Theoretical Analysis in Particle Swarm Optimisation

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Abstract: Nowadays, particle swarm optimisation (PSO) is considered as one of the most efficient techniques in solving optimisation problems. Implementing theoretical analyses in PSO can lead to a more deep understanding of its behavior and characteristics and may quicken the improvement in its computational behavior. However, almost all the existing findings on PSO are based on experimental observations and just in a few cases, theoretical analysis on PSO has been implemented. In this paper, theoretical analyses implemented on PSO are reviewed and some directions for future research are proposed.

Keywords: Particle swarm optimisation; optimisation; theoretical analysis.

1. Introduction

There are so many optimisation problems in various areas of science and engineering. For solving them, there exist twofold approaches; classical approaches and heuristic approaches. Classical approaches such as linear programming and non-linear programming are not efficient enough in solving optimisation problems. Since they suffer from curse of dimensionality and also require preconditions such as continuity and differentiability of objective function that usually are not met.

Heuristic approaches which are usually bio-inspired include a lot of approaches such as genetic algorithms, evolution strategies, differential evolution and so on. Heuristics do not expose most of the drawbacks of classical and technical approaches. Among heuristics, particle swarm optimisation (PSO) has shown more promising behavior.

PSO is a stochastic, population-based optimisation technique introduced by Kennedy and Eberhart (Kennedy and Eberhart, 1995). It belongs to the family of swarm intelligence computational techniques and is inspired of social interaction in human beings and animals (especially bird flocking and fish schooling).

Some PSO features that make it so efficient in solving optimisation problems are the followings:

- In comparison with other heuristics, it has less parameters to be tuned by user.
- Its underlying concepts are so simple. Also its coding is so easy.
- It provides fast convergence.
- It requires less computational burden in comparison with most other heuristics.
- It provides high accuracy.
- Roughly, initial solutions do not affect its computational behavior.
- Its behavior is not highly affected by increase in dimensionality.
- It is efficient in tackling multi-objectives, multi-modalities, constraints, discrete/integer variables.
- There exist many efficient strategies in PSO for mitigating “premature convergence.” Thus, its success rate is so high.

Despite the fact that implementing theoretical analyses in PSO can lead to a more deep understanding of its behavior and characteristics and may quicken the improvement in its computational behavior, just in a few cases, theoretical analyses on PSO have been conducted. It is undeniable that this aspect should be paid more attention by PSO research community. In this paper, the aim is to review all the implemented theoretical analyses on PSO and propose some directions for future research.

The paper is organised as follows; in section 2, an overview of PSO is provided. In section 3, all the implemented theoretical analyses on PSO are reviewed. Finally, drawing conclusions and proposing some directions for future research is implemented in section 4.

2. PSO Overview

PSO starts with the random initialisation of a population (swarm) of individuals (particles) in the n-dimensional search space (n is the dimension of problem in hand). The particles fly over search space with adjusted velocities. In PSO, each particle keeps two values in its memory; its own best experience, that is, the one with the best fitness value (best fitness value corresponds to least objective value since fitness function is conversely proportional to objective function) whose position and objective
value are called $P_i$ and $P_{best}$ respectively and the best experience of the whole swarm, whose position and objective value are called $g_b$ and $g_{best}$ respectively. Let denote the position and velocity of particle $i$ with the following vectors:

$$X_i = (X_{i1}, X_{i2}, ..., X_{id}, ..., X_{in})$$

$$V_i = (V_{i1}, V_{i2}, ..., V_{id}, ..., V_{in})$$

The velocities and positions of particles are updated in each time step according to the following equations:

$$v_{id}(t + 1) = v_{id}(t) + c_1 r_{id}(p_{id} - x_{id}) + c_2 r_{2d}(p_{gd} - x_{id})$$ (1)

$$x_{id}(t + 1) = x_{id}(t) + v_{id}(t + 1)$$ (2)

Where $c_1$ and $c_2$ are two positive numbers and $r_{id}$ and $r_{2d}$ are two random numbers with uniform distribution in the interval [0,1]. Here, according to (1), there are three following terms in velocity update equation:

1) The first term this models the tendency of a particle to remain in the same direction it has traversing and is called “inertia,” “habit,” or “momentum.”

2) The second term is a linear attraction toward the particle’s own best experience scaled by a random weight $c_1 r_{id}$. This term is called “memory,” “nostalgia,” or “self-knowledge.”

3) The third term is a linear attraction toward the best experience of the all particles in the swarm, scaled by a random weight $c_2 r_{2d}$. This term is called “cooperation,” “shared information,” or “social knowledge.”

The procedure for implementation of PSO is as follows:

1) Particles’ velocities and positions are initialised randomly, the objective value of all particles are calculated, the position and objective of each particle are set as its $P_i$ and $P_{best}$ respectively and also the position and objective of the particle with the best fitness (least objective) is set as $P_g$ and $g_{best}$ respectively.

2) Particles’ velocities and positions are updated according to equations (1) and (2).

3) Each particle’s $P_{best}$ and $P_i$ are updated, that is, if the current fitness of the particle is better than its $P_{best}$, $P_{best}$ and $P_i$ are replaced with current objective value and position vector respectively.

4) $P_g$ and $g_{best}$ are updated, that is, if the current best fitness of the whole swarm is fitter than $g_{best}$, $g_{best}$ and $P_g$ are replaced with current best objective and its corresponding position vector respectively.

5) Steps 2-4 are repeated until stopping criterion (usually a presupposed number of iterations or a quality threshold for objective value) is reached.

It should be mentioned that since the velocity update equations are stochastic, the velocities may become too high, so that the particles become uncontrolled and exceed search space. Therefore, velocities are bounded to a maximum value $V_{max}$, that is (Eberhart, Shi and Kennedy, 2001):

$$|V_{id}| > V_{max} \text{ then } V_{id} = \text{sign}(V_{id})V_{max}$$ (3)

Where $\text{sign}$ represents sign function.

However, primary PSO characterised by (1) and (2) does not work desirably; especially since it possesses no strategy for adjusting the trade-off between explorative and exploitative capabilities of PSO. Therefore, the inertia weight PSO is introduced to remove this drawback. In inertia-weight PSO, which is the most commonly-used PSO variant, the velocities of particles in previous time step is multiplied by a parameter called inertia weight. The corresponding velocity update equations are as follows (Shi and Eberhart, 1998), (Shi and Eberhart, 1999):

$$v_{id}(t + 1) = \omega v_{id}(t) + c_1 r_{id}(p_{id} - x_{id}) + c_2 r_{2d}(p_{gd} - x_{id})$$

$$x_{id}(t + 1) = x_{id}(t) + v_{id}(t + 1)$$ (4)

Inertia weight adjusts the trade-off between exploration and exploitation capabilities of PSO. The less the inertia weight is, the more the exploration capability of PSO will be and vice versa. Commonly, it is decreased linearly during the course of the run, so that the search effort is mainly focused on exploration at initial stages and is focused more on exploitation at latter stages of the run.

3. Theoretical Analyses in PSO

Almost all the existing findings and conclusions on PSO are based on experimental observations. However, implementing theoretical analyses in PSO can lead to a more deep understanding of its behavior and characteristics and may quicken the improvement in its computational behavior. But due to some reasons, the implementation of theoretical analysis is so difficult and has been rarely undertaken in PSO literature. Some of the reasons are as proceed.

- The forces among particles are stochastic that hinders utilising standard mathematical tools used in the analysis of dynamic systems.
- The PSO’s performance is strongly dependent on the used fitness function, whereas there exist so
many different types of fitness functions. Consequently, drawing conclusions applicable to all
type of fitness functions is so challenging.
- In PSO, the swarm consists of a large number of
  particles which understanding the dynamic of the
  whole is difficult.
- The inherent stochasticity existent is a crucial
  factor in PSO that makes its analysis so difficult.
- The memory that particles and swarm possess,
makes the analysis more challenging.

Due to the above-mentioned reasons, some
simplifying assumptions are used while conducting
theoretical analyses in PSO. Some commonly used
simplifying assumptions are ignorance of
stochasticity, assuming swarm as a single isolated
particle in one-dimension, assuming the coincidence
of personal and neighborhood best, assuming search
process in stagnation state, and also ignoring velocity
clamping and inertia weight. Needless to say, the
less the simplifying assumptions are, the more is the
validity and accuracy of drawn conclusions. So,
researchers are attempting to model PSO’s
characteristics as real as possible. In this paper,
theoretical analyses undertaken in specialised
literature are classified into two main groups; those
which do not consider the stochasticity of PSO and
those who consider it, which are called deterministic-
based (Ozcan and Mohan, 1998), (Shi and Eberhart,
2001), (Clerc and Kennedy, 2002), (Van den Bergh,
2002), (Brandstatter and Baumgartner, 2002),
(Trelea, 2003), (Liang and Suganthan, 2005),
(Yasuda and Iwasaki, 2003), (Blackwell, 2005),
(Blackwell, 2003), (Campana, Fasano and Pinto,
2006), (Campana, Peri and Pinto, 2006), (Zhao,
and Mao, 2009), (Samal, Konar, 2007), (Bratton,
and Blackwell, 2007) and stochastic-based analyses
(Clerc, 2006), (Kadirkamanathan, Selvarajah, and
Fleming, 2006), (Poli, et. al, 2007), (Poli, and
Broomhead, 2007), (Poli, 2007), (Zhang, Li, Zhao,
and Wang, 2009), (Helwig, and Wanka, 2007),
(Chen, and Jiang, 2010), (Ghosh, et. al, 2012),
(Pena, 2008), (Helwig and Wanka, 2008),
(Veramachaneni, Osadciw and Kamath, 2007).
They will be explained bellow.

3.1 Deterministic-based Theoretical Analyses in
PSO
In the first attempt for theoretical analysis in
PSO, in a highly approximate paradigm, PSO has
been modeled as a single, isolated, one-dimension
particle in stagnation state, while inertia weight,
velocity clamping are ignored. According to this
modeling, the particle’s trajectories were determined
(Ozcan and Mohan, 1998). This work was extended
by modeling multiple particles in multi-dimension
while \( P_i \) and \( P_g \) are not assumed to coincide (Shi and
Eberhart, 2001). According to this modeling, it is
concluded that the particles’ trajectories change with
\( C_1 \) and \( C_2 \) and the trajectories rely upon \( C = C_1 + C_2 \).
Under the same simplifying assumptions with
(Shi and Eberhart, 2001), later on, the swarm was
considered as a discrete-time linear dynamic system
wherein the dynamics of the state of the particles are
determined by finding eigenvalues and eigenvectors
of the state transition matrix and it is concluded that
the particles converge to equilibrium if and only if
the eigenvalues are less than unity. Consequently,
since eigenvalues are functions of PSO parameters,
the parameters guaranteeing PSO’s convergence are
determined (Clerc and Kennedy, 2002).

In a later work, under the same assumptions with
(Shi and Eberhart, 2001), it is concluded that the
particles are pulled toward the weighted some of
personal and neighborhood best (Van den Bergh,
2002).

In (Brandstatter and Baumgartner, 2002), an
analogy between PSO considered as a one-dimension
single particle in stagnation state and a damped mass-
spring oscillator is drawn, and notions of “damping
factor” and vibrational frequency” were invoked to
draw some guidelines for setting PSO’s parameters.

In (Trelea, 2003), in a work similar to (Clerc and
Kennedy, 2002) and under the same assumptions
with (Ozcan and Mohan, 1998), the dynamic
behavior and the convergence of PSO are analysed
with discrete time dynamic system theory.
Consequently, some guidelines for setting PSO’s
parameters are extracted.

In (Liang and Suganthan, 2005) and (Yasuda and
Iwasaki, 2003), under the same assumptions with
(Ozcan and Mohan, 1998), but considering inertia
weight, eigenvalue analysis is invoked for
determining PSO parameters which result in a stable
dynamic system.

In (Blackwell, 2005), (Blackwell, 2003) the
constricted PSO is modelled by a single multi-
dimensional particle and changes in spatial extent of
a particle is explored over time and it is concluded
that spatial extent is diminished exponentially over
time.

In (Campana, Fasano and Pinto, 2006) and
(Campana, Peri and Pinto, 2006) fully informed
particle swarm (FIPS) under the same assumptions
with (Ozcan and Mohan, 1998) is modeled as a
dynamic system and eigenvalue analysis is invoked
to determine that each setting for PSO parameters
leads to which type of computational behavior.
Moreover, some guidelines for initialising positions
and velocities are provided so that the most
orthogonality in particles’ trajectories is guaranteed.

In (Zhao, and Mao, 2009), PSO flight equations
are transformed into a linear difference equation and
the particle’s stability criteria as a function of PSO parameters are derived.

In (Samal and Konar, 2007) a closed-loop stability analysis of PSO dynamics is implemented by Jury’s test and root-locus technique. By Jury’s stability test proposes some settings for PSO parameters. An explicit modelling of nonlinearity in feedback path is presented. In this analysis, unlike previous analyses that combined acceleration coefficients in one term, their separate existence is considered and their suitable range for achieving stability is determined. In table 1.1 all different deterministic-based theoretical analysis in PSO, their pros, cons and their main findings are tabulated.

3.2 Stochastic-based Theoretical Analyses in PSO

This group of analyses model the inherent stochasticity of PSO. However, there exists so limited number of these analyses in the literature.

In (Clerc, 2006), assuming a single particle in one dimension, during stagnation with considering the stochasticity of PSO, the distribution of particle velocity is analysed. It is showed that particle’s new velocity is the combination of three following terms.

\[ V(t + 1) = K_1 V(t) - K_2 V(t - 1) + K_3 (P_i - P_g) \]

Where \( K_1 \) represents a forward force, \( -K_2 V(t - 1) \) represents a backward force and also \( K_3 (P_i - P_g) \) indicates the noise. \( K_1, K_2 \) and \( K_3 \) are stochastic variables whose distribution models are determined. These distribution models depend on PSO parameters. It is proved that \( E(K_1) = \omega - C^2 \), \( E(K_2) = 2\omega \), \( E(K_3) = 2\omega \) where \( \omega \) and \( C \) represent inertia weight and sum of acceleration coefficients respectively. By manipulating the distributions of \( K_1, K_2 \) and \( K_3 \) some conclusions and guidelines has been drawn.

In (Kadirkanathan, Selvarajah, and Fleming, 2006), PSO is modeled as a single, one-dimension particle in stagnation state in \( g_{\text{best}} \) PSO with inertia weight considering the stochasticity and lyapunov stability analysis is invoked to investigate particle’s stability. The particle is represented as a nonlinear feedback system, its transfer function is determined and its observability and controllability are proved. A lyapunov function for the system is determined and sufficient conditions on PSO parameters to guarantee convergence are derived. However, the derived conditions are so restrictive due to the conservative characteristic of lyapunov function.

In (Poli, et. al, 2007), a discrete markov chain model of bare-bones PSO assuming a single one-dimensional particle is devised that can approximate it on arbitrary continuous problems to any precision. The objective function is discretised using finite element grid which produces corresponding distinct states in algorithm. Iterating the transfer matrix gives precise information about the behavior of optimizer at each iteration. The experiments strongly support the findings of this theoretical analysis.

In (Poli, and Broomhead, 2007) and (Poli, 2007), considering all characteristics of real PSO but assuming it in a stagnation state, firstly, the exact dynamic equations for the moments of sampling distribution are devised, then according to the conducted statistical analysis, areas in parameter space leading to stability are identified.

In (Zhang, Li, Zhao, and Wang, 2009), based on dynamic characteristic analysis of eigenvalues in Z-plane and control theory rules, new guidelines for setting PSO parameters are provided. The assumptions are a single one-dimensional particle in stagnation state.

In (Helwig, and Wanka, 2007) \( g_{\text{best}} \) PSO is modelled in high-dimensional constrained search spaces considering the stochasticity. Consequently, according the theoretical analysis, best fly back strategy for handling constraints in different type of problems are put forward. For modelling, PSO is modelled as multiple multi-dimensional particles.

In (Chen, and Jiang, 2010), the PSO’s particle interaction behavior is analysed. Firstly, a statistical interpretation of PSO is provided in order to capture the stochastic behavior of the whole swarm. Based on statistical interpretation, the effect of particle interaction is investigated by focusing on social-only PSO model and the lower bounds of the expected particle norm are derived. The assumptions of this analysis are multiple particles, one-dimension, stochastic acceleration coefficients.

In (Ghosh, et. al, 2012) PSO has been modeled with multiple one-dimensional particles and \( l_{\text{best}} \) neighborhood. A state-space model of the swarm dynamics is presented and the necessary conditions that assure stability and asymptotic convergence of the particles’ dynamics are determined by Jury and Blanchard’s stability tests. Its salient aspect is modelling \( l_{\text{best}} \) neighborhood topology which had not conducted before. In table 1.2 all different stochastic-based theoretical analysis in PSO, their pros, cons and their main findings are tabulated.
Table 1.1: Different deterministic-based theoretical analysis in PSO, their pros, cons and main findings

<table>
<thead>
<tr>
<th>Reference Number</th>
<th>Modeling Assumptions</th>
<th>Main findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5]</td>
<td>Single, isolated, one-dimension particle in stagnation state, without stochasticity. $P_1$ and $P_2$ are assumed to coincide.</td>
<td>The particle’s trajectories were determined</td>
</tr>
<tr>
<td>[6]</td>
<td>Single, isolated, one-dimension particle in stagnation state, without stochasticity. $P_1$ and $P_2$ are not assumed to coincide.</td>
<td>The particles’ trajectories change with $C_1$ and $C_2$ and the trajectories rely upon $C = C_1 + C_2$.</td>
</tr>
<tr>
<td>[7]</td>
<td>Single, isolated, one-dimension particle in stagnation state, without stochasticity. $P_1$ and $P_2$ are not assumed to coincide.</td>
<td>Using eigenvalue analysis, the PSO parameters guaranteeing its s convergence are determined.</td>
</tr>
<tr>
<td>[8]</td>
<td>Single, isolated, one-dimension particle in stagnation state, without stochasticity. $P_1$ and $P_2$ are not assumed to coincide.</td>
<td>It is concluded that the particles are pulled toward the weighted some of personal and neighborhood best.</td>
</tr>
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<td>[9]</td>
<td>Single, isolated, one-dimension particle in stagnation state, without stochasticity. $P_1$ and $P_2$ are not assumed to coincide.</td>
<td>Using the analogy between PSO and a damped mass-spring oscillator, some guidelines for setting PSO’s parameters are derived.</td>
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<td>[10]</td>
<td>Single, isolated, one-dimension particle in stagnation state, without stochasticity. $P_1$ and $P_2$ are assumed to coincide.</td>
<td>Using discrete time dynamic system theory, some guidelines for setting PSO’s parameters are extracted.</td>
</tr>
<tr>
<td>[11]-[12]</td>
<td>Single, isolated, one-dimension particle in stagnation state, without stochasticity. $P_1$ and $P_2$ are assumed to coincide, but considering inertia weight.</td>
<td>Eigenvalue analysis is invoked for determining PSO parameters which result in a stable dynamic system.</td>
</tr>
<tr>
<td>[13]-[14]</td>
<td>Constricted PSO by a single multi-dimensional particle.</td>
<td>The changes in spatial extent of a particle is explored over time and it is concluded that spatial extent is diminished exponentially over time.</td>
</tr>
<tr>
<td>[15]-[16]</td>
<td>Fully informed particle swarm (FIPS), single, isolated, one-dimension particle in stagnation state, without stochasticity. $P_1$ and $P_2$ are assumed to coincide.</td>
<td>Eigenvalue analysis is invoked to determine that each setting for PSO parameters leads to which type of computational behavior. Moreover, some guidelines for initialising positions and velocities are provided so that the most orthogonality in particles’ trajectories is guaranteed.</td>
</tr>
<tr>
<td>[17]</td>
<td>Single, isolated, one-dimension particle in stagnation state, without stochasticity. $P_1$ and $P_2$ are assumed to coincide.</td>
<td>By solving linear difference equation, particle’s stability criteria are derived as a function of PSO parameters.</td>
</tr>
<tr>
<td>[18]</td>
<td>Single, isolated, one-dimension particle in stagnation state, without stochasticity. $P_1$ and $P_2$ are assumed to coincide. But unlike previous analyses that combined acceleration coefficients in one term, their separate existence is considered.</td>
<td>A closed-loop stability analysis of PSO dynamics is implemented by Jury’s test and root-locus technique. Jury’s stability test proposes some settings for PSO parameters. An explicit modelling of nonlinearity in feedback path is presented. Also, suitable range of $C_1$ and $C_2$ for achieving stability is determined.</td>
</tr>
</tbody>
</table>

Table 1.2: Different stochastic-based theoretical analysis in PSO, their pros, cons and main findings

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<tr>
<td>[20]</td>
<td>A single particle in one dimension, during stagnation with considering the stochasticity.</td>
<td>The distribution of particle velocity is analysed. It is showed that particle’s new velocity is the combination of three stochastic terms; a forward force, a backward force and a noise. The distribution models of static variables are determined. Consequently, some guidelines for tuning PSO parameters are provided.</td>
</tr>
<tr>
<td>[21]</td>
<td>A single, one-dimension particle in stagnation state in $g_{best}$ PSO with inertia weight considering the stochasticity.</td>
<td>Lyupanov stability analysis is invoked to investigate particle’s stability. The particle is represented as a nonlinear feedback system, its transfer function is determined and its observability and controllability are proved. Sufficient conditions on PSO parameters to guarantee convergence are derived.</td>
</tr>
<tr>
<td>[22]</td>
<td>Bare-bones PSO assuming a single one-dimensional particle.</td>
<td>A discrete markov chain model of bare-bones PSO is devised that can approximate it on arbitrary continuous problems to any precision. The objective function is discretised using finite element grid which produces corresponding distinct states in algorithm. Iterating the transfer matrix gives precise information about the behavior of optimizer at each iteration.</td>
</tr>
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<td>[23]-[24]</td>
<td>All characteristics of real PSO but assuming it in a stagnation state.</td>
<td>Firstly, the exact dynamic equations for the moments of sampling distribution are devised, then according to the conducted statistical analysis, areas in parameter space leading to stability are identified.</td>
</tr>
<tr>
<td>[25]</td>
<td>Single one-dimensional particle in stagnation state.</td>
<td>Based on dynamic characteristic analysis of eigenvalues in Z-plane and control theory rules, new guidelines for setting PSO parameters are provided.</td>
</tr>
<tr>
<td>[26]</td>
<td>$g_{best}$ neighborhood, multiple high-dimensional particles in constrained search space, considering the stochasticity.</td>
<td>According the theoretical analysis, best fly back strategy for handling constraints in different type of problems are put forward.</td>
</tr>
<tr>
<td>[27]</td>
<td>Multiple particles, one-dimension, stochastic acceleration coefficients, considering interaction among particles, ignoring cognitive part of flight equation.</td>
<td>PSO’s particle interaction behavior is analysed. Firstly, a statistical interpretation of PSO is provided. Based on it, the effect of particle interaction is investigated by focusing on social-only PSO model and the lower bounds of the expected particle norm are derived.</td>
</tr>
<tr>
<td>[28]</td>
<td>Multiple one-dimensional particles and $f$ best neighborhood. The salient aspect is modelling $f$ best neighborhood topology which had not conducted before.</td>
<td>A state-space model of the swarm dynamics is presented and the necessary conditions that assure stability and asymptotic convergence of the particles’ dynamics are determined.</td>
</tr>
</tbody>
</table>
4. Conclusions and Future Research Directions

In this paper, theoretical analyses implemented on PSO are reviewed. The aim of theoretical analysis is to model PSO as real as possible. But, to date, this aim has not been approached as desired. A few researches have been done on theoretical analysis in PSO and it should be paid more attention. In particular, the following aspects are highlighted as promising directions for future research on this area.

- Devising more efficient approaches for modelling stochasticity of PSO.
- Implementing theoretical analysis with assuming swarm in stagnation state.
- Implementing theoretical analysis on PSO with different neighborhood topologies like wheel, ring,…. etc.
- Implementing theoretical analysis on PSO with multi-objective objective function.
- Implementing theoretical analysis on PSO with multi-modal objective function.
- Implementing theoretical analysis on PSO with dynamic objective function.
- Implementing theoretical analysis on PSO in constrained environments.
- Implementing theoretical analysis on PSO with discrete/binary/integer variables.
- Implementing theoretical analysis on computational time of PSO.

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