A Stochastic Capacitated P-hub Location Problem: A Case Study of Iran

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Abstract: Robust optimization (RO) can be applied to location problems under uncertainty. In this paper, we present a robust optimization model for stochastic multi-objective operation of capacitated P-hub location problems (MCpHLP-s). Most existing approaches to p-hub location problems are restricted to deterministic environments. However, the volume of demand in networks and the amount of time required to process commodities in a hub are always different over the variety of conditions. We consider both through different scenarios. After the model presented, the goal programming approach is used to solve the problem for a particular case study.
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1. Introduction

Hubs are used as switching equipments in a variety of systems, such as transportation and multistage distribution. Hub location problems deal with locating hubs and assigning each demand point so that the traffic between an origin-destination pair can be routed. Defined as intermediates, hubs play particular role in distribution systems. Based on given problems, hubs can route and organize origin-destination traffic demands, thereby leading to reduced times and costs, and improved parameters.

Models developed on hub location problems are mostly applied to deterministic sets. O'Kelly (1987) presented the first recognized mathematical formula for a hub location problem by studying airline passenger networks. His formulation dealt with the single allocation p-median allocation problem. Research was followed by a variety of studies. Campbell (1994) developed the first integer linear programming formulation for single allocation p-median problems. Thereafter, hub location problems under certainty set have been broadly investigated. However, location –assignment problems in uncertain settings were first investigated by Ermoliev and Leonardi (1982) who developed some models for the location problem with formulation under uncertainty, and solved it by using uncertain programming devices. Louveaux (1986) reviewed the current uncertain models for location problems where locating facilities was considered as the first step in the decision-making process, while distribution pattern was the second step.

Among studies conducted on hub location-assignment problems under uncertainty settings, only seven valid cases have been published. The first article addressed hub location problems under uncertainty was presented by Marianov and Serra (2003). The authors applied the M/D/c queuing models with a capacity constraint to a plane on landing. The model dealt with the hub location optimization in airline networks. Later, Mohammadi et al.(2011) proposed the same model; differences were found in the capacity constraint added and the M/M/c queuing model applied. Yang (2009) developed a model for air traffic demand forecasting. The stochastic programming model was introduced for hub location problems in air traffic and flight path programming when the volume of demands varied over seasons. In the same year, Thaddeus et al. (2009) introduced a stochastic p-median model which could minimize the peak hour travel time by using random constraints to reach guaranteed service level. The problem concerned travel times in a stochastic process with normal distribution. In this line, Contreras et al. (2011) studied models for hub location problems with uncertain transportation demands or costs, but no capacity constraint. Considering uncertainty in the set-up costs and demands, Alumur et al. (2012) developed generic models for single and multiple allocation problems which capture different sources of uncertainty. Recently published study by Zhai et al. (2012) concerns a new two-stage stochastic programming approach for hub location problems with minimum-risk criterion in which random vectors are used to characterize uncertain demands. Through standardization, the author presents a deterministic binary programming problem corresponding to binary fractional programming problems.

Among studies conducted on robust optimization hub location problems, only two papers have been published. Huang and Wang (2009) presented a robust model for hub location, minimizing total transportation costs. Without capacity constraints, the model was solved by a multi-objective genetic algorithm. Makui et al. (2012)also developed a robust optimization algorithm for Yang model with capacity constraints.

The current paper is organized as the follow; section 2 gives a brief history of the robust optimization applied here. Section 3 describes and formulates the problem. And finally, a particular case study is analyzed and solved by goal programming in section 4.

2. Robust Optimization

In this paper, the framework Mulvey and Ruszczynsk (1995) developed for the robust optimization is used for modeling the problem. The framework consists of two robustness approaches: solution robustness and model robustness. The first means that the solution for all scenarios must be approximate to the optimum solution, while the latter refers to feasibility of the solution for all scenarios. However, no solution, both feasible and optimum, could be generally obtained under any scenario. Therefore, the concept of multi-criteria decision-making (MCDM) can be applied to balance solution robustness and model robustness. Feng and Rakesh (2010) developed the LP model including random parameters, as below:

$$\mathrm{Min} c^T x + d^T y$$

Subject to:

$$Ax = b,$$

$$Bx + Cy = e,$$

$$x, y \geq 0$$

Where x is the decision variable vector, y is the control variable vector, and B, C, e are the random values. Assume that the set S=1,2,…s may give different scenarios for the random parameters, and each scenario has a probability value p^s, ($\sum_s p^s=1$). Note that the model could be infeasible for any scenario s. Therefore, δ^s is defined as the feasible value. If the model is feasible, then δ^s is equal to zero. Otherwise, it will find a positive value. The robust optimization model will then be given as below:

$$Min\ \sigma\left(x, y^1, \ldots, y^S\right) + \omega\rho(\delta^1, \ldots, \delta^S)$$

Subject to:

$$Ax = b,$$

$$B^S x + C^S y^S + \delta^S = e^S \quad \forall s \in S,$$

$$x \geq 0, y^S \geq 0, \delta^S \geq 0, \quad \forall s \in S$$

The first part of the above objective function considers the solution robustness, and the second part deals with the model robustness. Mulvey and Ruszczynsk (1995) defined the robust optimization model for the first part as: $\sigma(0) = \sum_{s \in S} p^S \psi^S + \lambda \sum_{s \in S} p^S (\psi^S - \sum_{s' \in S} p^{S'} \psi^{S'})^2$

Where λ is the weight value allocated for the variance solution. The less sensitive to change in data the solutions under different scenarios, the more increase the λ value shows. Yu and Li (2000) converted the above quadratic equation into an absolute value, and presented it by some modifications as the following:

$$Min \sum_{s \in S} p^S \psi^S + \lambda \sum_{s \in S} p^S \left[\left(\psi^S - \sum_{s' \in S} p^{S'} \psi^{S'}\right) + 2\theta^S\right]$$

Subject to:

$$\psi^S - \sum_{s \in S} p^S \psi^S + \theta^S \geq 0, \quad \forall s \in S,$$

$$\theta^S \geq 0, \quad \forall s \in S,$$

The second part of the objective function related to the model robustness includes the penalties applied in the control constraints. Here, we use the coefficient ω as the weight to balance two parts of the objective function. Therefore, the objective function can be presented as:

$$Min \sum_{s \in S} p^S \psi^S + \lambda \sum_{s \in S} p^S \left[\left(\psi^S - \sum_{s' \in S} p^{S'} \psi^{S'}\right) + 2\theta^S\right] + \omega \sum_{s\ S} p^S \delta^S$$

3. Modeling

3.*1. Stochastic Multi-objective Capacitated p-hub Location Problem* (MCpHLP-s)

The idea for uncertainty model, developed here, is inspired from the model proposed by Yang (2009). Assume that there is a given number of cities (n), and also, there are some volumes of demand for commodities between two cities (D_{ij}). The number of P-hub locations, chosen among the present cities, should be established in order to handle the distribution system. Commodities transporting from specific origins to specific destinations can utmost go through two hubs. A capacity (U_k) has been defined for each hub. Commodities are to be processed in each hub, so it takes some time (T_{kl}) to perform. Obviously, if commodities transport to their destination through a route with no hub, then the time value will equal zero. If they go through one hub, then $T_{kk} = T_k$. And, if they transport from two hubs, then $T_{kl} = T_k + T_l$. The distance transported from the origin to the destination, (d_{iklj}), is equal to the sum of distances from the origin to a hub, from that hub to another, and from the latter to the destination,($d_{ik} + d_{kl} + d_{lj}$) , all are the inputs for the problem.

In this model, we assume two parameters; namely, the demand for commodities between origin-destination pairs (D_{ij}) and the processing time for each hub (T_k) under uncertainty and in scenarios. Other parameters depended to these two uncertain cases are also defined as scenarios.

Table 1. Notations of model

variable	Description
M	*A big positive number*
p	*number of hubs which must be established*
n	*number of nodes in network*
β^s	*Maximum of arc length in scenario s*
U_k	*capacity of hub at node k*
F_k	*fixed cost of establishing a hub at node k*
T_k^s	*time hub k takes to process one unit of flow in scenario s*
P_k	*fixed time to initiate the service at hub k*
D_{ij}^s	*demand from location i to location j in scenario s*
d_{ij}	*distance between node i and node j*
C_{ij}	*the unit transportation cost for the non-stop service between i and j (per each distance unit)*
C_{iklj}^s	*the unit transportation cost for hub-connected service from i to j and transshipped at hubs k and l in scenario s(per each distance unit)*
Z_k	*equal 1 if a hub located at node k and otherwise 0*
x_{ij}^s	*equal 1 if the demand is transported through the non-stop path i–j for scenario s, otherwise 0*
x_{iklj}^s	*equal 1 if the demand is transported from i to j and transshipped at hubs k and t for scenario s, otherwise 0*

Parameters applied in the model are summarized in Table 1. The model is formulated as below:

MC*p*HLP-s:

$$Min \sum_{k=1}^{n} F_k Z_K + \sum_{i=1}^{n}\sum_{j=1}^{n} D_{ij}^s d_{ij} C_{ij} x_{ij}^s + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} D_{ij}^s d_{iklj}^s C_{iklj}^s x_{iklj}^s \qquad (1)$$

$$Min \max_{i,j,k,l} \max_{i,j,k,l} \left(d_{ij} x_{ij}^s, d_{ik}^s x_{iklj}^s, d_{kl}^s x_{iklj}^s, d_{lj}^s x_{iklj}^s \right) \qquad (2)$$

$$Min \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} T_{kl}^s D_{ij}^s x_{iklj}^s + \sum_{k=1}^{n} P_k Z_k \qquad (3)$$

Subject to:

$$\sum_{k=1}^{n} Z_k = p \qquad (4)$$

$$\sum_{k=1}^{n}\sum_{l=1}^{n} x_{iklj}^s + x_{ij}^s = 1 \quad \forall i, j \; i \neq j \quad D_{ij}^s \neq 0, \quad (5)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} D_{ij}^s x_{iklj}^s \leq U_k Z_k \quad \forall k \; i \neq j \quad (6)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} D_{ij}^s x_{ilkj}^s \leq U_k Z_k \quad \forall k \; i \neq j \quad (7)$$

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} \left(x_{iklj}^s + x_{ilkj}^s \right) - \sum_{i=1}^{n}\sum_{j=1}^{n} x_{ikkj}^s \right] \leq MZ_k \quad \forall k \; i \neq j \quad (8)$$

$$M \left[\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} \left(x_{iklj}^s + x_{ilkj}^s \right) - \sum_{i=1}^{n}\sum_{j=1}^{n} x_{ikkj}^s \right] \geq Z_k \quad \forall k \; i \neq j \quad (9)$$

$$z_k, x_{ij}^s, x_{iklj}^s \in \{0,1\} \quad \forall i, j, k, l \quad i \neq j \quad (10)$$

The objective function (1) minimizes the sum of fixed establishing hubs costs and transporting commodities costs. The objective function (2) minimizes maximum length of the arc established. By the term "arc", we mean a direct path created between two places. If a path, for example, is created with two stops at hubs between the origin and the destination, then three arcs will be established; one between the origin and hub 1, another between hub 1 and hub 2, and one last between hub 2 and the destination. The objective function (3) minimizes the total time values spent for processing commodities, and also for preparing established hubs. Clearly, the above objective functions are in conflict with each other. Constraint (4) allows us to establish maximum p-hubs. Constrain (5) is to ensure the transportation of commodities from the origin to the destination. Constraints (6) and (7) are related to the capacity. Constraint (8) indicates that if there is no hub in the node k, then the node must not perform as a hub. Constraint (9) makes it necessary to go through the hub when a hub placed on the node k. Constraint (10) defines the problem decision variables.

The objective function (2) is the *MiniMax*. To make a linear objective function, the objective function (2) is replaced by the function (11), also the constraints (12)-(15) are added to the problem:

$$Min \beta^s \qquad (11)$$

$$\beta^s \geq d_{ij} x_{ij}^s \quad \forall i, j \quad i \neq j \qquad (12)$$

$$\beta^s \geq d_{ik}^s x_{iklj}^s \quad \forall i, j, k, l \quad i \neq j \qquad (13)$$

$$\beta^s \geq d_{kl}^s x_{iklj}^s \quad \forall i, j, k, l \quad i \neq j \qquad (14)$$

$$\beta^s \geq d_{lj}^s x_{iklj}^s \quad \forall i, j, k, l \quad i \neq j \qquad (15)$$

3.2. *Robust Optimization Formulation*

In this section, the model MC*p*HLP-s, proposed in 3.1, is developed using Mulvey's robust optimization methodology where uncertain parameters are under discontinued scenario. For simplicity, the objective functions are first abbreviated as below:

$$TC^S\ (transfer\ costs) = \sum_{i=1}^{n}\sum_{j=1}^{n} D_{ij}^S d_{ij} C_{ij} x_{ij}^S + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} D_{ij}^S d_{iklj}^S C_{iklj}^S x_{iklj}^S$$

$$FC^S\left(fix\ costs\right) = \sum_{k=1}^{n} F_k Z_k$$

$$PT^S\left(processing\ times\right) = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} T_{kl}^S D_{ij}^S x_{iklj}^S$$

$$ST^S\left(setup\ times\right) = \sum_{k=1}^{n} P_k Z_k$$

According to the above definitions, the robust optimization model is formulated as:

$$MinZ_1 = \sum_s p^S\left(TC^S+FC^S\right) + \lambda_1\sum_s p^S\left[\left(TC^S+FC^S\right)-\sum_{s'} p^{s'}\left(TC^{s'}+FC^{s'}\right)+2\theta_1^S\right] + \omega\sum_{s,i,j} p^S\delta_{ij}^S, \quad (16)$$

$$MinZ_2 = \sum_s p^S\left(\beta^S\right) + \lambda_2\sum_s p^S\left[\left(\beta^S\right)-\sum_{s'} p^{s'}\left(\beta^{s'}\right)+2\theta_2^S\right], \quad (17)$$

$$MinZ_3 = \sum_s p^S\left(PT^S+ST^S\right) + \lambda_3\sum_s p^S\left[\left(PT^S+ST^S\right)-\sum_{s'} p^{s'}\left(PT^{s'}+ST^{s'}\right)+2\theta_3^S\right], \quad (18)$$

Subject to:

$$\left(TC^S+FC^S\right)-\sum_s p^S\left(TC^S+FC^S\right)+\theta_1^S \ge 0, \forall s, \quad (19)$$

$$\left(\beta^s\right)-\sum_s p^s\left(\beta^s\right)+\theta_2^s \ge 0, \forall s, \quad (20)$$

$$\left(PT^S+ST^S\right)-\sum_s p^S\left(PT^S+ST^S\right)+\theta_3^S \ge 0, \forall s, \quad (21)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n}\left(D_{ij}^s-\delta_{ij}^s\right)x_{iklj}^s \le U_k Z_k \quad \forall k \quad i\ne j \quad (22)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n}\left(D_{ij}^S-\delta_{ij}^S\right)x_{ilkj}^S \le U_k Z_k \quad \forall k \quad i\ne j \quad (23)$$

$$\theta_1^s, \theta_2^s, \theta_3^s, \delta_{ij}^s \ge 0 \qquad \forall s,i,j \quad (24)$$

Constraints (4), (5), (8), (9), (10) and (12)-(15).

The first and second parts of (16), (17) and (18) represent the mean and variance for the objective functions. The third part of (16) indicates the amount of model robustness with respect to the uncertainty of the constraints (22) and (23) under each scenario. Constraints (19), (20) and (21) are applied to make the model linear as the given definition. The constraints (22) and (23), the control constraints, are defined the same as the constraints (6) and (7). The difference is that δ_{ij}^s would be a positive value when the scenario reaches an infeasible solution. Otherwise, $\delta_{ij}^s = 0$. Furthermore, Constraint (24) defines non-zero variables.

4. A Given Case Solution

4.1 *Solution Process*

As addressed in the previous section, the robust Optimization model can be concerned as a multi-objective mixed integer programming. Also, three objective functions are all implicitly in contrast. Hence, the application of goal programming (GP) with the aim to solve multi-objective models can find an alternative problem with a single objective function. The GP is more direct and flexible method which allows the manipulation of various scenarios by changing target or weight values. However, it seems that the GP is effective for solving multi-objective problems with heterogeneous functions. This method also needs no objective function to scale, just the goals must be pre-determined.

As (Romero 2004) stated, for the pre-emptive goal programming model, the framework can be formulated as:

$$Lex\ Min\ a = \left[\sum_{k\in h_1}\left(\alpha_k d_k^+ + \beta_k d_k^-\right),\ldots,\sum_{k\in hj}\left(\alpha_k d_k^+ + \beta_k d_k^-\right),\ldots,\sum_{k\in h_Q}\left(\alpha_k d_k^+ + \beta_k d_k^-\right)\right] \quad (25)$$

Subject to:

$$f_i(x) \sim 0, i = 1,2,\ldots,q \qquad (26)$$

$$g_k(x) - d_k^+ + d_k^- = b_k, \quad k\in h_j, \quad j\in\{1,2,\ldots Q\} \quad (27)$$

$$d_k^+, d_k^- \ge 0, \quad k\in h_j, \quad j\in\{1,2,\ldots Q\} \quad (28)$$

Where,

$\sqcup$: =, ≥ or ≤

h_j: Index set of goals placed in the *j*th priority level

α_k : Weighting factor for positive deviation

β_k : Weighting factor for negative deviation

$f_i(x)$: System constraint

$g_k(x)$: Goal constraint;

b_k: Aspiration level of the goal k

d_k^+ : Positive deviation

d_k^- : Negative deviation.

Considering the above framework, the goal programming formulation for the proposed robust optimization approach may be written as follows:

$$LexMinimize\{\alpha_1 d_1^+ + \alpha_2 d_2^+ + \alpha_3 d_3^+\} \quad (29)$$

Subject to:

$$\sum_s p^s \left(TC^s + FC^s\right) + \lambda_1 \sum_s p^s \left[\left(TC^s + FC^s\right) - \sum_{s'} p^{s'} \left(TC^{s'} + FC^{s'}\right) + 2\theta_1^s\right] + \omega \sum_{s,i,j} p^s \delta_{ij}^s + d_1^- - d_1^+ = GOAL_1 \quad (30)$$

$$\sum_s p^s \left(\beta^s\right) + \lambda_2 \sum_s p^s \left[\left(\beta^s\right) - \sum_{s'} p^{s'} \left(\beta^{s'}\right) + 2\theta_2^s\right] + d_2^- - d_2^+ = GOAL_2 \quad (31)$$

$$\sum_s p^s \left(PT^s + ST^s\right) + \lambda_3 \sum_s p^s \left[\left(PT^s + ST^s\right) - \sum_{s'} p^{s'} \left(PT^{s'} + ST^{s'}\right) + 2\theta_3^s\right] + d_3^- - d_3^+ = GOAL_3 \quad (32)$$

Constraints (4), (5), (8), (9), (10), (12)-(15) and (19)-(24).

That α_i is weight of i[th] objective function. According to Romero framework (2004), constraints (30), (31) and (32) are goal constraints. The resultant single-objective model (MIP) can be easily solved by different linear model solution software, like Lingo and Gams.

4.2. Case Study

In order to evaluate the model, data was collected from *Shirin Asal Co.*, an Iranian chocolate producer. Demand rate for chocolate products varies season to season, hence four scenarios were developed; namely, spring, summer, fall, and winter. The main production plant is located in *Tabriz* (the origin), with 36 distribution offices (the destinations) across the country, each with different demand rate. The country can be divided into three regions of West, Central and East; here, the research focuses on west region covering 14 destinations. The company management seeks to establish two hubs among these cities (14 destinations). Table 2 shows the unit transportation cost (package size 50 * 25 * 25 cm) for the non-stop service per each distance unit (Km); the unit transportation cost for the hub-connected service per each distance unit (Km) in different scenarios; and, possibility to set each scenario. Table 3 represents distance between cities in western region, where Table 4 provides demands and processing times in each scenario. Finally, Table 5 covers capacity, and fixed costs and times to establish hubs for each city.

Table 2. Primary data set

The number of nodes	*14*
number of hubs which should be established	*2*
the unit transportation cost for the non-stop service (per each km)	*9.5 Rials*
the unit transportation costs for hub-connected service (per each km) (spring/summer/fall/winter)	*(2.9,2.7,3.9,4.1)Rials*
Probability sets (spring/summer/fall/winter)	*(0.25,0.25,0.25,0.25)*

Table 3. Distances between cities (kms)

	Rasht	*Kermansha h*	*Tabriz*	*Tehran*	*Zanjan*	*Qazvin*	*Hamadan*	*Urmia*	*Ardebil*	*Sanandaj*	*Shahrekord*	*Ilam*	*Karaj*	*Arak*
Rasht	-	590	485	325	348	185	401	739	266	565	868	774	285	577
Kermanshah		-	588	526	414	433	189	582	791	136	731	184	538	365
Tabriz			-	599	280	455	609	308	219	452	1142	772	574	785
Tehran				-	319	150	337	907	591	501	543	710	50	239
Zanjan					-	175	329	588	377	278	862	598	282	505
Qazvin						-	244	763	451	453	584	617	106	303
Hamadan							-	610	667	164	568	373	354	176
Urmia								-	527	446	1178	766	729	786
Ardebil									-	655	1134	975	552	843
Sanandaj										-	732	320	523	340
Shahrekord											-	719	579	392
Ilam												-	706	514
Karaj													-	322
Arak														-

Table 4. Demands and processing times in each scenario

City	Demand scenarios(Unit)				Processing time scenarios(Day/Unit)			
	spring	*summer*	*fall*	*winter*	*spring*	*summer*	*fall*	*winter*
Rasht	9205	7899	21848	26510	0.00068	0.00058	0.00162	0.00196
Kermanshah	10459	11256	18751	19628	0.00077	0.00083	0.00139	0.00145
Tabriz	48022	39529	65890	94831	0.00355	0.00292	0.00487	0.00701
Tehran	14412	12571	44070	91906	0.00107	0.00093	0.00326	0.00679
Zanjan	10590	11732	20402	21218	0.00078	0.00087	0.00151	0.00157
Qazvin	4424	5995	10848	12502	0.00033	0.00044	0.00080	0.00092
Hamadan	6270	4802	9273	9505	0.00046	0.00035	0.00069	0.00070
Urmia	17022	16006	24951	23234	0.00126	0.00118	0.00184	0.00172
Ardebil	13764	19839	19281	16767	0.00102	0.00147	0.00143	0.00124
Sanandaj	7996	6105	11330	10807	0.00059	0.00045	0.00084	0.00080
Shahrekord	6142	5721	10320	9065	0.00045	0.00042	0.00076	0.00067
Ilam	4044	4135	6689	7881	0.00030	0.00031	0.00049	0.00058
Karaj	18519	22050	41018	48957	0.00137	0.00163	0.00303	0.00362
Arak	4272	3726	9402	10287	0.00032	0.00028	0.00070	0.00076

Table 5. Capacity, fixed times and costs for each city

City	Fixed costs (*Rial*)	Fixed times(P_k) (*Day*)	Capacity (*Unit*)
Rasht	1,300,800,000	656	275200
Kermanshah	616,800,000	210	154200
Tabriz	1,286,560,000	434	321640
Tehran	1,941,120,000	570	38528
Zanjan	1,005,440,000	412	201360
Qazvin	495,360,000	510	123840
Hamadan	470,400,000	364	97600
Urmia	440,320,000	395	110080
Ardebil	495,360,000	412	123840
Sanandaj	412,800,000	512	103200
Shahrekord	550,400,000	462	137600
Ilam	330,240,000	486	82560
Karaj	2,143,840,000	892	460960
Arak	331,340,000	384	82835

The modeling and solution processes for the above problem were performed by the software Lingo in a PC with Core2duo 2.00 GHz CPU and 4 GB of RAM with $\lambda_1 = \lambda_2 = \lambda_3 = 1$, $GOAL_1$=1,000,000,000, $GOAL_2$=400, $GOAL_3$=500 and $\omega = 200$. The value ω will impose significant effects on the solutions. If $\omega = 0$, for example, then the maximum value δ_{ij}^s will be obtained. In this case, the average costs reach their minimum values.

Figures 1-4 show the findings obtained from each scenario. The paths created include no hub-stop and hub-stop routs. The result reveals that two hubs should be established between *Zanjan* and *Hamadan*. As seen from the figures, the number of hub-stop routes will be increased when demand rates increase. Because in this case, rising hub-stop services can help more cost savings. The number of hub-stop routes for each scenario (spring, summer, fall, and winter) is 7, 7, 9, and 10, respectively. The result indicates that the states of routes with hub and no hub are the same in spring and summer scenarios. The reason is that demands and costs are relatively close during these seasons. For fall scenario, some routes have one hub-stop, other routes with two hub-stops. To distinct the routes clearly, Table 6 shows the routes of origin of *Tabriz*; and the parameters *k* and *I* as hub 1 and hub 2, respectively, and *j* as the destination.

The average amount of construction and transportation costs is 2,004,506,701 Rls; the average maximum arc length established is 598.75 Km; the average total processing time is 960.5 day; and the average sum of the values of δ_{ij}^s is 11422.

Table 6. Transport paths in fall

i	*k*	*l*	*j*
tabriz	-	-	*Rasht*
tabriz	*hamadan*	-	*Kermanshah*
tabriz	-	-	*Tabriz*
tabriz	*zanjan*	-	*Tehran*
tabriz	-	-	*Zanjan*
tabriz	*zanjan*	-	*Qazvin*
tabriz	*zanjan*	-	*Hamadan*
tabriz	-	-	*Urmia*
tabriz	-	-	*Ardebil*
tabriz	*zanjan*	-	*Sanandaj*
tabriz	*zanjan*	*hamadan*	*Shahrekord*
tabriz	*zanjan*	*hamadan*	*Ilam*
tabriz	*zanjan*	-	*Karaj*
tabriz	*hamadan*	-	*Arak*

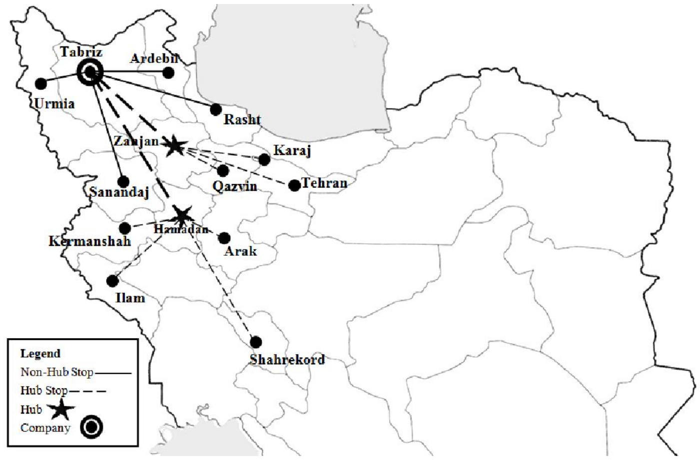

Figure 1. Network in spring

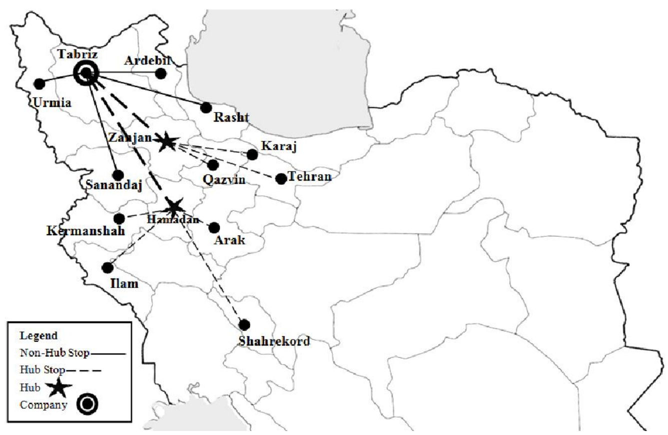

Figure 2. Network in summer

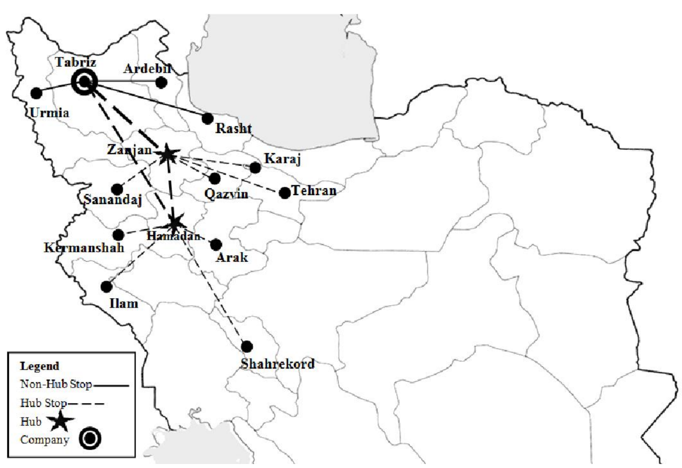

Figure 3. Network in fall

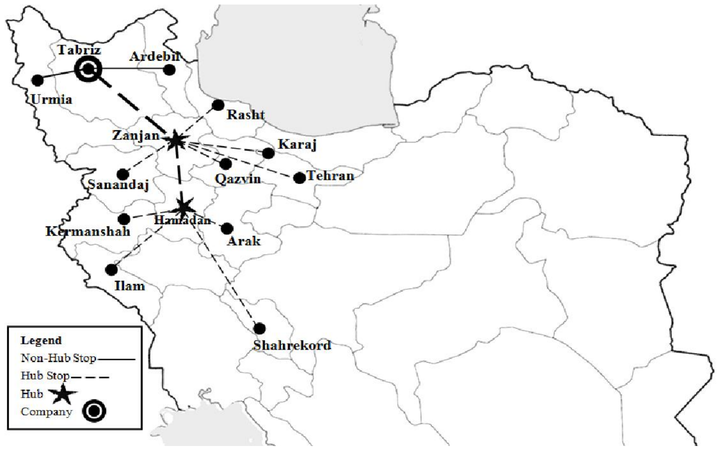

Figure 4. Network in winter

As seen before, ω may affect the values of objective functions and δ_{ij}^{s} . Figures 5 and 6 show the effect in the present model. The value of objective functions will be increased when ω increases, whereas the value of δ_{ij}^{s} decreases.

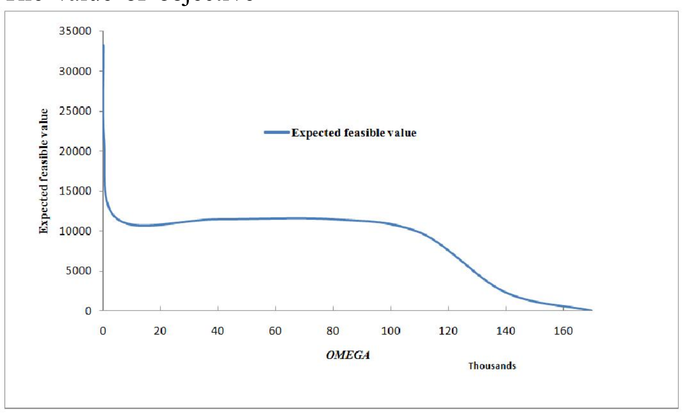

Figure 5. Trade-off for model robustness vs. expected feasible value

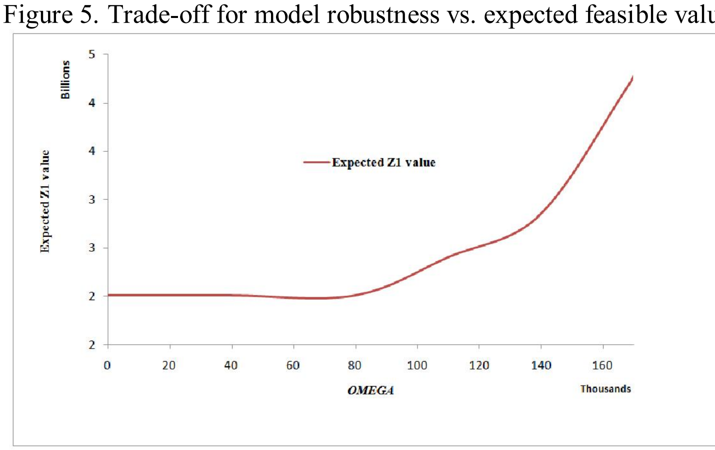

Figure 6. Trade-off for model robustness vs. expected Z_1 value

4. Conclusion

A robust optimization model was proposed for multi-objective operation of capacitated P-hub location problems (MCpHLP). Here, simultaneous minimization of three objective functions was applied; namely, total costs, the maximum length of the arc established, and total processing times. By using the goal programming method, the robust multi-objective model was converted into a single-objective problem. Also, different scenarios were developed for the volume of demand, processing time and related costs. The objectives of solution and model robustness can be achieved by simultaneously optimizing the robust approach. The benefit of using the proposed model is that a close approximation to the real-world can be made.

Since a case study was used to validate the model, it can be concluded that the model was designed based on real conditions and seasonal demands. The model is designed for general cases with different origin and destination points between each pair the traffic is possible, although the case study included only one city as the origin. The results indicated that the model robustness increased, but the solution robustness decreased. However, choosing the best ω with trade-off between these may put the decision maker in ideal conditions.

For future studies, it is recommended to apply meta-heuristic techniques to solve large-size problems. Moreover, the model can be extended by including uncertainty parameters based on known distribution functions and using alternative methods.

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