Applying Logarithmic Fuzzy Preference Programming and VIKOR Methods for Supplier Selection: A Case Study

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Abstract: Supplier selection plays a key role in supply chain management and deals with evaluation, ranking and selection of the best option from a pool of potential suppliers especially in the presence of conflicting criteria. The aim of this study is applying a new integrated method for supplier selection. In this paper, the weights of each criterion are calculated by using of Logarithmic Fuzzy Preference Programming. After that, the VIKOR method is utilized to rank the alternatives. Then we select the best supplier based on these results.

Keywords: Supplier Selection, Logarithmic fuzzy preference programming (LFPP), VIKOR, Fuzzy set.

1. Introduction

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1. Introduction

Selecting the right suppliers significantly reduces the purchasing costs and improves corporate competitiveness therefore supplier selection one of the most important decision making problems. During recent years supply chain management and supplier selection process have received considerable attention in the literature. Supplier selection is a multi-criteria problem and there are not a lot of efficient techniques or algorithms that address this problem. However three major groups of methods in the literature are mathematical programming models, cost based models, and categorical models. Since supplier selection problems usually have several objectives such as maximization of quality or maximization of profit or minimization of cost, the problem can be modeled using mathematical programming. Weber and Current (1993) proposed a multi-objective approach to supplier selection to aim at minimizing the price, maximizing the quality and on time delivery using systems’ constraints and policy constraints in a mixed integer model. Ghodsypour and O’Brien (1998) proposed an integration of AHP and linear programming to consider both tangible and intangible factors in choosing the best suppliers and placing the optimum order quantities among them such that the total value of purchasing becomes maximum. Çebi and Bayraktar (2003) structure the supplier selection problem as an integrated lexicographic goal programming and AHP model including both quantitative and qualitative conflicting factors. Wang, Huang, and Dismkes (2004) use AHP and preemptive goal programming based multi-criteria decision-making methodology is then developed to take into account both qualitative and quantitative factors in supplier selection. Wang and Yang (2009) search supplier selection in a quantity discount environment using multi objective linear programming, AHP, and fuzzy compromise programming. The rest of the paper is organized as follows: The following section presents a concise treatment of the basic concepts of fuzzy set theory. Section 3 presents the methodology of Logarithmic fuzzy preference programming and VIKOR. The application of the proposed framework to Supplier selection is addressed in Section 4. Finally, conclusions are provided in Section 5.

2. Fuzzy Set Theory

Fuzzy set theory was first developed in 1965 by Zadeh; he was attempting to solve fuzzy phenomenon problems, including problems with uncertain, incomplete, unspecific, or fuzzy situations. Fuzzy set theory is more advantageous than traditional set theory when describing set concepts in human language. It allows us to address unspecific and fuzzy characteristics by using a membership function that partitions a fuzzy set into subsets of members that “incompletely belong to” or “incompletely do not belong to” a given subset.

2.1. Fuzzy Numbers

We order the Universe of Discourse such that U is a collection of targets, where each target in the Universe of Discourse is called an element. Fuzzy number \( A \) is mapped onto U such that a random \( x \rightarrow \mu_A(x) \rightarrow [0,1] \). If another element in U is greater than \( x \), we call that element under \( A \).

The universe of real numbers \( R \) is a triangular fuzzy number (TFN) \( A \), which means that for \( x \in R, \mu_A(x) \in [0,1] \). If another element in U is greater than \( x \), we call that element under \( A \).

The universe of real numbers \( R \) is a triangular fuzzy number (TFN) \( A \), which means that for \( x \in R, \mu_A(x) \in [0,1] \), and

\[
\mu_A(x) = \begin{cases} 
\frac{(x - L)}{(M - L)}, & L \leq x \leq M, \\
\frac{(U - x)}{(U - M)}, & M \leq x \leq U, \\
0, & \text{otherwise,}
\end{cases}
\]
Note that \( \bar{A} = (L, M, U) \), where \( L \) and \( U \) represent fuzzy probability between the lower and upper boundaries, respectively, as in Fig. 1. Assume two fuzzy numbers \( \tilde{A}_1 = (L_1, M_1, U_1) \), and \( \tilde{A}_2 = (L_2, M_2, U_2) \); then,

1. \( \tilde{A}_1 \oplus \tilde{A}_2 = (L_1, M_1, U_1) \oplus (L_2, M_2, U_2) = (L_1 + L_2, M_1 + M_2, U_1 + U_2) \)
2. \( \tilde{A}_1 \otimes \tilde{A}_2 = (L_1, M_1, U_1) \otimes (L_2, M_2, U_2) = (L_1 L_2, M_1 M_2, U_1 U_2) \)
3. \( \tilde{A}_1 - \tilde{A}_2 = (L_1, M_1, U_1) - (L_2, M_2, U_2) = (L_1 - L_2, M_1 - M_2, U_1 - U_2) \)
4. \( \tilde{A}_1 \times \tilde{A}_2 = (L_1, M_1, U_1) \times (L_2, M_2, U_2) = (L_1, M_1, U_1) \times (L_2, M_2, U_2) \)

2.2. Fuzzy Linguistic Variables

The fuzzy linguistic variable is a variable that reflects different aspects of human language. Its value represents the range from natural to artificial language. When the values or meanings of a linguistic factor are being reflected, the resulting variable must also reflect appropriate modes of change for that linguistic factor. Moreover, variables describing a human word or sentence can be divided into numerous linguistic criteria, such as equally important, moderately important, strongly important, very strongly important, and extremely important. For the purposes of the present study, the 5-point scale (equally important, moderately important, strongly important, very strongly important and extremely important) is used.

3. Research Methodology

In this paper, the weights of each criterion are calculated using LFPP. After that, VIKOR is utilized to rank the alternatives. Finally, we select the best Supplier based on these results.

3.1. The LFPP-based nonlinear priority method

In this method for the fuzzy pairwise comparison matrix, Wang et al (2011) took its logarithm by the following approximate equation:

Maximise \( \lambda \)

\[
\begin{array}{l}
\text{Subject to } \left\{ \mu_{ij} \left( \ln \left( \frac{w_i}{w_j} \right) \right) \geq \lambda, i = 1, \ldots, n - 1; j = i + 1, \ldots, n \right\} \\
\quad w_i \geq 0, i = 1, \ldots, n,
\end{array}
\]  

(8)

Or as

Maximise \( 1 - \lambda \)

\[
\begin{array}{l}
\text{Subject to } \left\{ \ln w_i - \ln w_j - \lambda \ln \left( \frac{m_{ij}}{l_{ij}} \right) \geq \ln l_{ij}, i = 1, \ldots, n - 1; j = i + 1, \ldots, n \right\} \\
\quad \text{and } - \ln w_i + \ln w_j - \lambda \ln \left( \frac{u_{ij}}{m_{ij}} \right) \geq - \ln u_{ij}, i = 1, \ldots, n; j = i + 1, \ldots, n.
\end{array}
\]  

(9)

\[
\ln \tilde{a} = (\ln l_{ij}, \ln m_{ij}, \ln u_{ij}), \ i,j = 1, \ldots, n \}
\]

That is, the logarithm of a triangular fuzzy judgment \( a_{ij} \) can still be seen as an approximate triangular fuzzy number, whose membership function can accordingly be defined as

\[
\mu_{ij} \left( \ln \left( \frac{w_i}{w_j} \right) \right) = \begin{cases} 
\ln \left( \frac{w_i}{w_j} \right) - \ln l_{ij}, & \ln \left( \frac{w_i}{w_j} \right) \leq \ln m_{ij} \\
\ln m_{ij} - \ln l_{ij}, & \ln m_{ij} \leq \ln \left( \frac{w_i}{w_j} \right) \leq \ln u_{ij} \\
\ln u_{ij} - \ln m_{ij}, & \ln u_{ij} \geq \ln \left( \frac{w_i}{w_j} \right) 
\end{cases}
\]

(7)

Where \( \mu_{ij} \left( \ln \left( \frac{w_i}{w_j} \right) \right) \) is the membership degree of \( \ln \left( \frac{w_i}{w_j} \right) \) belonging to the approximate triangular fuzzy judgment \( \tilde{a} = (\ln l_{ij}, \ln m_{ij}, \ln u_{ij}) \). It is very natural that we hope to find a crisp priority vector to maximize the minimum membership degree \( \lambda = \min \left\{ \mu_{ij} \left( \ln \left( \frac{w_i}{w_j} \right) \right) | i = 1, \ldots, n - 1; j = i + 1, \ldots, n \right\} \). The resultant model can be constructed (Wang et al, 2011) as

\[
\begin{array}{l}
\text{Subject to } \left\{ \ln w_i - \ln w_j - \lambda \ln \left( \frac{m_{ij}}{l_{ij}} \right) \geq \ln l_{ij}, i = 1, \ldots, n - 1; j = i + 1, \ldots, n \right\} \\
\quad - \ln w_i + \ln w_j - \lambda \ln \left( \frac{u_{ij}}{m_{ij}} \right) \geq - \ln u_{ij}, i = 1, \ldots, n; j = i + 1, \ldots, n.
\end{array}
\]  

(9)
It is seen that the normalization constraint \( \sum_{i=1}^{n} w_i = 1 \) is not included in the above two equivalent models. This is because the models will become computationally complicated if the normalization constraint is included. Before normalization, without loss of generality, we can assume \( w_i \geq 1 \) for all \( i = 1, \ldots, n \) such that \( \ln w_i \geq 0 \) for \( i = 1, \ldots, n \). Note that the nonnegative assumption for \( \ln w_i \geq 0 \) (i = 1, ..., n) is not essential. The reason for producing a negative value for \( \lambda \) is that there are no weights that can meet all the fuzzy judgments in \( \tilde{A} \) within their support intervals. That is to say, not all the inequalities \( \ln w_i - \ln w_j - \lambda \ln \left( \frac{m_{ij}}{t_{ij}} \right) \geq \ln l_{ij} \) or \(- \ln w_i + \ln w_j - \lambda \ln \left( \frac{u_{ij}}{m_{ij}} \right) \geq - \ln u_{ij} \) can hold at the same time. To avoid \( k \) from taking a negative value, Wang et al (2011) introduced nonnegative deviation variables \( \delta_{ij} \) and \( \eta_{ij} \) for \( i = 1, \ldots, n-1; j = i + 1, \ldots, n \), such that they meet the following inequalities:

\[
\ln w_i - \ln w_j - \lambda \ln \left( \frac{m_{ij}}{t_{ij}} \right) \geq \ln l_{ij}, i = 1, \ldots, n-1; j = i + 1, \ldots, n \\
- \ln w_i + \ln w_j - \lambda \ln \left( \frac{u_{ij}}{m_{ij}} \right) \geq - \ln u_{ij}, i = 1, \ldots, n; j = i + 1, \ldots, n (10)
\]

It is the most desirable that the values of the deviation variables are the smaller the better. Wang et al (2011) thus proposed the following LFPP-based nonlinear priority model for fuzzy AHP weight derivation:

\[
J = (1-\lambda)^2 + M \sum_{i=1}^{n} \sum_{j=i+1}^{n} (\delta_{ij}^2 + \eta_{ij}^2)
\]

Minimize

\[
\begin{cases}
    x_i - x_j - \lambda \ln \left( \frac{m_{ij}}{t_{ij}} \right) + \delta_{ij} \geq \ln l_{ij}, i = 1, \ldots, n-1; j = i + 1, \ldots, n, \\
    -x_i + x_j - \lambda \ln \left( \frac{u_{ij}}{m_{ij}} \right) + \eta_{ij} \geq - \ln u_{ij}, i = 1, \ldots, n; j = i + 1, \ldots, n, \\
    \lambda, x_i \geq 0, i = 1, \ldots, n \\
    \delta_{ij}, \eta_{ij} \geq 0, i = 1, \ldots, n-1; j = i + 1, \ldots, n
\end{cases}
\]

where \( x_i = \ln w_i \) for \( i = 1, \ldots, n \) and \( M \) is a specified sufficiently large constant such as \( M = 10^9 \). The main purpose of introducing a big constant \( M \) into the above model is to find the weights within the support intervals of fuzzy judgments without violations or with as little violations as possible.

### 3.2. The VIKOR Method

The VIKOR method is a compromise MADM method, developed by Opricovic S and Tzeng (Opricovic, 1998; Opricovic, S. and Tzeng, G. H., 2002) started from the form of Lp-metric:

\[
L_{pi} = \left\{ \frac{\sum_{j=1}^{n} w_j (f_j^+ - f_{ij}) / (f_j^+ - f_j^-) \}^{1/p} \right\}^{1/p} \leq + \infty ; i = 1, 2, \ldots, I.
\]

The VIKOR method can provide a maximum “group utility” for the “majority” and a minimum of an individual regret for the “opponent” (Opricovic, 1998; Opricovic, S; Tzeng, G. H., 2002; Serafin Opricovic & Gwo-Hshiung Tzeng, 2004). 3.2.1. Working Steps of VIKOR Method

1) Calculate the normalized value

Assuming that there are \( m \) alternatives, and \( n \) attributes. The various \( i \) alternatives are denoted as \( x_i \). For alternative \( x_j \), the rating of the \( j \)th aspect is denoted as \( x_{ij} \), i.e. \( x_{ij} \) is the value of \( j \)th attribute. For the process of normalized value, when \( x_{ij} \) is the original value of the ith option and the jth dimension, the formula is as follows:

\[
f_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}}, i = 1, 2, \ldots, m ; j = 1, 2, \ldots, n \quad (12)
\]

2) Determine the best and worst values

For all the attribute functions the best value was \( f_j^+ \) and the worst value was \( f_j^- \), that is, for attribute \( J=1-\)n, we get formulas (13) and (14)

\[
f_j^+ = \max f_{ij}, i = 1, 2, \ldots, m \quad (13)
\]

\[
f_j^- = \min f_{ij}, i = 1, 2, \ldots, m \quad (14)
\]

Where \( f_j^+ \) the positive ideal solution for the jth criteria is, \( f_j^- \) is the negative ideal solution for the jth criteria. If one associates all \( f_j^+ \), one will have the optimal combination, which gets the highest scores, the same as \( f_j^- \).

3) Determine the weights of attributes

The weights of attribute should be calculated to express their relative importance.

4) Compute the distance of alternatives to ideal solution

This step is to calculate the distance from each alternative to the positive ideal solution and then get
the sum to obtain the final value according to formula (15) and (16).

\[ S_i = \sum_{j=1}^{n} w_j (f_i^j - f_{i0})/(f_i^* - f_{i0}^*) \]  \hspace{1cm} (15)

\[ R_i = \max_j \left[ w_j (f_i^j - f_{i0})/(f_i^* - f_{i0}^*) \right] \]  \hspace{1cm} (16)

Where \( S_i \) represents the distance rate of the \( i \)-th alternative to the positive ideal solution (best combination), \( R_i \) represents the distance rate of the \( i \)-th alternative to the negative ideal solution (worst combination). The excellence ranking will be based on \( S_i \) values and the worst rankings will be based on \( R_i \) values. In other words, \( S_i, R_i \) indicate \( L_{+i} \) and \( L_{-i} \) of \( L_0 \)-metric respectively.

5) Calculate the VIKOR values \( Q_i \) for \( i=1,2, \ldots , m \), which are defined as

\[ Q_i = v \left( \frac{S_i - S^*}{S^* - S} \right) + (1 - v) \left( \frac{R_i - R^*}{R^* - R} \right) \]  \hspace{1cm} (17)

where \( S^* = \max_i S_i, S^* = \min_i S_i \), \( R^* = \max_i R_i \), \( R^* = \min_i R_i \), and \( v \) is the weight of the strategy of “the majority of criteria” (or “the maximum group utility”). \( (S - S^*)/(S^* - S) \) represents the distance rate from the positive ideal solution of the \( i \)-th alternative’s achievements. In other words, the majority agrees to use the rate of the \( i \)-th \( (R - R^*)/(R^* - R) \) represents the distance rate from the negative ideal solution of the alternative; this means the majority disagree with the rate of the \( i \)-th alternative. Thus, when \( v \) is larger (\( \geq 0.5 \)), the index \( Q_i \) will tend to majority agreement; when \( v \) is less (\( < 0.5 \)), the index \( Q_i \) will indicate majority negative attitude; in general, \( v = 0.5 \), i.e. compromise attitude of evaluation experts.

6) Rank the alternatives by \( Q_i \) values

According to the \( Q_i \) values calculated by step (4), we can rank the alternatives and to make-decision.

4. A Numerical Application of Proposed Approach

The paper has been conducted in Electrofan Company. This company is a large, well known manufacturer that Working in LPG and CNG industry in Iran. The large number of criteria that should typically be considered in selecting the best supplier, Using the structure of the five criteria as the base and synthesizing the other literature, in current study dimension including Capacity (\( C_1 \)), Delivery (\( C_2 \)), Quality (\( C_3 \)), Shipment Accuracy (\( C_4 \)) and Availability of Raw materials (\( C_5 \)). In addition, there are six suppliers include \( A_1, A_2, A_3, A_4, A_5 \) and \( A_6 \). In this paper, the weights of criteria are calculated by using of LFPP, and these calculated weight values are used as VIKOR inputs. Then, after VIKOR calculations, evaluation of the alternatives and selection of supplier is realized.

Logarithmic Fuzzy Preference Programming:

In LFPP, firstly, we should determine the weights of each criterion by utilizing pair-wise comparison matrices. We compare each criterion with respect to other criteria. You can see the pair-wise comparison matrix for criteria in Table 1.

Table 1: comparison matrix

<table>
<thead>
<tr>
<th></th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
<th>( C_4 )</th>
<th>( C_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>(1,1)</td>
<td>(2,4)</td>
<td>(1,3)</td>
<td>(3,4,5)</td>
<td>(1,4,5)</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>(4,4,1,2)</td>
<td>(1,1)</td>
<td>(1,4,5)</td>
<td>(2,3,4)</td>
<td>(1,4,5)</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>(1,1)</td>
<td>(2,4)</td>
<td>(1,3)</td>
<td>(3,4,5)</td>
<td>(1,4,5)</td>
</tr>
<tr>
<td>( A_4 )</td>
<td>(1,4,5)</td>
<td>(1,2,3,4)</td>
<td>(1,2,3,4)</td>
<td>(1,2)</td>
<td>(2,4)</td>
</tr>
<tr>
<td>( A_5 )</td>
<td>(1,4,5)</td>
<td>(1,2,3,4)</td>
<td>(1,2,3,4)</td>
<td>(1,2)</td>
<td>(2,4)</td>
</tr>
<tr>
<td>( A_6 )</td>
<td>(1,4,5)</td>
<td>(1,4,5)</td>
<td>(1,4,5)</td>
<td>(1,4,5)</td>
<td>(1,4,5)</td>
</tr>
</tbody>
</table>

After forming the model (11) for the comparison matrix and solving this model using Genetic algorithms, the weight vector is obtained as follow:

\[ w^T = (0.284337, 0.169417, 0.065963117, 0.288269, 0.192014)^T \]

VIKOR:

The weights of the criteria are calculated by LFPP up to now, and then these values can be used in VIKOR. So, the VIKOR methodology must be started at the second step. Thus, weighted normalized decision matrix can be prepared. This matrix can be seen from Table 2.

Table 2. The weighted normalized decision matrix

<table>
<thead>
<tr>
<th>( A_i - C_j )</th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
<th>( C_4 )</th>
<th>( C_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>0.040128</td>
<td>0.010319</td>
<td>0.998737741</td>
<td>0.002145</td>
<td>0.028312</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>0.029168</td>
<td>0.226438</td>
<td>0.972255313</td>
<td>0.036066</td>
<td>0.03597</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>0.034362</td>
<td>0.928506</td>
<td>0.5811057</td>
<td>0.045613</td>
<td>0.075262</td>
</tr>
<tr>
<td>( A_4 )</td>
<td>0.445023</td>
<td>0.437878</td>
<td>0.599585114</td>
<td>0.032574</td>
<td>0.499654</td>
</tr>
<tr>
<td>( A_5 )</td>
<td>0.229275</td>
<td>0.888532</td>
<td>0.37005764</td>
<td>0.07569</td>
<td>0.114663</td>
</tr>
<tr>
<td>( A_6 )</td>
<td>0.164703</td>
<td>0.418122</td>
<td>0.857028739</td>
<td>0.055191</td>
<td>0.062029</td>
</tr>
</tbody>
</table>

By following VIKOR procedure steps and calculations, the ranking of Suppliers are gained. The results and final ranking are shown in Table 3.

Table 4: Final evaluation of the alternatives

<table>
<thead>
<tr>
<th>( i )</th>
<th>( E_i )</th>
<th>( Fr_i )</th>
<th>( P_i )</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>0.926543</td>
<td>0.288269</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>0.760809</td>
<td>0.284337</td>
<td>0.852731</td>
<td>5</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>0.631303</td>
<td>0.274563</td>
<td>0.712304</td>
<td>4</td>
</tr>
<tr>
<td>( A_4 )</td>
<td>0.300627</td>
<td>0.169</td>
<td>0.048792</td>
<td>1</td>
</tr>
<tr>
<td>( A_5 )</td>
<td>0.373227</td>
<td>0.156103</td>
<td>0.057995</td>
<td>2</td>
</tr>
<tr>
<td>( A_6 )</td>
<td>0.547432</td>
<td>0.191666</td>
<td>0.331692</td>
<td>3</td>
</tr>
</tbody>
</table>

According to result, if the best one is needed to be selected, then the alternative \( A_4 \) must be chosen.

5. Conclusions

Nowadays, the problem of supplier selection has emerged as an active research field where numerous research papers have been published around this area within last few years. Supplier
selection is a broad comparison of suppliers using a common set of criteria and measures to identify suppliers with the highest potential for meeting a firm’s needs consistently and at an acceptable cost. Selecting the right suppliers significantly reduces the purchasing costs and improves corporate competitiveness therefore supplier selection one of the most important decision making problems. In this study, we have combined LFPP and VIKOR approaches to select the best supplier. The results of the current study indicate that A_4 is the best supplier for this company.

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