Transient Response of Vertical Magnetic Dipole above a two layer medium

Adel A. S. Abo Seliem[1] and Fathia Alseroury[2]

[1]Department of Mathematics, Faculty of Science, University of Kafr El-Sheikh, Egypt
[2]Department of Physics, Faculty of Science for Girls, King Abdulaziz University, Jeddah21412, KSA
adel_atta60@yahoo.com

Abstract: The duct model used here is that Kahan and Eckart [1950]. The source of the electromagnetic field is taken to a vertical magnetic dipole in the upper surface layer, with an arbitrary time – varying moment, we can determine the transient behavior of the electric field strength at any distance above the duct .The paper presents a method which allows the calculation of the atmospheric distortion of radar pulse, provided that the influence of the atmosphere is to transfer the transmitted signal through a duct. The polarization of the primary source, whose moment varies arbitrarily in time, is chosen in such a way that it allows the exact determination of the electric field strength at some field point above the duct layer. From the physical point of view, Cahniard's idea is applicable as it is based on evaluating the field in a series of image sources of the primary source. The step – function solution of the problem can then be determined as infinite integrals over finite integrals. Two cases would be distinguished on the basis of the distance between the receiving and transmitting ends and whether it is greater or lesser than the total reflection distance.

[Adel A. S. Abo Seliem and Fathia Alseroury. **Transient Response of Vertical Magnetic Dipole above a two layer medium]** Journal of American Science 2012;8(9): 317-324]. (ISSN: 1545-1003). http://www.jofamericanscience.org. 46

Key Words: Transmitted signal; Electromagnetic field

1. Introduction

Historically, in the problem of electromagnetic radiation from a vertical magnetic dipole situated at a certain height above a plane earth, all field quantities are usually assumed to vary harmonically in time. Summerfield [1], calculated the electromagnetic radiation from an electric vertical dipole, located above the plane interface of two media.

Many writers Wait [2] Moore [3] and Durrani [4] have considered this problem, the aim of the present work is to extend the study – state to transient excitation when no restrictions on the distance between receiving and transmitting ends are made. two integral transforms are applied to analyze the transient field of vertical electric dipole above a dielectric layer the distinction of different cases where the distance between the receiving and transmitting end are greater and lesser than the total reflection distance studied Abo Seliem [5] .

The problem has been studied by Arutaki and Chiba [6] and Abo Seliem [7] .This Integral is estimated by using the steepest descent method , along the count our Γ and around the branch cuts , from the obtained results the Saddel point method shows that the reflected waves and integrals Abo Seliem [8], the component of electric field strength is also arbitrary for the excitation function F(t) = t at some fixed but arbitrary position from the point of observation in the half-space .The problem has been studied by Abo-seliem [7] and this integration is extended by using the steepest descent method along the contour Γ and around the branch – cuts . From the obtained results, the Saddle point method shows that the reflected waves integrals.

2. Formulation of the problem

As shown in Fig. 1, the duct model of Kahen and Eckart [9]. A dielectric layer is assumed of relative permittivity ε_1 over laying an infinitely conducting plane earth which is confined by the plane z = 0 of a rectangular coordinate system. The source of the field is assumed to be a vertical electric dipole in the medium 1 at the point x = y =0, z =d > 0 whose moment is given by.

$\bar{P}_m = \{0,0,F(t)\delta(x,y,z-d)\}$, t being the time variable and δ the three dimensional –distribution. Regarding F(t) , we make the assumptions F(t) = 0 for t $\leq$ 0 and $\frac{dF(t)}{dt}$ =0

At, t = 0 the electric field $\vec{E}(x,y,z;t)$ of this vertical magnetic dipole has only horizontal component and co -forms to following wave equation.

$$(\nabla^2 - v^{-2}\frac{\partial^2}{\partial t^2})\vec{E}(x,y,z;t) = \nabla \cap \frac{\partial}{\partial t} P_m \quad \textbf{(1)}$$

Where v_i denotes the phase velocity in the medium. The application of a Laplace transform in time and a two – dimensional Fourier transform in horizontal coordinates x, y leads under consideration of the initial, boundary and transform of $\vec{E}(x, y, z; t)$ being the variable in the transform space we get for $h \prec z \prec \infty$

3. Method of Solution

The solution point is the wave equation for the electrical field $\vec{E}(x, y, z; t)$ in the two media

$$[\frac{\partial^2}{\partial z^2} - s^2 \gamma_i^2]\vec{E}(\alpha, \beta, z, s) = \begin{cases} 0 & for \quad i = 2 \\ jsf(s)\delta(z-d)(\beta, -\alpha, 0) & for \quad i = 1 \end{cases} \qquad (2)$$

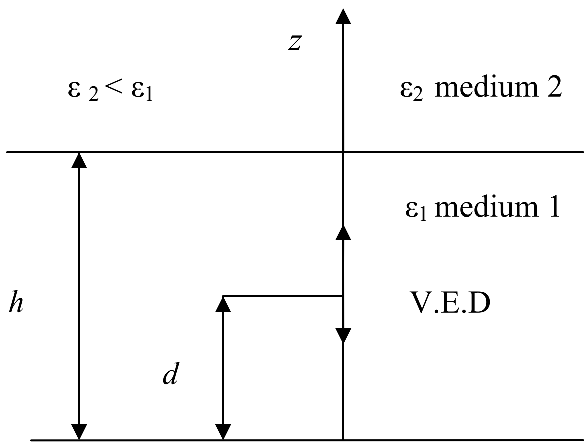

Fig. (1) Geometry of the problem

Where $j^2 = -1$, $\gamma_i^2 = (\alpha^2 + \beta^2 + v_i^{-2}), i = 1,2 \quad with\ \mathrm{Re}(\gamma_i) \geq 0$ in the medium 1 This an integral representation results of the Laplace transform of electric field in terms of two – dimensional inverse Fourier integral.

$$E_x^{(i)}(x, y, z; s) = -\frac{s^3 f(s)}{8\pi^2} \int \int_{-\infty}^{+\infty} j\beta \left[\frac{e^{-s\gamma_1|z-d|}}{\gamma_1} + \frac{(1 + c_{12}e^{-2s\gamma_1(h-d)})e^{-s\gamma_1(z+d)}}{\gamma_1(1 + c_{12}e^{-2\gamma_1 h})} + \frac{c_{12}(1 + c_{12}e^{-2s\gamma_1 d})e^{-s\gamma_1(2h-z-d)}}{\gamma_1(1 + c_{12}e^{-2s\gamma_1 h})} \right] e^{js(\alpha x + \beta y)} d\alpha d\beta \qquad (3)$$

With the reflection coefficient at the upper duct boundary is $C_{12} = \frac{\gamma_1 - \gamma_2}{\gamma_1 + \gamma_2}$. Here α and β are variables in the transform space of the two – dimensional Fourier transform f(s) is the Laplace transform of F(t).The components $E_y^{(i)}(x, y, z, s)$ can be determined from $E_x^{(i)}(x, y, z, s)$ by substituting $\frac{j\beta}{\gamma_1}$ which precedes the square brackets in (3), with $-\frac{j\alpha}{\gamma_1}$, also, in the duct $E_x^{(i)}(x, y, z, s)$ and $E_y^{(i)}(x, y, z, s)$ can be determined, we discus only $E_x^{(i)}(x, y, z, s)$, it follow that by using polar coordinate ,we get

$$E_x^{(i)}(p,\phi,z;s)=-\frac{s^3 f(s)}{8\pi^2}\int\int_{-\infty}^{+\infty} j\lambda\sin'\phi'\left[\frac{e^{-s\gamma_1|z-d|}}{\gamma_1}+\frac{(1+c_{12}e^{-2s\gamma_1(h-d)})e^{-s\gamma_1(z+d)}}{\gamma_1(1+c_{12}e^{-2\gamma_1 h})}+\right.$$

$$\left.\frac{c_{12}(1+c_{12}e^{-2s\gamma_1 d})e^{-s\gamma_1(2h-z-d)}}{\gamma_1(1+c_{12}e^{-2s\gamma_1 h})}\right]e^{js\lambda p\cos(\phi-\phi y)}\lambda d\lambda d\phi' \qquad (4)$$

And

$\gamma_i^2=(\lambda^2+v_i^{-2}), i=1,2 \qquad with\ \mathrm{Re}(\gamma_i)\geq 0$

By using Bessel representation, where $j_0(\lambda sp)$ is the Bessel function of order zero next, we transform $j_0(\lambda sp)$ into a Hankel function $H_0^{(1)}(\lambda sp)$ to change the semi- infinite integral in (4) into a fully infinite integral

$$E_x^{(i)}(p,\phi,z;s)=-\frac{s^3 f(s)}{8\pi^2}\int_{-\infty}^{+\infty}\left[\frac{e^{-s\gamma_1|z-d|}}{\gamma_1}+\frac{(1+c_{12}e^{-2s\gamma_1(h-d)})e^{-s\gamma_1(z+d)}}{\gamma_1(1+c_{12}e^{-2\gamma_1 h})}+\right.$$

$$\left.\frac{c_{12}(1+c_{12}e^{-2s\gamma_1 d})e^{-s\gamma_1(2h-z-d)}}{\gamma_1(1+c_{12}e^{-2s\gamma_1 h})}\right]\frac{H_0^{(1)}}{\gamma_1}\lambda^2 d\lambda \qquad \mathbf{(5)}$$

4- Discussion of the integrals

Referring to equation (5)the first term of the integral represents the direct wave originating from the source and traveling towards the observating point the second with $e^{-s\gamma_1(z+d)}$ the dimensional wave towards the point of observation and we treat the second term

$$E_x^{(s)}(\rho,\phi,z;s)=-\frac{s^3 f(s)}{8\pi^2}\int_{-\infty}^{+\infty}\left[\frac{((\gamma_1+\gamma_2)+(\gamma_1-\gamma_2)e^{-2s\gamma_1(h-d)})e^{-s\gamma_1(z+d)}}{(\gamma_1+\gamma_2)+(\gamma_1-\gamma_2)e^{-2\gamma_1 h})}\right]$$

$$\frac{H_0^{(1)}(\lambda ps)}{\gamma_1}\lambda^2 d\lambda \qquad (6)$$

Where

$\gamma_i^2=(\lambda^2+v_i^{-2}), i=1,2 \qquad with\ \mathrm{Re}(\gamma_i)\geq 0$

.Tte integrand (6) is not single – valued due to the square roots of γ_i (i=1,2) that it. The integrand has four values corresponding to the four combinations of sings γ_1 and the Riemman surface has four sheets. We demand that the path of integration the convergence of our integral should be an the permissible sheet only, we put $\lambda=j\nu_1^{-1}\sin\alpha$ so γ_i is transformed as $\gamma_1=j\nu_1^{-1}\cos\alpha$ .Therefore the branch cuts corresponding to γ_0 vanich and we have $\gamma_1=\sqrt{-\nu_1^{-2}\sin^2\alpha-\nu_2^{-2}}$ .The branch cuts $R(\gamma_i)=0$ and other situation on illustrated in Fig(2) the permissible sheets of -plane is mapped into a band zero which is put between $R(\gamma_i)=0$ on the z-plane.

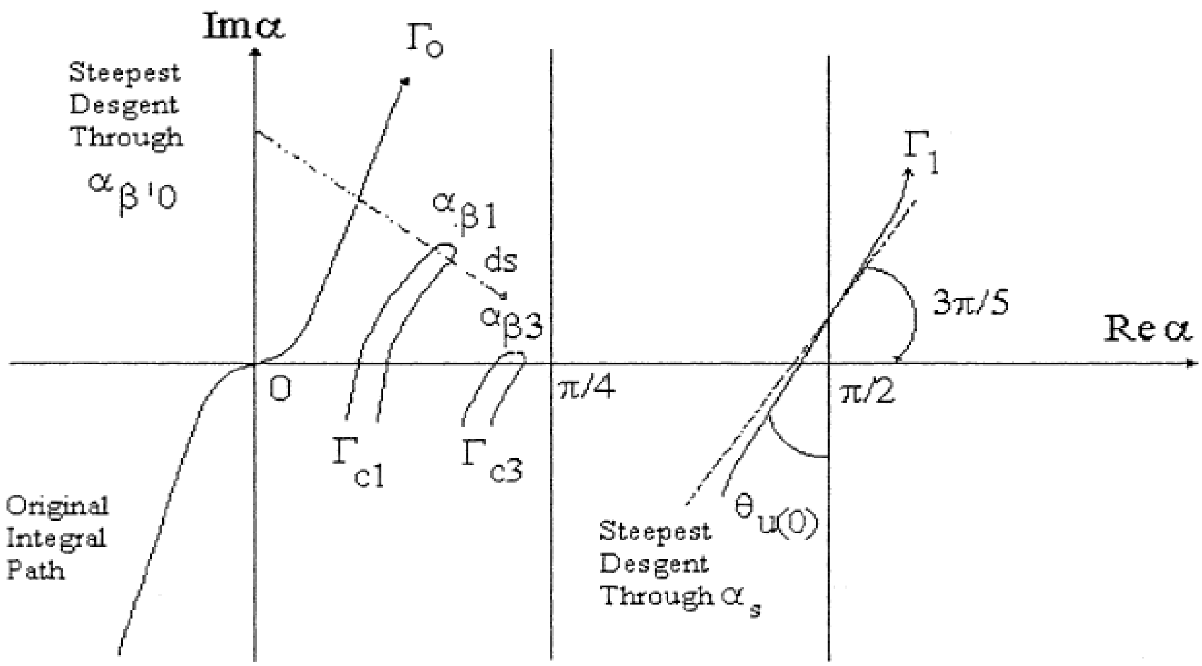

Fig. (2) Branch cuts, the steepest descent paths and the poles in the positive λ- plane

We treat the far field, so that the Hankel function can be transform into the asymptotic expansion ,the $E_x^{(i)}(x,y,z,s)$ will be as follows;

The above integral cannot be solved exactly , I we applied then the Saddle point method, in order to change the integral pathes which it the steepest descent through Γ_1

$$E_x^{(s)}(\rho,\phi,z;s) = -\frac{s^3 f(s)}{2\pi} e^{-\frac{3}{2}\pi} \sqrt{\frac{2}{\pi s p}}\, v_1^{-1} \int_{\Gamma_1} \phi(\alpha) \sin^2\alpha \exp(-j v_i^{-1} \sin(\alpha+\theta_i) d\alpha \quad (7)$$

Where

$$\phi = [\frac{((N_1+\cos\alpha)+(N_1-\cos\alpha)\exp(-2js v_1^{-1}(h-d)\cos\alpha)}{((N_1+\cos\alpha)+(N_1-\cos\alpha)\exp(-2js v_1^{-1}(h)\cos\alpha))}] \quad (8)$$

And $N_1 = \sqrt{-\sin^2\alpha + n_1^2}$, $n_1 = \frac{v_1}{v_2}$ and the integral in the complex r- plane corresponding to $\lambda_i, -\infty \to \infty$.

By Taylor expansion , we get

$$\phi = \sum [\frac{(-N_1+\cos\alpha)}{(N_1+\cos\alpha)}]^m \exp(-2js v_1^{-1} m(h)\cos\alpha + \sum [\frac{(-N_1+\cos\alpha)}{(N_1+\cos\alpha)}]^{m+} \exp(-2js v_1^{-1} m+1(h-d)\cos\alpha]$$

$$\phi = \sum_{m=0}^{\infty} \phi_u(m)\exp(-2js v_1^{-1} m(h)\cos\alpha) + \sum_{m=0}^{\infty} \phi_d(m)\exp(-2js v_1^{-1} m+1(h-d)\cos\alpha) \quad (9)$$

Where

$$\exp(-2js\nu_1^{-1}(h)\cos\alpha)\left|{-N_1+\cos\alpha}\middle/{N_1+\cos\alpha}\right| \prec 1 along \Gamma_0, -N_1+\cos\alpha=0$$

$$E_x^s = E_u + E_d = \sum_{m=0}^{\infty} E_u(m) + \sum_{m=0}^{\infty} E_d(m) \quad (10)$$

where

$$E_*(m) = c\int_{\Gamma_1} \phi_*(\alpha)\sin^2\alpha\exp(-j\nu_i^{-1}R_*(m)\sin(\alpha+\theta_i(m))d\alpha$$

(11)

And $R_u = \sqrt{r^2+(z+d+2mh)^2}$,

$R_d = \sqrt{r^2+(z-d+2(1+m)h)^2}$

$$\cos\theta_*(m) = \frac{r}{R_*(m)},$$

$$c = \frac{jf(s)s^3}{2\pi}\sqrt{\frac{2}{\rho\pi s v^2}}\exp(\frac{3j}{4}\pi) and * = u,d$$

5-Evaluation of the integral along the contour and around the branch cuts.

As $E_*(0)$ is the dominating term in E_x^s it will be initially estimated ,Jones [10]

$$E_u(0) = c\int_{\Gamma_1} \sin^2\alpha\exp(-j\nu_i^{-1}R_u(0)\sin(\alpha+\theta_u(m))d\alpha \quad (12)$$

$$E_d(0) = c\int_{\Gamma_1} \frac{-N_1+\cos\alpha}{N_1+\cos\alpha}\sin^2\alpha\exp(-j\nu_i^{-1}R_d(0)\sin(\alpha+\theta_d(m))d\alpha \quad (13)$$

The above integral cannot be solved exactly, we applied to the saddle point method, in order to change the integral path $\Gamma_i to \Gamma_1$ which is the steepest descent through $\alpha_1 = \frac{\pi}{3} - \theta_u(0)$ and we must consider the poles and stride our branch cuts. If $R_u(0) and R_d(0)$ are large $\theta_u(0) = \theta_1$ h, can be assumed to be nearly zero thus

$$E_u(0) = c\int_{\Gamma_1}\sqrt{\frac{\pi}{s\nu_1^{-1}R_u(0)}}\exp(-js\nu_i^{-1}R_u(0)\frac{\exp(-js\nu_i^{-1}R_u}{R_1} \quad (14)$$

$$E_d(0) = c\int_{\Gamma_1}\sqrt{\frac{\pi}{s\nu_1^{-1}R_d(0)}}\exp(-js\nu_i^{-1}R_d(0) \quad (15)$$

According $E_{uc}(0) and E_{ds}(0)$ is vanishing rapidly, when r becomes large. generally, we suppose the following

$$E_x^s = \sum_{m=0}^{\infty} E_{uc}(m) + \sum_{m=0}^{\infty} E_{sd}(m)$$

$E_{uc}(m) and E_{ds}(m)$ can be estimated similarly to $E_{uc}(0) and E_{ds}(0)$

$$E_{cu}(m) = c\int_{\Gamma_1}\sqrt{\frac{\pi}{s\nu_1^{-1}R_{cu}(m)}}\exp(-js\nu_i^{-1}R_{uc}(m) \quad (16)$$

$$E_{dsu}(m) = c\int_{\Gamma_1}\sqrt{\frac{\pi}{s\nu_1^{-1}R_{dsu}(m)}}\exp(-js\nu_i^{-1}R_{dsc}(m) \quad (17)$$

And the integral path strides over the two branch cuts in the complex plane, we obtain the $E_{uc1}(0) and E_{ds1}(0)$,where

$$E_{uc1}(0) = c\int_{\arcsin\frac{\nu_1}{\nu_2}}^{\frac{\pi}{4}-j\alpha} \sin^2\alpha\exp(-j\nu_i^{-1}R_u(0)\sin(\alpha+\theta_u(0))d\alpha \quad (19)$$

$$E_{ds1}(0) = c\int_{\arcsin\frac{v_1}{v_2}}^{\frac{\pi}{4}-j\alpha} \frac{-N_1+\cos\alpha}{N_1+\cos\alpha}\sin^2\alpha\exp(-jv_i^{-1}R_d(0)\sin(\alpha+\theta_d(0))d\alpha \quad (20)$$

The steepest descent through the branch point $\alpha_{\beta 1}$ is considered Brekhovsikh[11], the trace of the steepest descent is obtained from $\operatorname{Im}\{jv_1^{-1}\sin(\alpha+\theta_u(0))\}=\operatorname{Im}\{jv_1^{-1}\sin(\alpha_{\beta 1}+\theta_u(0))\}=\frac{1}{v}$ and the real part of exponent decreases most rapidly along this path, we deform the function of α in the integral into a function of s using Tayler expansion .the new variable s through the equation is $\sin\alpha = n_i + is^2$ where $n_i = \sin\alpha_{\beta 1}, \cos d\alpha = 2jsd\tau$, we get;

$$E_{uc1} = \sum_{m=0}^{\infty} E_{uc} = \sum_{m=0}^{\infty} \frac{js^2 f(s)}{2\pi r^2}\frac{v_1^2}{v_2^2}\exp(-jv_i^{-1} - js\sqrt{v_1^2-v_2^2}(z+d+2mh)(m+1)(\frac{1-n_1}{1+n_1})^m \quad (21)$$

$$E_{dsc1} = \sum_{m=0}^{\infty} E_{dsc} = \sum_{m=0}^{\infty} \frac{js^2 f(s)}{2\pi r^2}\frac{v_1^2}{v_2^2}\exp(-jv_i^{-1} - js\sqrt{v_1^2-v_2^2}(z+d+(2m=1)h)(m+1)(\frac{1-n_1}{1+n_1})^{m+1} \quad (22)$$

6. **Physical meaning and numerical calculations**

$E(r,z,s)$ Is the transform of $E(r,z,t)$. By using the inverse Laplace transform of $f(s)=\frac{1}{s}$ and equation (12) , (13) , (21 and (22) we have the complete solution of $E_x^{(1)}$

$$E_x^{(1)}(r,t) = E_x^{prim)}(r,t) + E_x^{(s)}(r,t) \quad (23)$$

Where

$$E_x^{prim}(r,t) = \begin{cases} 0 & t \succ \frac{\tau}{v_1} \\ \frac{2}{\pi}\delta(1-\frac{t}{v_1}) & t \prec \frac{\tau}{v_1} \end{cases} \quad (24)$$

And

$$E_x^{s}(r,t) = \sum_{m=0}^{\infty} E_{uc1}(m) + \sum_{m=0}^{\infty} E_{dc1}(m) + \sum_{m=0}^{\infty} E_{uc}(m) + \sum_{m=0}^{\infty} E_{ds}(m) \quad (25)$$

Where

$$E_{uc} = \frac{-2j}{\pi}\sum_{m=0}^{\infty}\frac{v_1^2}{2r^2 v_2}(2m+1)\left(\frac{1-n_1}{1+n_1}\right)^m \delta(t-t_1)_s \quad (26)$$

Where

$$t_1 = \frac{r}{v_1} + \frac{(z=d+2mh)}{v_2}$$

And,

$$E_{dc} = \frac{j}{\pi}\sum_{m=0}^{\infty}\frac{v_1^2}{2r^2 v_2}(2m+1)\left(\frac{1-n_1}{1+n_1}\right)^{m+1} \delta(t-t_2)_{ds} \quad (27)$$

where

$$t_2 = \frac{r}{v_1} + \frac{(z-d+(2m+1)h)}{v_2}$$

And

$$E_{us}(r,t) = \frac{j}{\pi}\sum_{m=0}^{\infty}\frac{v_1^{-2}}{r^{-\frac{1}{2}}}(r^2+(z+d+2mh)^2)^{-\frac{1}{2}}(2m+1)\left(\frac{1-n_1}{1+n_1}\right)^m \delta(t-t_3)_s \quad (28)$$

where $$E_{dss}(r,t) = \frac{j}{\pi}\sum_{m=0}^{\infty}\frac{v_1^{-2}}{r^{-\frac{1}{2}}}(r^2+(z-d+2(m+1)h)^2)^{-\frac{1}{2}}(m+1)\left(\frac{1-n_1}{1+n_1}\right)^{m+1} \delta(t-t_4)_{ds} \quad (29)$$

$$t_3 = \frac{(r^2 + (z + d + 2mh))^2)^{\frac{1}{2}}}{v_1}$$

where

$$t_4 = \frac{(r^2 + (z - d + 2(m+1)h))^2)^{\frac{1}{2}}}{v_1}$$

Where

$\left(\frac{1-n_1}{1+n_1}\right)$ Is the coefficient equivalent to the reflection one at the surface in the duct .If the height duct is h=20m and the difference of relative permutations in the boundary $\Delta\varepsilon = 4.10^{-4}$, the height of the primary source and the point of observation are taken as z=d=15m, the $n_1 = \frac{v_1}{v_2} = 1.00033$,if r=4000m.

7. Physical meaning and numerical calculations

Figure describes the relationship between Im|y| and t at t < 6 x 10 $^{-3}$s .The absolute value of the z-component of the electric field strength increasing with increase the time. The absolute value of the z-component of the electric field strength is increasing when the spherical distance between the source and the point of observation is very small. ($|E| \ \alpha \frac{1}{R}$).

We note that the absolute value of the electric field strength is dependent of R. At t > 6 x 10 $^{-3}$ s .The absolute value of the electric field strength is constant for each values of R = 5 km ,10 km, 15 km , 20 km and 25 km .

We note that: - the saturation curve when the time increases.

Figures from 3 to 6): describes the relationship between Log |Abs|y||and t.

1- At time < 5 x 10^{-3} s .The value of Log of the electric field strength is negative (-ve) value.

2- At time (t) < 6x 10^{-3} s. Log value of the electric field strength increasing by increase the time.

3- At time (t) > 6x 10^{-3} s. Log value of the electric field strength is constant (+ve) for all value.

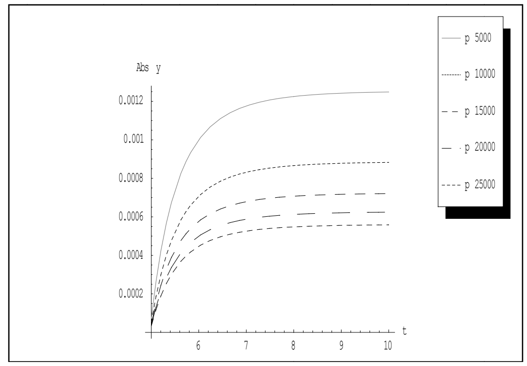

Fig.(3) describes the relationship between Abs[y]and t

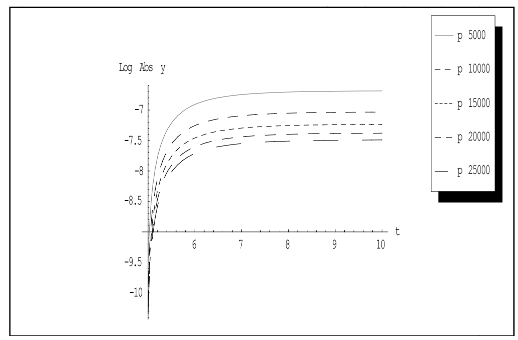

Fig.(4) describes the relationship between Log |Abs|y||and t

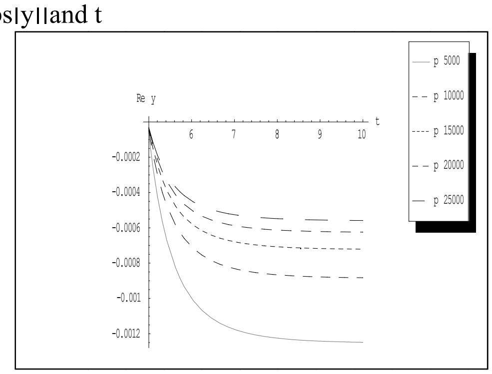

Fig.(5) describes the relationship between Re|y|and t

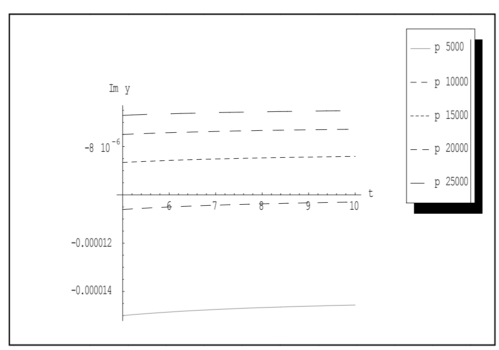

Fig.(6) describes the relationship between Im|y| and t

The real value of the electric field strength is constant taken negative (-ve) value for all values of R .Figures (3 to 6.): describes the relationship between imaginary value of the electric field strength and the time .In this figures (saturation relationship) where value of imaginary is constant at varying time for each value of R but we note that at R = 5 km and 10 km the saturation curves are negative (-ve) value. But for R = 15 km, 20 km and 25 km the saturation curve are positive (+ ve) value.

8. Conclusion

The integral represent of the physical point of view; also, the integral is evaluated by two mathematical methods: Residue and Saddle point method. A disadvantage of the method is not be able to use to calculate the potential in the dielectric half – space outside the layer in similar manner.

Corresponding author
Adel A. S. Abo Seliem
Department of Mathematics, Faculty of Science, University of Kafr El-Sheikh, Egypt
adel_atta60@yahoo.com

9. References

[1] A.Sommerfeld ,Uber die Ausbreitung der wellen in der drahlosen telegraphic ,Ann.d.Phys.Lpz **28**,pp665-736(1909).
[2] J.R. Wait, Electromagnetic waves in stratified Media ,Pergumon Press, Oxford ,pp.608(1970).
8/10/2012
[3] R.K.Moore,W.E.Blair ,Dipole radiation in a conducting half-space ,J Res Nat. Bar standards. **65D** PP.547-583(1961).
[4] S.H.Durrani. Air to under sea communication with magnetic dipole ,trans.IEEE. **Ap-12**(1964).
[5] A.A. Abo Seliem, The transient reponse above an evaporation duct , J.Phys . D. Appl. Phys **31**, pp3046-3050(1998) .
[6] A.Arutaki,J.Chiba,Commincation in a three – layered conducting media with a vertical magnetic dipole , Trans.IEEE,Antennas Propagat. **28** ,pp.551-556(1980).
[7] A.A. Abo Seliem Propagation of the transient electromagnetic field above atmospheric surface duct Applied Mathematics and Computation, 145, 25 December),vol.9 ,no. 2 ,pp.631-639(2003).
[8] A.A. Abo Seliem, Evaluation of the transient electromagnetic field in evaporation duct, Applied Mathematics and Computation ,vol.151, pp.411-421(2004) .
[9] T. Kahan, G .Eckart , Theorie de la propagation des ondes electromagne-tiques le guide d'ondes atmospheriques ,Ann.Phys.vol.5,pp.641-705(1950).
[10] D.S. Jones, Theory of electromagnetism, Pergamon Press, London, England, (1964) .
[11] L.M. Brekhovsikh, Wave in Layered Media. Academic. Press. New York. (1960) .