Formalization of Oil and Gas Seismic Survey using Z-notation

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Abstract: Formal methods are mathematical tools use for modeling and verification of hardware system. These tools increase the quality and reliability of a system. In this work we present to model the system use for locating oil and gas reservoirs using z-notation. Oil and gas are usually found in various types of subsurface traps. Seismology (the science concerned with the finding oil and gas), involves the measurement of sound waves reflected back to the surface from rock layers. This is complex and critical task, so by formalizing this we can attain accuracy. This modeling can develop a system which is accurate and verified and efficient.

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1. Introduction

In computer science and software engineering, formal methods are techniques and tools based on discrete mathematics, i.e., predicate logic, set theory, function, relation and graph theory, use for specification and verification of software and hardware systems [10][15] . The term “formal method” refers to formal notations and development techniques that support the rigorous development of systems [[25]-[28]]. The ability to prove properties of a system, prior to implementation, is a key benefit of formal methods. Proof is essentially a form of reasoning or argument that uses the syntactic rules of a notation to determine the validity of a theorem or hypothesis [10]. Petroleum reserves are usually found in various types of subsurface traps, to trace out these traps is a real problem. The science concerned with finding of oil traps is called Seismology. Seismology involves creating sound waves on the surface that penetrates downwards into the rock layers. Each layer reflects the sound wave back to the surface, where sensitive instruments record and measures the intensity of reflections. It is extremely expensive to find and produce oil and gas. Wells can run in the millions of US dollars, and offshore platforms can cost over a billion US dollars [14]. Digging for well is dependent on accuracy of results we get from this survey. It is very critical from monetary point of view. Processing of data from experiment should be processed accurately. In this research, we formalize the seismic survey to discuss the use of Z-notion to obtain accuracy and reliability. Field survey will be done to gather requirements, and then those requirements are formalized for strength and reliability.

2. Terminology for Seismic Survey

2.1. Refraction and Reflection Waves

Rays and Wave front: A wavefront is the locus (a line, or, in a wave propagating in dimensions a surface) of points having the same phase [5]. Wave fronts are propagating from the mass equally in all directions. These are spherical in shape. Rays are perpendicular to wavefronts [1].

Wave Conversion: Earth is composed of different layers. When the rays strike the boundaries of different materials having different densities, usually reflect back or refract into lower layers. Types of waves are also converted. A P-Wave converts into S-Wav

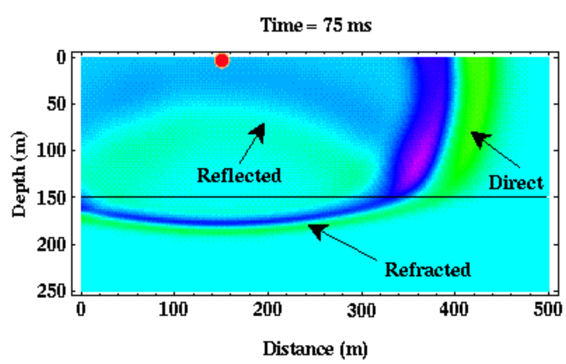


Figure 1Wave Conversion

Path of Seismic Body Waves: Different types of rays are detected at receiver. We can calculate the path of the seismic waves by knowing the length of path, wave speeds and time interval of that ray from source to receiver. The time required by Seismic waves is least time, according to Fermat's Principle, which states "Elastic waves travel between two points along paths requiring the least time."

Seismograms: Output of receivers is a chart to record the oscillations of ground vibrations. This is called Seismogram. From this recording, we can do further processing on it and make a conclusion about earth inner surface. Geophones are usually used as receiver to record vibrations.

Energy Source: Two types of energy source are used. For impulsive source, Dynamite is used and for sweep generation, vibroseis is used.

Recording: Geophones are used for recording..

Refraction Waves: A refraction wave penetrates into lower layer having different densities. They refract with angle equal to incidence angle on striking the boundary. Both layers have different velocities.

Snell's Law: Snell's law gives the relationship between incidence and refracted angle when wave refracts the boundary of any layer

$$\frac{\sin i}{\sin r} = \frac{V_i}{V_r}$$

Where i is angle of incidence and r is angle of reflection. Vi is the incident wave Speed and Vr is the speed of refracted or reflected wave.

Critical Refraction: According to Snell's law, the incident rays refract at angle of refraction on boundaries. Some of the rays in them refract at 90 degrees, which shows that how the critical angle depends on wave speed. Criticalrefraction is possible only if the refracted ray move faster than incident ray.

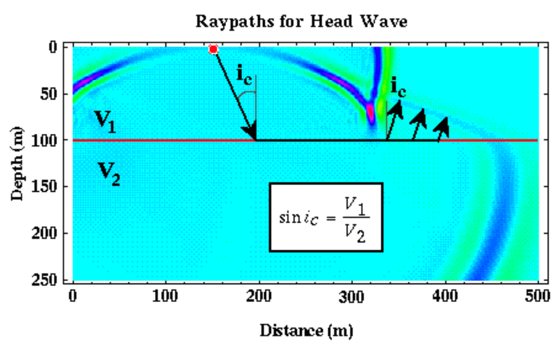


Figure 2 Critical Refraction

Huygen's Principle: A critically refracted ray moves along the top of the lower boundary. It produces the P and SV- waves that refract back to upper layer [1]. That is due to Huygen's Principle.

Critical Distance: Critical Distance is minimum distance to get the refracted distance. Distance between the source and receiver must be greater than the critical distance.

Time Intervals: For the purpose of seismic survey the first pulse, reaching geophones is usually used, these are called the first arrivals. The seismologist measures the distances of geophones and the time of arrival at geophones of first arrivals.

t-x Curve: Plot of points of distances against time on a graph. This graph is called the t-x Curve or travel-time curve.

Velocities: We can calculate the velocities of the seismic wave using t-x curve. The slope of the curves gives us the velocities by using following formula:

$$slope = \frac{\Delta x}{\Delta t} = \frac{1}{V}$$

where x is the distance and t is the time of traveling wave. The graph has different straight lines for different types of waves. The straight line that intercepts y-axis at time T, is called the intercept time. The two straight lines on graph intersects at some point, the x-axis position against that intersection is called the Crossing point.

Analyze t-x curve: There are different straight lines on the graph. The first straight line that is passing through the origin is showing the direct waves. It shows the direct waves reaching the geophones that have traveled straight along the surface from the source. The next straight line is representing the refracted waves. All refracted waves follow same path to geophones. Travel time for direct and refracted waves is directly proportional to distance from it's source.

Intercept Time is obtained by analyzing a graph: There are straight lines on the graph, which are not passing through the origin of a graph. These lines when extend back, it will pass through the vertical axis at the Time value, T. This time value is called Intercept time.

Crossing Distance: Different types of waves have different types of lines on graph. The point where two lines intersect is called Crossing Distance Xc. This distance indicates that the direct waves and refracted waves reach the receiver at the same time.

Layer Thickness: Two approaches are used to calculate the layer thickness. One is using the crossing distance, having formula:

$$h = \frac{X_c}{2}\left(\frac{V_2 - V_1}{V_2 + V_1}\right)^{1/2}$$

where h is the layer thickness, Xc is the crossing distance and V is the velocity.

Reciprocity: If the positions of a source and a receiver are interchanged, the travel time remains the same. This fact is called condition of reciprocity [1]. By changing the source and receiver positions we get the two graphs opposite to each other. These opposite travel time curves indicate identical velocities and intercept times. But for critical refracted waves, this graph is not identical.

Reversed t-x Curve: For horizontal surfaces, the upward and downward parts of the refracted waves of refracted waves are similar. But for dipping layers, this condition is invalid. We get the dipping layer properties by these opposite refracted waves. Two seismograms are recorded by reverse positions of source and receiver. This procedure is called 'Reversing a line' and survey done in this manner is called 'Reversed Refraction Survey'. After doing the reversed refraction survey, we get the conclusion for layer surface. Following types of surfaces should be concluded from this reversed survey:

Straight Layer: If the corresponding straight line on opposite travel time curves have identical slopes, the layer is horizontal [1].

Multi Surfaces: Same procedure is used to check the different layers as of single layer. In travel time curve, the multi surfaces are represented by different straight lines. The reciprocal of the slopes of all straight lines gives us the different velocities.

Dipping Layer: If the corresponding straight line on opposite travel time curves have different slopes, the layer is dipping [1]. For dipping layers the upward and downward time of the refracted layer is different. Direct waves are not affected by dipping refractor. The refractor dips downward towards the source at which the intercept time is largest and for which the crossing distance is greatest.

Faults: Until now, we didn't discuss about the discontinuous boundaries. In the travel time curve is displaced to the right of the actual offset of the layer boundary [1]. It is separated in two pieces by gap of time difference.

3.2. Reflection Waves Details

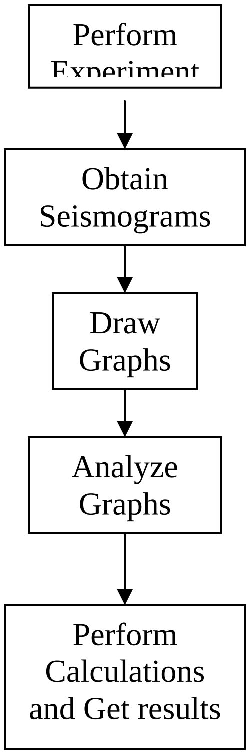
The waves that don't penetrate into lower layers are reflected layers. They reflect back to upper layers. They are useful to detect the deeper layers with small geophone spread. Another advantage is that they don't depend on velocities of different layers. A property of every substance is its acoustical impedance, which is the product of density and seismic wave velocity, ρV, Seismic waves will reflect from any boundary where the acoustical impedance changes [1]. Their recognition is little difficult and often weak recorded. The product of a seismic reflection survey is called a seismic section, which is combination of seismograms.

Reflection Travel Time Curve: The reflection time curve is of first arrivals of reflected waves. It is between the time of wave reached the receiver and distance of the geophones from the source. The reflected waves describe an arc on the travel time curve. These are arrived at the source as well. The reflection travel time curve lies above the direct waves travel cut but approaches it asymptotically with increasing distance. Critically reflected wave is a wave of zero distance along the refractor. This wave has energy of both refracted and reflected wave; it produces a pulse of higher amplitude on the seismogram. The direct waves reach the receivers before the reflection waves, that is why the reflected waves curve lies above the direct waves curve. With increase in distance, this difference diminishes, so the two curves approaches asymptotically. Obtain reflection arrival times at receiver distances from the seismogram. Square the values and plot on x2-t2 graph. Draw a straight line through the alignment of points in graph and calculate the velocity from the slope of the graph. Calculate the Minimum Travel Time by getting the lowest point in hyperbola.

Single Layer: If the hyperbola comes in the middle of the graph then it's a single layer. Direct wave curve is the asymptote of the reflection time curve [1].

Slopping Surface: If the hyperbola graph of travel time does not come in the middle of the graph then it's slopping surface.

4. Seismic Model and their Formal Specification



5. Specification:

All the specification is done using Z-Notation. The Z notation is based upon set theory and mathematical logic. Waves are used in seismic survey, which are of two types mentioned above. These waves refract or reflect back by striking any layer boundary. Waves hit the boundary with some incidence angle, refract into lower layer with some angle and reflect back with some angle. Some refracted waves refracted at 90◦ by Huygen's Principle. Sets are defined for these types:

[*IncidenceAngles, ReflectedAngles, RefractedAngles, Distances, Times, Velocities, Waves, Layers, InterceptTimes, CriticalAngle, CrossingDistances, LayerThickness*]

All incidence angles, reflected and refracted angles must be of type real numbers and should be between 0◦ and 90◦.

$\forall IncidenceAngles: \mathbb{R} \bullet IncidenceAngles \geqslant 0 \wedge IncidenceAngles \leqslant 90$

$\forall ReflectedAngles: \mathbb{R} \bullet ReflectedAngles \geqslant 0 \wedge ReflectedAngles \leqslant 90$

$\forall RefractedAngles: \mathbb{R} \bullet RefractedAngles \geqslant 0 \wedge RefractedAngles \leqslant 90$

Distances are set of the distances between source and receiver.

$\forall Distances: \mathbb{R} \bullet Distances \geqslant 0$

Time is the set of first arrivals of the waves.

$\forall Times: \mathbb{R} \bullet Times \geqslant 0$

This is for refracted waves

$\forall CriticalAngle: \mathbb{R} \bullet CriticalAngle = 90$

From Curves

$\forall InterceptTimes: \mathbb{P}\ Times \bullet InterceptTimes \subseteq Times$

$\forall CrossingDistances: \mathbb{P}\ Distances \bullet CrossingDistances \subseteq Distances$

Refracted Waves:

RefractedWaves

$LayerBelongs: Waves \leftrightarrow Layers$
$WaveIncAng: Waves \rightarrow\!\!\!\!\!+ IncidenceAngles$
$WaveRefAng: Waves \rightarrow\!\!\!\!\!+ RefractedAngles$

$\text{ran } LayerBelongs \subseteq Layers$
$\exists WaveVelocity1, WaveVelocity2: Waves \rightarrow\!\!\!\!\!+ Velocities$
$\bullet\ WaveVelocity1 \neq WaveVelocity2$
$\text{ran } WaveIncAng \subseteq IncidenceAngles$
$\text{ran } WaveRefAng \subseteq RefractedAngles$
$\exists LayerVelocity1, LayerVelocity2: Layers \rightarrowtail\!\!\!\!\!+ Velocities$
$\bullet\ LayerVelocity1 \neq LayerVelocity2$

Refracted Waves must belong to two layers. They must strike boundary with some incidence angle and refract into lower layer with refracted angle. Both layers have different velocities.

According to Huygen's Principle, the refracted whose refract at 90 degrees reflect back to upper layer, that's why we receive those waves.

CriticalRefracted

$RefWave: RefractedWaves \rightarrow\!\!\!\!\!+ CriticalAngle$

$\text{ran } RefWave \subseteq CriticalAngle$

Get the first arrival waves and calculate the distance between source and receiver. Time should also be recoded during experiment. Plot a t-x curve for distances and times.

TravelTimeCurve

$GraphPoint: Distances \leftrightarrow Times$
$Vel!: Velocities$
$IT: \mathbb{P}\ InterceptTimes$
$CD: \mathbb{P}\ CrossingDistances$

From the Travel Time Curve, we get many straight lines; the one which is passing through the origin is giving the information of direct waves.

```
DirectWaves
  TravelTimeCurve
  ─────────
  ∃d: Distances; t: Times •
        d ↦ t ∈ GraphPoint ∧ d ↦ t = (0,0)
```

If the corresponding straight line on opposite travel time curves have identical slopes, the layer is horizontal. The travel time curve must have one intercept line.

```
RefractionSingleLayer
  TR: TravelTimeCurve
  RTR: TravelTimeCurve
  ─────────
  TR = RTR
  ∃it: ℙ InterceptTimes
    • it ⊂ TR. IT ∧ (∀it1: ℙ InterceptTimes | it1 ⊂ TR. IT • it = it1
```

Same procedure is used to check the different layers as of single layer. In travel time curve, the multi surfaces are represented by different straight lines. The reciprocal of the slopes of all straight lines gives us the different velocities. Travel Time is given by:

$$T_n = 2\sum_{k=1}^{n} \frac{h_k}{V_k} \cos i_{k(n+1)}$$

Where h is the layer thickness and V is the velocity. This formula is used to calculate the layer thickness of different layers.

$$h_n = \left[\frac{T_n}{2} - \sum_{k=1}^{n-1} \frac{h_k}{V_k} \cos i_{k(n+1)}\right] \frac{V_n}{\cos i_{n(n+1)}}$$

More than one intercept times are required.

```
RefractionMultiLayers
  TR: TravelTimeCurve
  RTR: TravelTimeCurve
  h!: LayerThickness
  V!: Velocities
  ─────────
  TR = RTR
  ∃it1, it2: InterceptTimes • it1 ≠ it2
```

If the corresponding straight line on opposite travel time curves have different slopes, the layer is dipping.[1]

```
RefractionDippingLayer
  TR: TravelTimeCurve
  RTR: TravelTimeCurve
  h!: LayerThickness
  V!: Velocities
  ─────────
  TR ≠ RTR
```

5.1.1. Reflected Waves

```
ReflectionWaves
  LayerBelongs: Waves ⇸ Layers
  WaveIncAng: Waves ⇸ IncidenceAngles
  WaveRefAng: Waves ⇸ ReflectedAngles
  LayerVelocity1, LayerVelocity2: Layers ⤔ Velocities
  ─────────
  ran LayerBelongs ⊆ Layers
  ran WaveIncAng ⊆ IncidenceAngles
  ran WaveRefAng ⊆ ReflectedAngles
  ∃WaveVelocity1, WaveVelocity2: Waves ⇸ Velocities
    • WaveVelocity1 = WaveVelocity2
```

Reflection waves reflect back from the boundary and they belong only to single layer.

The Travel Time curve for reflected waves is formed by taking square of distances and times.

```
  DoubleDistance: ℝ ↔ ℝ
  ─────────
  ∀d1, d2: ℝ •
      d1 ↦ d2 ∈ DoubleDistance ⇔ d1 + d1 = d2
```

```
  DoubleTimes: ℝ ↔ ℝ
  ─────────
  ∀t1, t2: ℝ • t1 ↦ t2 ∈ DoubleTimes ⇔ t1 + t1 = t2
```

The x2- t2 graph for reflected waves is represented like

```
TravelTimeCurveReflection
  GraphPoint: DoubleDistance → DoubleTimes
```

Calculate the minimum point on the graph for further analysis.

```
MinTravelTime
  Points : TravelTimeCurveReflection
  minPoint : ℙTravelTimeCurveReflection
             ⇸ TravelTimeCurveReflection
  ─────────
  minPoint = (λmp : ℙTravelTimeCurveReflection
             | mp ≠ ∅ •
               (μ m: mp | ∀ p:mp | p ≠ m • p>m))
```

If the hyperbola comes in the middle of the graph then it's a single layer. Direct wave curve is the asymptote of the reflection time curve.[1]

ReflectionSingleLayer
MP : MinTravelTime
h! : LayerThickness
V! : Velocities

MP = (0,0)

If the hyperbola graph of travel time does not come in the middle of the graph then it's slopping surface.

Travel Time

$$t_x^2 = \left(\frac{2h\cos\alpha}{V_1}\right)^2 + \left(\frac{x+2h\sin\alpha}{V_1}\right)^2$$

where h is the layer thickness, α is the dipping angle, x is the distance of source and receiver and V is the velocity.

ReflectionSloppingSurface
MP : MinTravelTime
h! : LayerThickness
V! : Velocities

MP ≠ (0,0)

6. Verification of the Specification:

All the specification is implemented in Z/Eves 2.1 Tool. Its snapshot is given for proof:

7. Conclusions

Seismic survey is a complex task. It needs accuracy because a lot of investment is required for later processes, e.g., digging of well. Here, we have represented the seismic survey system using mathematics notations. We have minimized the errors in the system by verification using mathematical rules. In this paper, we didn't consider the noises occurred during the survey. Many types of noises are induced during experiment and filters are available for their reduction. We will formalize those filtration processes as well.

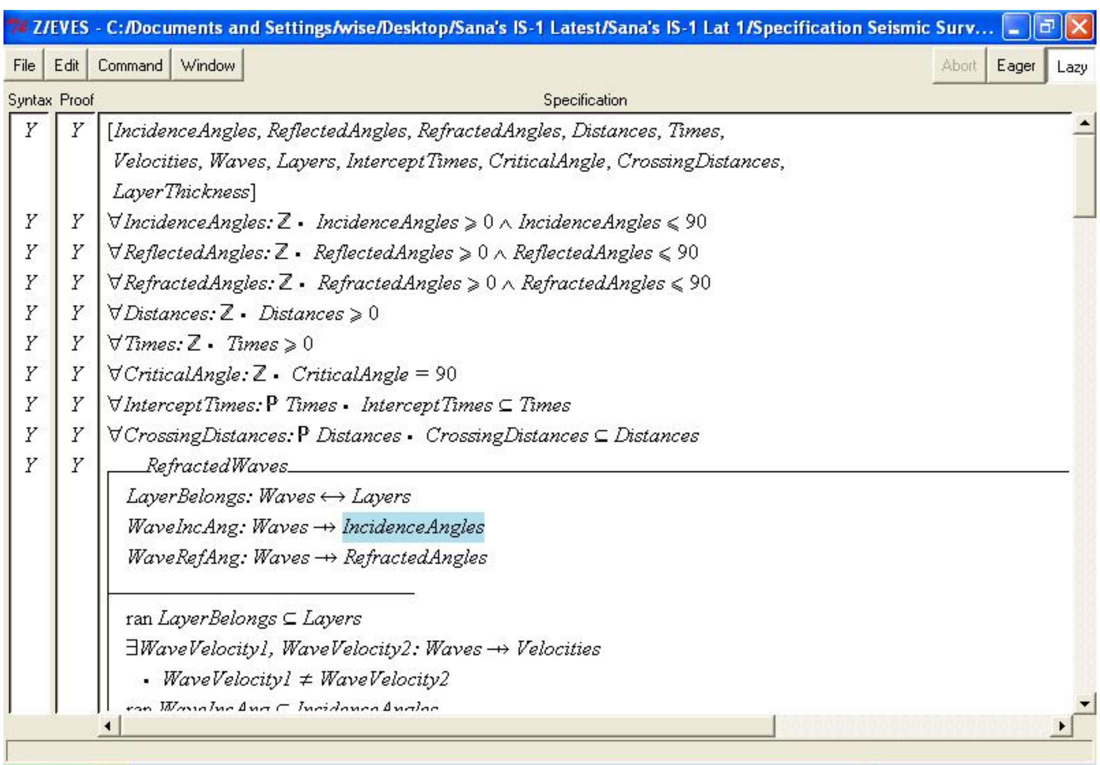


Figure 3. Specification and Proof

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