Structural Analysis Methods for Petri Net based Control Systems: a Review

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Abstract: Petri net (PN) formalism has been widely used for the design and implementation of supervisory control systems and the structural analysis methods for PNs have been successfully used for their analysis. This paper focuses on the recent developments in the area of structural analysis techniques for the PN based control systems. Theoretical developments in the area of PN based control applications with the practical experiences are discussed and further identified the research trends in this area. In addition a brief overview of siphon based analysis methods for controlled systems and initiation of elementary siphon based analysis is also presented. Clear link between the finding of siphons and the net-reduction has been established from the literature and its application for analysis is also discussed.

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1. Introduction

A control system, which is used to control the sequences of events, is a component to coordinate, direct or command the behavior of other components of the system. Predetermined principals and desired behavior of the system to be controlled define its control. In addition, control variables are defined to describe these principals and the involvements through the control variables communicate to requirements imposed on the system behavior.

The size and complexity of automated systems to be controlled, e.g. flexible manufacturing systems, distributed computer systems and distributed database systems, often require a number of processes containing distinct operations. Such systems can be called as supervised systems, in which supervisors are used to coordinate the operations of various subsystems so that overall system's specification is definite. The design of discrete event supervisory controllers for the systems is important step to construct the intelligent control systems. In designing the control system, in essence, it is important to know and be able to identify the required behavior of the controlled system. For this purpose it is necessary to use formal methods to specify and verify the required behavior of a system [87-100]. Further, the controller is required to specify and analyze with the formalism. Thereafter, the final control program describing the operations of the system through a sequence of instructions is implemented and verified by the formalism applied [1].

A system to be controlled is usually studied at logical and performance levels and the models of the former are used to describe qualitative properties and control the sequences of events by ignoring their occurrence time [2]. The avoidance of prohibited states or sequences of operations for the purpose of deadlock avoidance are typical problems for the control of the system [3]. Supervisory control theory of Ramadge and Wonham [4] for the control of discrete event systems is the most important approach at the logical level. Moreover, behavior of the system can be controlled by a supervisor that prevents event occurrences in order to satisfy a control specification [5, 6].

Petri net (PN) formalism is extensively used for the modeling, control, analysis and design of discrete event systems [7, 8, 9, 10, 11, 12] and it has been recognized as flexible modeling technique because it is well suited for modeling the multifarious constraints. Due to its graphical representation, the PN model can be developed and enhanced from the logical sequencing of the system. Further, PN modeling incorporates concurrency [13], non-determinism, timing information [14] and resource-sharing [15] and capture structural interactions to model deadlocks, conflicts [16], buffer-sizes [17] and precedence relations [18]. In addition, PNs have the salient feature to perform qualitative and quantitative analysis of the modeled system due to their underlying mathematical foundation [19].

After performing the modeling of physical system, the main power of PNs as mathematical tool is its support for analysis to study the dynamic behavior [20]. Generally, the analysis of PNs can be classified into the state space based methods [7, 8] and structural analysis including the net reduction and refinement [21].

The state space enumeration method which requires the construction of reachability graph [8, 22, 23, 24] is important and fundamental approach for verification and qualitative analysis of the PN model. Further, it provides complete and detailed information about the dynamic behavior of PN models. One of the main advantages of reachability graph method is that its application is not limited to a certain sub-class of PN [25]. However, the size of reachability graph for PN models is main hurdle to apply this method because its generation requires memory and time at least proportional to the number of reachable markings and it suffers from state explosion problem [26]. Moreover, complete state enumeration is needed for state based analysis methods while the computational complexity has been a major problem when generating the state space for the PN model [2].

On the contrary, the structural analysis methods for PN models has advantage over state space generation methods because the static structure of PN facilitates to study marking-independent properties which depend on the place-transition relationship of underlying net by the flow relation [27]. The underlying static structural has a potential to provide important information about the dynamic behavior of the modeled system. So the behavioral properties that are of foremost interest and less easily analyzable in the analysis of systems may be reduced to easier-to-investigate structural properties [28]. Therefore, structural analysis methods grasped the attention of the researchers and have been widely used for PN based control applications. This paper reveal the major findings in theoretical as well as application perspective of structural analysis techniques for the PN based control systems and overviews the recent developments in this area.

The paper is organized as follows. Section 1 was about introduction and purpose of this review. Section 2 is about related terminology used rest of the paper and structure of the PN. In Section 3 and 4, siphons, traps and minimal siphons are addressed and analysis of PN based control systems are overviewed by taking such kind of structural objects into account. Net reduction based techniques are surveyed in Section 5. Section 6 provides the further research directions in this research area and some concluding remarks are given in Section 7.

2 Fundamentals of Petri nets

In this section, some basic definitions and notations of PN are described. The concepts about structural properties are also explained. The related terminology and notations are mostly taken from [22, 23].

Definition 1: (Petri net) A marked Petri net (Place/Transition Net) is a 5-tuple PN = (P, T, F, W, M_0) where:

$P = \{p_1, p_2, \ldots.., p_n\}$ finite set of places for n>0,

$T = \{t_1, t_2, \ldots\ldots, t_m\}$ finite set of transitions for m>0,

$F \subseteq (P \times T) \cup (T \times P)$ is a set of arcs (flow relation),

$W = F \rightarrow \{1, 2, 3, \ldots\ldots\}$ is the weight function,

$M_0 = P \rightarrow \{0, 1, 2, 3, \ldots..\}$ is the initial marking,

$P \cap F = \Phi$ and $P \cup F \neq \Phi$

A PN structure (P, T, F, W) is denoted by N, so that a PN with specific initial marking is denoted by (N, M_0).

Definition 2 (Pre-set and post-set) For $x \in P \cup T$, $pre(x) = \{y \mid (y, x) \in F\}$ and $post(x) = \{y \mid (x, y) \in F\}$ are called the *pre-set* (*input set*) and *post-set* (*output set*) of x, respectively. For a set $X \subseteq P \cup T$, $pre(X) = \cup_{x\in X} pre(x)$ and $post(X) = \cup_{x\in X} post(x)$.

The structural properties do not depend on the initial marking of a PN but only depend on the PN topology or structure. The most important structural question about the analysis of PNs is the determination of place and transition invariants.

Definition 3: (*Siphons, traps and minimal siphons*) A set of places $S \subseteq P$ is called a *siphon* if $pre(S) \subseteq Post(S)$ and it is called a *trap* if $post(S) \subseteq pre(S)$. A siphon S is called *minimal* if there does not exist S' such that $S' \subset S$.

Definition 4: (Boundedness and safeness) for all $p_i \in P$ of PN (N, M_0), PN is b-bounded if $\forall M_k \in R(M_0) : M_k(p_i) \le b$ and said to be safe if $M_k(p_i) \le 1$. Moreover, a PN is said to be structurally bounded if it is bounded for any finite initial marking M_0.

Definition 5: (Deadlock, deadlock-free and liveness) A transition $t_j \in T$ is said to be dead transition at marking $M_k \in R(M_0)$ if there is no reachable marking to make transition t_j enabled. A marking $M_k \in R(M_0)$ is said to be deadlock if $\forall t_j \in T$, t_j is dead. A PN (N, M_0) is said to be deadlock-free if and only if there is no deadlock. A PN (N, M_0) is said to be live if there is no dead transition at any reachable marking.

Definition 6: (Liveness) a transition $t_j \in T$ is said to be live if and only if there is a marking M_k' reachable from each $M_k \in R(M_0)$ such that M_k'

enables t_j and PN (N, M_0) is live if $\forall t_j \in T$: t_j is live. Further, PN is said to be structurally live if there exists a live initial marking for *N*.

3 SIPHONS AND TRAPS AS STRUCTURAL OBJECTS IN PETRI NETS

This section addresses the siphons and traps as structural objects and their role in the analysis of Petri net based control systems. Siphons and traps are the structural objects that occupy marking invariants. However, the invariant laws associated with them do not hold under any reachable marking, but once they become true they remain true for any consequently reachable markings. Once a siphon loses all its tokens, it remains empty. Similarly a trap remains marked once it becomes marked by a token in it. For the structural analysis of a PN siphons and traps have been extensively investigated and used. They also play an important role in the liveness analysis of a net, particularly in ordinary ones.

Siphons and traps can simply be described as a set of places as given: assume that S is subset of the place set P of a given PN *N* is called a P-siphon or a P-trap, respectively, if and only if •S ⊆ S• or S• ⊆ •S, where •S = {u | N has an edge from u to an element of S} and S• = {v |N has an edge from an element of S to v}. In general, P-siphons or P-traps are simply called siphons or traps, respectively. A place set which has both a siphon and a trap is known as a siphon-trap (ST).

Minimal Siphon and Minimal Trap: As discussed in Definition 3, a minimal siphon is a siphon such that it's any proper subset is not a siphon. If a minimal siphon does not contain any marked trap then it will be a strict minimal siphon. A strict minimal siphon may become empty during the marking progress. Hence, to make a net live is to control such siphons so that they never be empty. These siphons can be well controlled by adding a control place (called monitor), as described in [41]. Unfortunately, by using this method the number of such siphons grows exponentially with the size of a PN and thus leads to very complex control structure for a sizable system. Nuemerous methods have been developed to enumerate the siphons in the literature, [30, 31, 32 and 33] are few examples of such methods. To reduce this control complexity, Li and Zhou [45] introduced the concept of elementary siphons whose control can prevent all other siphons from being emptied.

Example of Siphons and traps: For example in the net shown in Figure 1 given below is modeling the production of two product types J1 and J2 by the help of two machine types M1 and M2. S1 = {p3, p4, p5, p7} is a siphon since •S1 = {t2, t3, t5, t6} ⊂ {t1, t2, t3, t4, t5, t6} = S1•. S1 is also a minimal and basis siphon. S2 = {p1, p2, p3} is siphon as well as a trap (or a P-invariant) because •S1 = S1• = {t1, t2, t3}. S3 = {p1, p2, p3, p4, p5, p7} is a siphon but not a basis siphon since S3 = S1 U S2. S2 is both a minimal and basis trap.

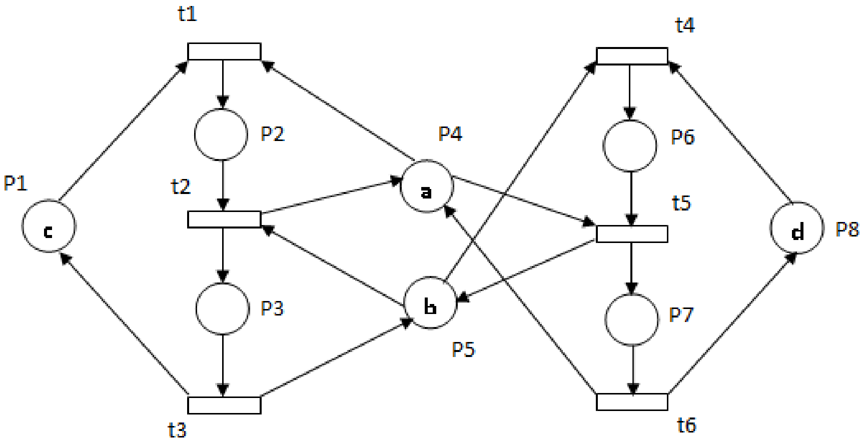

Figure 1: Petri net model of a resource-sharing manufacturing system [34].

Table 1. Interpretation of places of the net given in Fig. 1

Places	Interpretation
P1	Job1's raw part ready
P2	M1 processing J1
P3	M2 processing J1
P4	M1 available
P5	M2 available
P6	M2 processing J2

P7	M1 processing J2
P8	Job2's raw part ready

As a set of place elements, a siphon as a structural object in Petri nets is also known as a structural deadlock. Therefore, the relationship between liveness and siphons becomes strong and apparent when we investigate the practical discrete event system (DES) including a variety of resource allocation systems in an existing technological domain. Moreover the siphon based categorization of liveness and liveness-enforcing management for DESs modeled with PNs is usually considered to be one of the most exciting developments in the last decade from both theoretical and practical points of view.

One of the simplest methods to find a siphon/trap is to reduce the net, because it is easy to find siphon from reduced net as from complete net. As we are interested to find siphon, so the siphons of a PN contained in a particular set of places $\tilde{P}$, all the places not in $\tilde{P}$ can be discarded from the net, thus significantly the problem can be signified.

Let G = (P, T, F) be a PN and $\tilde{P} \subset$ P. The reduction function red is defined as follows:

$\tilde{G} = red$ (G, $\tilde{P}$), where the reduced net $\tilde{G} = (\tilde{P}, \tilde{T}, \tilde{F})$ is defined by:

$\tilde{T} = \{t \in T \mid (\bullet t \cup t\bullet) \cap \tilde{P} \neq \phi\}$ and

$\tilde{F}(p,t) = F(p,t),\ \tilde{F}(t,p) = P(t,p),\quad \forall p \in \tilde{P},\ \forall t \in \tilde{T}$

Lemma 1:[35] Let G = (P, T, F) be a PN and $\tilde{P} \subset$ P. The set of siphons of G contained in $\tilde{P}$ coincides with a set of siphons of the reduced net $\tilde{G}$ = red (G, $\tilde{P}$).

Proof: Consider a set of places S' $\subseteq \tilde{P}$ in G and let $\tilde{S}'$ be the corresponding set of places in $\tilde{G}$. Now, •S' = • $\tilde{S}'$ and S'• = $\tilde{S}'$ •, since all the transitions connected to places in $\tilde{P}$ are preserved by the red operator in net $\tilde{G}$, as well as the corresponding arcs.

The siphon search problem can then be handled by appropriate net reduction. So in fact it is an easiest way to recognize a siphon in a sub-net with a certain structure, than to find the same siphon in the original unreduced net.

4 SIPHON/TRAP BASED APPLICATIONS

The control problem is equivalent to the behavioral control which is directly related to prevent the forbidden states or sequence of events to avoid the deadlock situation and for the purpose of liveness enforcement.

Siphons are structural objects in PNs which occupy marking invariants, play an important role in the deadlock analysis and liveness enforcement for supervisory control systems. Further, most of the results in the literature to assess the liveness of the system are based on siphons. While investigating the discrete event control system, the relationship between siphons and liveness appears strong and effective in an existing technical domain. Therefore, the siphon-based characterization of liveness and liveness-enforcing supervision for PN based control systems is considered to be one of the most interesting developments in the last decade from both theoretical and practical points of view [2].

The work of Ezpeleta et al. [36] introduced the deadlock control policy for a class of PNs known as S3PR based on complete siphon enumeration for plan model. The proposed policy in [36] led to add new places to unmarked siphons.

Minimal siphons and traps are often much smaller than the number of all siphons and traps. Jeng and Peng in [37] presented an algorithm for generating minimal siphons and traps of Petri nets for the purpose of reachability and deadlock freeness. The authors used different combination of places in the proposed algorithm by applying the depth-first search.

The integrated model has been developed by the dynamic behavior of each resource using the resource control net and state machines in [38]. The authors presented the siphon based conditions to ensure the reversibility and liveness in the developed integrated model. Further, the relatioins between circular structure, circular-wait and siphons has also been established in [38].

The work of M. P. Fanti in [39] developed the connections between the PN model results and graph-theoretic approach to address the deadlock problem. In this way the authors established the correspondence between empty siphons and maximal-weight zero-out degree strong components in order to characterize the deadlock occurrence.

Empty or unmarked siphons have also been addressed in the work of Huang et al. [40]. A deadlock prevention algorithm based on iterative approach of adding ordinary and weighted control places to avoid the siphons being unmarked has been proposed by the authors. Further, the relation of the proposed method to liveness and reversibility for the controlled net has been established.

A siphon based reversibility and strong reversibility has been verified for a class of nets called process net with resources in [41]. An important Petri net-based method to prevent deadlocks arising in flexible manufacturing systems (FMS) is to add control places and related arcs to strict minimal siphons (SMS) such that no siphon can be emptied. Its disadvantage lies in that the method often adds too many additional places to the net.

Till year 2002, most of the authors addressed the deadlock prevention policy by adding the control places and related arcs to empty-able siphons or minimal siphons. This policy has two fold issues. First of all it needs the enumeration of siphons which has high computational complexity. Moreover by adding too many control places and arcs in the actual model, the final model becomes a large and complex. Li and Zhou in [42, 43] addressed these issues and introduced the elementary siphons. Further, the authors presented the ways to minimize the addition of places while achieving the same control purpose.

An algorithm has been proposed to find and optimal set of elementary siphons for given set of all siphons in [44]. Further, it has been shown that different set of elementary siphons has different set of reachable states using same control synthesis method.

Elementary siphons as a proper subset of strict minimal siphons (SMS) has been introduced by Z. Li and M. Zhou [45] and applied for the purpose of minimizing the control places to avoid the deadlock situation in flexible manufacturing systems. Liveness enforcing PN supervisors has been generated by adding the monitors to the partial siphons and theory of regions to control the deadlock states in [46].

Z. Li and M. Zhou in year 2004 presented some theoretical results to investigate the conditions for weakly dependent siphons and proved that the number of count for elementary siphons is bounded by the small number of places and transitions in the PN model [47]. The deadlock prevention policy based on adding the monitors to the set of minimal siphons has been frequently addressed in several articles appeared in year 2004 by Z. Li et al. [48, 49, 50, 51, 52, 53, 54, 55, 56, 57].

Later in year 2006, Y. Huang et al. [58] addressed the deadlock prevention policy for a class of PN called S^3PGR^2. The authors developed an algorithm based on siphons and proposed an iterative method of added two kind of control places, ordinary control places and weighted control places, to the siphons.

To avoid the complete siphon enumeration, Z. Li and M. Zhou [59] generated the siphons that need to be controlled by applying the integer programming method. In this way they developed liveness-enforcing supervisors by rearranging the output arcs of the monitors. The concept of elementary siphons along with the mixed integer programming to detect the empty-able siphons for the purpose of deadlock control is presented in [60].

In year 2007 and 2008 Z. Li et al. [3, 61, 62, 63, 64, 65, 66] further divided the set of siphons into the elementary and dependent ones to achieve highly permissive solutions using as few control places as possible. In these articles, monitors were added to elementary siphons and control-induced siphons to make the structurally simple liveness-enforcing PN supervisors.

The set-covering approach has been modified by Piroddi et al. [67] to make the smaller size of the control subnets. The overall computation time and memory has been reduced by avoiding the full siphon enumeration which made the proposed approach to large-size models. The control policy proposed by Piroddi et al. in [67] has been refined by Chao et al. [68] by reducing the number of control arcs and weighted control arcs. Further, the authors improved the approach of Piroddi et al. by reducing the token count in the PN model.

Mixed integer programming technique has been refined in year 2010 by Liu and Li [69] to detect the current deadly marked siphons for a class of nets called system of sequential systems with shared resources (S^4R).

The method of extracting minimal siphons proposed by Li et al. extract one minimal siphon for given maximal unmarked siphon using mixed integer programming approach. Recently, S. Wang et al. [70] proposed an extraction algorithm to extract all the strict minimal siphons ones. Further, S. Wang et al. [71, 72] proposed necessary and sufficient condition for marked optimally controlled strict minimal siphons. The number of monitors added to strict minimal siphons has been reduced for deadlock prevention.

5 NET REDUCTION BASED APPLICATIONS FOR STRUCTURAL ANALYSIS

As discussed in Section 3, the siphon search problem can then be handled by the appropriate net reduction. Reduction techniques are usually used to transform the original large scale model of the system into a smaller and simpler one by preserving the properties of interest to be analyzed. The net reduction is a constructive approach to make possible the analysis of a complex system. The objective behind this approach is to preserve the concerned properties, simplify the complex PN model into subnets. On the other hand, a reduction technique may be used to develop an abstract PN model into a more refined one, i.e., serving as a synthesis technique. There are some limitations about reducibility of PN, either subnets may not exist or are difficult to find. Some rules are hard to apply.

Net reduction and refinement is important structural analysis technique and used by several researchers for control application in the literature. Three reduction rules and heuristic algorithms have been proposed for designated PN in the work of Murata and Komodo [73]. These rules preserved the liveness property of original net and applied for sequence control specification.

The transformation theorems have been presented by using the concept of live and bounded circuit for synthesis and reduction in [74, 75]. The theorems further have been applied for the synthesis of liveness and boundedness preserving generalized PNs. A stepwise reduction and approximation method for generalized stochastic Petri nets (GSPNs) in order to reduce their state space has been presented in [76]. Further, subnets have been reduced to simpler structure by preserving the token flow rates. Petri net reduction techniques and their relationship to top-down and bottom-up synthesis techniques have been reviewed in [77].

The timed Petri net has been analyzed for cycle time by converting it into an equivalent timed marked graph by a reduction technique. In addition, the standard procedure to find the cycle time for marked graphs has been applied in [78].

A closed-loop desired marking control problem for the automatic assembly system using the supervisory control technique has been studied in [79]. The authors further discussed the supervisor reduction technique.

The work of M. C. Zhou [80] presented the reduction of marked graphs to reduce the complexity problem to find the loops inside the large scale models. A stepwise reduction theory and an algorithm for timed marked graph is proposed in this paper.

The application of PN reduction for deadlock analysis can be found in the work of Shatz at al. [81] where they introduced a number of reduction rules applicable to Ada nets (PN models for Ada tasking).

Influencing nets corresponding to forbidden states for the controlled timed PN models has been introduced in [82].The authors presented some equivalence relations between subnet of PN and concluded that replacing a subnet by a simpler equivalent subnet obtained by reduction rules can reduce the sets of inequalities to solve.

H. J. Huang et al. [83] presented three kinds of property-preserving subnet reduction methods to verify that the reduced model has the same desirable properties as the original one. The authors considered the transformations of a transition-bordered path to a single transition, a place-bordered path to a single place and a place-subnet to a single place. For each reduction method, the conditions have been presented for ensuring the preservation of the properties liveness, boundedness and reversibility.

A set of component-level reduction rules for timed PNs has been presented in the work of J. Wang et al. [84]. The reduction rules transformed a timed PN component to a simple one by preserving the external observable timing properties.

A technique to build the reduced model and its constraint graph in order to check the existence of feasible control has been presented in [85] and associated control for the original model has been established in a hierarchical manner. To ease the determination of a feasible control for the system, the authors took advantage of the reduction of complexity provided by the application of transformations rules considered in the research.

A verification and reduction method of cyclic structure based on transition invariant subnet of workflow-net has been presented in [86]. The proposed method has been used to point out the error in the cyclic structure and further used to reduce the sound cyclic structure.

6 RESEARCH DIRECTIONS IN THE PETRI NET ANALYSIS

From the overview of the literature presented above following points can be concluded.

- *Enumeration of siphons:* the number of siphons grows quickly beyond practical limits and in the worst case grows exponentially fast with respect to the PN size [19, 29] and siphon-based liveness enforcing approaches become restricted and degraded. In addition, they suffer from the computational complexity problem since it is known that in general the complete siphon enumeration in a PN is NP-complete. To deal with this problem, partial calculation of siphons, e.g., minimal siphons, elementary siphons, dependant elementary siphons etc. have been proposed by the researchers.
- *Addition of monitors/control places:* siphons usually lead to a much more structurally complex liveness-enforcing supervisor than the PN model that is originally constructed. To deal with the problem of minimizing the addition of control places the concept of elementary siphon, set-covering approach and control-induced siphon and their extensions have been presented by the researchers. However, these solutions are not generally optimal and most of the authors proposed for a specific class of the PN.
- Net-reduction techniques for the computation of siphons, minimal and elementary siphons can be the further research direction in the area of the analysis of Petri net models.

7 CONCLUSIONS

In this paper, siphon/trap based approaches for structural analysis of PN applications to control systems have been discussed. Structural analysis methods for the analysis of PN based control systems discusses in this paper have been categorized into the siphon based methods and net reduction methods. In this survey we tried to emphasize the merits and the demerits of these techniques. Various classes of PNs have been developed for purpose of control of the system and structurally analyzed thoroughly in the literature, both with respect to modeling power and computational complexity for various problems of analysis. It has been established that the use of siphon/trap based techniques for analysis and performance evaluation of PNs is a rich field that is known under the name of structural analysis of PN

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