management, Shahrood University of Technology ,
SHAHROOD, IRAN
E-mail: mirzaei.afshin@gmail.com

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