Revenue malmquist index with considering inflation by using FDH models in DEA

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Abstract: Revenue Malmquist Index explains change of Revenue productivity of Decision Making Units (DMUs) in two periods. The Trade Offs approach is an advanced tool for the improvement of the discrimination of Data Envelopment Analysis (DEA) models. They used CRS models in DEA for computing this index, since the convexity assumption is strong condition for computing, so for solving this problem in this paper we use Free Disposal Hull (FDH) models in DEA for computing Meta Revenue Malmquist Index. Also In this paper Revenue Malmquist Index is evaluated considering in fact that relative importance of inputs and outputs in different periods are different. In the papers concerning Revenue Mamlquist Index this fact is not considered, which is very important from managerial point of you.
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1 Introduction

Data Envelopment Analysis (DEA) is a mathematical programming technique that measures the relative efficiency of Decision Making Units (DMUs) with multiple inputs and outputs. Charnes and et al.(1978) first proposed DEA as an evaluation tool to measure and compare the relative efficiency of DMUs. Their model assumed Constant Returns to scale (CRS, the CCR model) and the model with Variable Return to Scale (VRS, the BCC model) was developed by banker and et al.(1984). Podinovski et al.(2004) suggests the incorporation of production Trade Offs in to DEA (TO) models under these circumstances, but weight restriction and Trade Offs are most commonly used by Decision Makers. The Malmquist Index is the most important Index for measuring the relative productivity change of DMUs in multiple time periods. For the first time, the Mamquist Index was introduced by Caves and et.al (1982), later DEA was used by Fare, Grosskopf, Lindgren and Ross (FGLR, fare et al.1992), and (FGNZ, Fare et al.1994) for measuring the Mamquist Index.

The structure of the paper is as follows. In section 2 we describe Free Disposal Hull (FDH) Models in DEA and in section 3 we explain Revenue Efficiency and Revenue Malmquist Index for DMUs in different models of DEA (CRS, VRS, TO). We explain the method for measuring Revenue Malmquist Index with variable relative importance as a function of time in different period by using FDH Models of DEA in section 4. The last section summarizes and concludes.

2 Free Disposal Hull (FDH) Models

Considering the observed output vector as $Y_j \in R^s$ and the input vector as, $X_j \in R^m$, we assume that the inputs and outputs are nonnegative and, $X_j \neq 0, Y_j \neq 0$ for DMU_j ; $j = 1,2,\dots,n.$

The basic motivation for introducing FDH model is to make sure taht the efficiency evaluation are effected from only actually observed performances. For using FDH in DEA models, Deprins, Simar and Tulknes make some assumptions and extends the axioms of PPS in the following manner (for more details about FDH Models see[5, 6]):

Assumption:

1-The main point for making production possibility set is removing convexity axiom.

Extended axioms:

1- (Nonempty). The observed; $(X_j, Y_j) \in T$, $j = 1,2,\dots,n.$

2- (Proportionality). If $(X,Y) \in T$, then $(\lambda X, \lambda Y) \in T$ for all ≥ 0.

3- (Free disosability). If $(X,Y) \in T, \bar{X} \leq X, \bar{Y} \geq Y$, then $(X,Y) \in T$.

4- (Minimum extrapolation). T is the smallest set that satisfies axiom 1-3. (Where T is, $T = \{(X,Y) \mid$ output vector $Y \geq 0$ can produced from input vector $X \geq 0\}$).

Now, the PPS can be defined on the basis of the following the minimal PPS (PPSFDH−CRS) that satisfies axioms (1-4) is:

$$PPS_{FDH-CCR} = \cup_{i=1}^{m}\{(x,y)\ \ x \geq \lambda_j^i x_j\ \ y \leq \lambda_j^i y_j\ \ \lambda_j^i \geq 0\ \ (j = 1,2,\dots,n)\}$$

Based on, $PPS_{FDH-CCR}$ for assessing the efficiency of DMU_k ($k = 1,2,\dots,n$) that is defined from this PPS, we have following model:
DEA model with FDH technology and input orientation:

$$\text{Min} \quad \text{Max}\theta_j^k$$

$$\begin{aligned} \text{S.t} \quad & \lambda_{jk}x_{ij} \leq \theta_j^k x_{ik} \quad & i = 1,2,\dots,m \quad & (a) \\ & \lambda_{jk}y_{rj} \geq y_{rk} \quad & r = 1,2,\dots,s \quad & (b) \quad (1) \\ & \lambda_{jk} \geq 0 \quad & j,k = 1,2,\dots,n & \end{aligned}$$

By computing λ_{jk} from constraint (b) we will have:

$$\lambda_{jk} \geq \frac{y_{rk}}{y_{rj}} \qquad r = 1,2,\dots,s$$

Let $\lambda_{lk} = \max\left\{\frac{y_{rk}}{y_{rj}} \mid r = 1,2,\dots,s\right\} = \frac{y_{lk}}{y_{lj}}$ (2)

So $\theta_j^k \geq \frac{\lambda_{lk}{}^* x_{ij}}{x_{ik}}$

$i = 1,2,\dots,m$

$$\theta_j^{*k} = \max\left\{\frac{\lambda_{lk}{}^* x_{ij}}{x_{ik}}\right\}$$

$i = 1,2,\dots,m$ (3)

Therefore $\theta^{*k} = \text{Min}\,\theta_j^{*k}$

$j = 1,2,\dots,n$ (4)

Similarly, we can compute efficiency of DMU_k in V RS model of FDH, by following way:

$$\begin{aligned} & \text{Min}\,\theta \\ \text{S.t} \quad & X\lambda \leq \theta X_k \\ & Y\lambda \geq Y_k \qquad (5) \\ & 1\lambda = 1 \\ & \lambda_j \in \{0,1\} \end{aligned}$$

Model (5) is mix integer programming, λ is integer variable and θ is free variable.

3 Revenue Efficiency and Revenue Malmquist Index For *DMUs* In Different Models Of DEA

Assuming that there are n DMUs each with m inputs and s outputs, we evaluate the Revenue Efficiency of DMU_o, o ∈ {1, 2,…,n} in the following way:

$$\begin{aligned} & PY^{(CRS)} = \text{Max} \sum_{k=1}^{s} p_{ko}y_k \\ \text{S.t} \quad & \sum_{j=1}^{n}\lambda_j x_{ij} \leq x_{io} \qquad & i = 1,2,\dots,m \\ & \sum_{j=1}^{n}\lambda_j y_{kj} \leq y_k \qquad & k = 1,2,\dots,s \qquad (6) \\ & \lambda_j \geq 0 \qquad & j = 1,2,\dots,n \\ & y_k \geq 0 \qquad & k = 1,2,\dots,s \end{aligned}$$

Where j is the DMU index j = 1,2,…., n, k the output index , k = 1,2,…, s and i the input index i= 1,2,…. , m y_{kj} value of the kth output for the jth DMU, x_{ij} the value of the ith input for the jth DMU and $p = (p_1, p_2, \dots, p_s)$ is the common unit output price or unit Revenue vector . Let the optimal solution obtained from solving model (1) be (y^*, λ^*), then the Revenue Efficiency is defined in ratio from:

$$E_R(\text{Revenue Efficincy}) = \frac{py_o}{py^*} = \frac{\sum_{k=1}^{s} p_{ko}y_{ko}}{\sum_{k=1}^{s} p_{ko}y^*_k} \quad (7)$$

It is alleged that $0 \leq E_R \leq 1$; moreover, $DMU_o = (x_0, y_0)$ is revenue efficient if and only if $E_R = 1$.(For more details see Farrell (1957)). By a similar way , we can compute the Revenue Efficiency of DMU_o in VRS model of DEA by addition a constraint of $\sum_{j=1}^{n}\lambda_j = 1$ to model (6).

Supposing there are l Trade Offs, we shall represent the Trade Offs in from (D_{if}, Q_{kf}) where $i = 1,2,\dots,m$, $k = 1,2,\dots,s$ and $f = 1,2,\dots,l$ (for more details about Trade Offs model of DEA see Podinovski (2004)). We evaluate the Revenue Efficiency of DMU_o o ∈ {1,2,…,n} in Trade Offs model of DEA according to the following model :

$$\begin{aligned} & PY^{(CRS)} = \text{Max} \sum_{k=1}^{s} P_{ko}y_k \\ \text{S.t} \quad & \sum_{j=1}^{n}\lambda_j x_{ij} + \sum_{f=1}^{l}\pi_f d_{if} \leq x_{io} \\ & i = 1,2,\dots,m \\ & \sum_{j=1}^{n}\lambda_j y_{kj} + \sum_{f=1}^{l}\pi_f q_{kf} \geq y_k \\ & k = 1,2,\dots,s \qquad (8) \\ & \lambda_j \geq 0 \qquad j = 1,2,\dots,n \\ & \pi_f \geq 0 \qquad f = 1,2,\dots,l \\ & y_k \geq 0 \qquad k = 1,2,\dots,s \end{aligned}$$

Therefore the Revenue Efficiency of DMU_o in

Trade Offs model of DEA is :

$$E_R^{TO}(\text{Revenue Efficiency}) = \frac{py_o}{py^*} = \frac{\sum_{k=1}^{s} p_{ko} y_{ko}}{\sum_{k=1}^{s} p_{ko} y_k^*} \quad (9)$$

The computation of $PY_t^{t(CRS)}, E_{R(t)}^{t(CRS)}$ (DMU in period t and frontier period t) and $PY_{t+1}^{t+1(CRS)}, E_{R(t+1)}^{t+1(CRS)}$ (DMU in period t+1 and frontier period t+1) are like (6) and (7) where (x_{ij}^t, y_{kj}^t) and $(x_{ij}^{t+1}, y_{kj}^{t+1})$ are substituted for (x_{ij}, y_{kj}) for all i, k, j. In a similar way we can compute $E_{R(t)}^{t(VRS)}, E_{R(t+1)}^{t+1(VRS)}$. (The computation of $PY_t^{t(TO)}, E_{R(t)}^{t(To)}$ and $PY_{t+1}^{t+1(TO)}, E_{R(t+1)}^{t+1(TO)}$ are like (8), (9) where (x_{ij}^t, y_{kj}^t) and $(x_{ij}^{t+1}, y_{kj}^{t+1})$ are substituted for (x_{ij}, y_{kj}) for all i, k, j and, by addition a constraint of $\sum_{j=1}^{n} \lambda_j = 1$ to model (6).

DEA model with CRS technology and input orientation.

Frontier period = t + 1 and DMU$_o$ in period t .

$$PY_t^{t+1(CRS)} = \text{Max} \sum_{k=1}^{s} p_{ko}^t y_k^t$$

$$\text{S.t} \quad \sum_{j=1}^{n} \lambda_j^{t+1} x_{ij}^{t+1} \le x_{io}^t$$

$$i = 1,2, \ldots, m$$

$$\sum_{j=1}^{n} \lambda_j^{t+1} y_{kj}^{t+1} \ge y_k^t$$

$$k = 1,2, \ldots, s \quad (10)$$

$$\lambda_j^{t+1} \ge 0 \qquad j = 1,2, \ldots, n$$

$$y_k^t \ge 0 \qquad k = 1,2, \ldots, s$$

Therefore, the Revenue Efficiency for DMU$_o$ in period t and frontier period= t + 1 is:

$$E_{R(t)}^{t+1(CRS)} = \frac{\sum_{k=1}^{s} p_{ko}^t y_{ko}^t}{\sum_{k=1}^{s} p_{ko}^t y_k^{*(t)}} \quad (11)$$

DEA model with CRS technology and input orientation.

Frontier period=t and DMU$_o$ in period t + 1 .

$$PY_{t+1}^{t(CRS)} = \text{Max} \sum_{k=1}^{s} p_{ko}^{t+1} y_k^{t+1}$$

$$\text{S.t} \quad \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \le x_{io}^{t+1} \qquad i = 1,2, \ldots, m$$

$$\sum_{j=1}^{n} \lambda_j^t y_{kj}^t \ge y_k^{t+1}$$

$$k = 1,2, \ldots, s \quad (12)$$

$$\lambda_j^t \ge 0 \qquad j = 1,2, \ldots, n$$

$$y_k^{t+1} \ge 0 \qquad k = 1,2, \ldots, s$$

Hence, the Revenue Efficiency for DMU$_o$ in period t + 1 and frontier period= t is:

$$E_{R(t+1)}^{t(CRS)} = \frac{\sum_{k=1}^{s} p_{ko}^{t+1} y_{ko}^{t+1}}{\sum_{k=1}^{s} p_{ko}^{t+1} y_k^{*(t+1)}} \quad (13)$$

DEA model with Trade Offs technology and input orientation .

Frontier period = t + 1 and DMU$_o$ in period t .

$$PY_t^{t+1(TO)} = \max \sum_{k=1}^{s} p_{ko}^t y_k^t$$

$$\text{S.t} \quad \sum_{j=1}^{n} \lambda_j^{t+1} x_{ij}^{t+1} + \sum_{f=1}^{l} \pi_f^{t+1} d_{if}^{t+1} \le x_{io}^t$$

i=1,2,…,m

$$\sum_{j=1}^{n} \lambda_j^{t+1} y_{kj}^{t+1} + \sum_{f=1}^{l} \pi_f^{t+1} q_{kf}^{t+1} \ge y_k^t$$

$$k = 1,2, \ldots, s \quad (14)$$

$$\lambda_j^{t+1} \ge 0 \qquad j = 1,2, \ldots, n$$

$$\pi_f^{t+1} \ge 0 \qquad f = 1,2, \ldots, l$$

$$y_k^t \ge 0 \qquad k = 1,2, \ldots, s$$

Therefore, the Revenue Efficiency for DMU$_o$ in period t and frontier period t + 1 is:

$$E_{R(t)}^{t+1(TO)} = \frac{\sum_{k=1}^{s} p_{ko}^t y_{ko}^t}{\sum_{k=1}^{s} p_{ko}^t y_k^{*(t)}} \quad (15)$$

DEA model with Trade Offs technology and input orientation.

Frontier period=t and DMU$_o$ in period t + 1.

$$PY_{t+1}^{t(TO)} = \text{Max} \sum_{k=1}^{s} p_{ko}^{t+1} y_k^{t+1}$$

$$\text{S.t} \quad \sum_{j=1}^{n} \lambda_j^t x_{ij}^t + \sum_{f=1}^{l} \pi_f^t d_{if}^t \le x_{io}^{t+1}$$

$$i = 1,2, \ldots, m$$

$$\sum_{j=1}^{n} \lambda_j^t y_{kj}^t + \sum_{f=1}^{l} \pi_f^t q_{kf}^t \ge y_k^{t+1}$$

$$k = 1,2, \ldots, s \quad (16)$$

$$\lambda_j^t \ge 0 \qquad j = 1,2, \ldots, n$$

$$\pi_f^t \ge 0 \qquad f = 1,2, \ldots, l$$

$$y_k^{t+1} \ge 0 \qquad k = 1,2, \ldots, s$$

So, the Revenue Efficiency for DMU$_o$ in period t + 1 and frontier period t is:

$$E_{R(t+1)}^{t(TO)} = \frac{\sum_{k=1}^{s} p_{ko}^{t+1} y_{ko}^{t+1}}{\sum_{k=1}^{s} p_{ko}^{t+1} y_k^{*(t+1)}} \quad (17)$$

Consider the following equations:

$$REC = \frac{E_{R(t+1)}^{t+1(CRS)}}{E_{R(t)}^{t(CRS)}} \quad (18)$$

$$PREC = \frac{E_{R(t+1)}^{t+1(VRS)}}{E_{R(t)}^{t(VRS)}} \quad (19)$$

$$RTC = [\frac{E_{R(t)}^{t(CRS)}}{E_{R(t)}^{t+1(CRS)}} \times \frac{E_{R(t+1)}^{t(CRS)}}{E_{R(t+1)}^{t+1(CRS)}}]^{\frac{1}{2}} \quad (20)$$

$$SREC = [\frac{E_{R(t)}^{t(VRS)}}{E_{R(t)}^{t(CRS)}} \times \frac{E_{R(t+1)}^{t+1(CRS)}}{E_{R(t+1)}^{t+1(VRS)}}] \quad (21)$$

Where REC is Revenue Efficiency Change, PREC is Pure Revenue Efficiency Change, RTC is Revenue Technology Change and SREC is Scale Revenue Efficiency Change.

The Malmquist Index and its FGLR and FGNZ decompositions are as follows (for more details, see Fare and et al., 1992, 1994). By similar way we can compute Revenue Malmquist Index.

Revenue Malmquist Index (RMI) = REC × RTC (22)

Revenue Malmquist Index (RMI) = PREC × SREC ×RTC (23)

We define:

$$EREC = \frac{E_{R(t+1)}^{t+1(TO)}}{E_{R(t)}^{t(TO)}} \quad (24)$$

$$ERTC = [\frac{E_{R(t)}^{t(TO)}}{E_{R(t)}^{t+1(TO)}} \times \frac{E_{R(t+1)}^{t(TO)}}{E_{R(t+1)}^{t+1(TO)}}]^{\frac{1}{2}} \quad (25)$$

$$RREC = [\frac{E_{R(t)}^{t(CRS)}}{E_{R(t)}^{t(TO)}} \times \frac{E_{R(t+1)}^{t+1(TO)}}{E_{R(t+1)}^{t+1(CRS)}}] \quad (26)$$

Where EREC is Expanded Revenue Efficiency Change, ERTC is Expanded Revenue Technology Change and RREC is Regulation Revenue Efficiency Change. So

$$\text{Expanded Revenue Malmquist Index (ERMI)} = EREC \times ERTC \quad (27)$$

Or

$$\text{Expanded Revenue Malmquist Index (ERMI)} = REC \times RREC \times ERTC \quad (28)$$

By adding VRS technology, and by using PREC and SREC, we will have another decomposition of the ERMI :

$$\text{Expanded Revenue Malmquist Index (ERMI)} = PREC \times SREC \times RREC \times ERTC \quad (29)$$

If $RMI_j > 1$ or $EMRI_j > 1$, it shows DMU_j had progress.

If $RMI_j < 1$ or $EMRI_j < 1$, it shows DMU_j had regress.

If $RMI_j = 1$ or $EMRI_j = 1$, it shows DMU_j had not changing.

3. Revenue Malmquist Index for DMUs In Different Models Of DEA With Variable Relative Importance As A Function Of Time In Different Period by using FDH Models in DEA

With having previous assumption,

DEA model with FDH-CRS technology and input orientation .

Frontier period = t and DMU_o in period t .

$$PY_t^{t(FDH-CRS)} = Max\ \theta_j^{ot} = \sum_{k=1}^{s} p_{ko}^t \beta_{ko}^t y_k^t$$

$$j = 1,2,\dots,n$$

$$S.t \quad \lambda_{jo}^t \alpha_{ij}^t x_{ij}^t \le \alpha_{io}^t x_{io}^t \quad i = 1,2,\dots,m$$

$$\lambda_{jo}^t \beta_{kj}^t y_{kj}^t \ge y_k^t \quad k = 1,2,\dots,s \quad (30)$$

$$\lambda_{jo}^t \ge 0 \quad j = 1,2,\dots,n$$

$$y_k^t \ge 0 \quad k = 1,2,\dots,s$$

Therefore, the Revenue Efficiency for DMU_o in period t and frontier period=t is:

$$\theta_{R(t)}^{t(FDH-CRS)} = \frac{\sum_{k=1}^{s} p_{ko}^t \beta_{ko}^t y_{ko}^t}{PY_t^{t(FDH-CRS)}} \quad (31)$$

Where α_{ij}^t the variation of multiplier of the ith input for DMU_j in period t and β_{kj}^t the variation of multiplier of rth output for DMU_j in period t. The computation of $PY_{t+1}^{t(CRS)}$, $\theta_{R(t)}^{t+1(CRS)}$ (DMU in period t + 1 and frontier period= t + 1) and are like (30) and (31) where $(x_{ij}^{t+1}, y_{kj}^{t+1})$ are substituted for (x_{ij}^t, y_{kj}^t) for all i, k, j .

In a similar way we can compute $E_{R(t)}^{t(VRS)}$, $\theta_{R(t+1)}^{t+1(VRS)}$. (The computation of $PY_{t+1}^{t+1(TO)}$, $\theta_{R(t)}^{t+1(TO)}$ are like (32), (33) where $(x_{ij}^{t+1}, y_{kj}^{t+1})$ are substituted for (x_{ij}^t, y_{kj}^t) for all $, k, j$.

DEA model with Trade Offs technology and input orientation in FDH models of DEA.

Frontier period = t and DMU_o in period t .

$$PY_t^{t(FDH-TO)} = Max\ \theta_j^{ot} = \sum_{k=1}^{s} p_{ko}^t \beta_{ko}^t y_k^t$$

$$j = 1,2,\dots,n$$

$$S.t \quad \lambda_{jo}^t \alpha_{ij}^t x_{ij}^t + \sum_{f=1}^{l} \pi_{ft} \alpha_{if}^t d_{if}^t \le \alpha_{io}^t x_{io}^t$$

$$i = 1,2,\dots,m$$

$$\lambda_{jo}^{t}\beta_{kj}^{t}y_{kj}^{t}+\sum_{f=1}^{l}\pi_{ft}\beta_{kf}^{t}q_{kf}^{t}\geq y_{k}^{t}$$

$$k=1,2,\ldots,s \quad (32)$$

$$\lambda_{jo}^{t}\geq 0 \qquad j=1,2,\ldots,n$$

$$\pi_{ft}\geq 0 \qquad f=1,2,\ldots,l$$

$$y_{k}^{t}\geq 0 \qquad k=1,2,\ldots,s$$

Therefore, the Revenue Efficiency for DMU_o in period t and frontier period t is :

$$\theta_{R(t)}^{t(FDH-TO)}=\frac{\sum_{k=1}^{s}p_{ko}^{t}\beta_{ko}^{t}y_{ko}^{t}}{PY_{t}^{t(FDH-TO)}} \quad (33)$$

DEA model with FDH-CRS technology and input orientation .
Frontier period = t + 1 and DMU_o in period t .

$$PY_{t}^{t+1(FDH-CRS)}=\text{Max}\ \theta_{j}^{ot}=\sum_{k=1}^{s}p_{ko}^{t}\beta_{ko}^{t}y_{k}^{t}$$

$$j=1,2,\ldots,n$$

$$\text{S.t} \qquad \lambda_{jo}^{t+1}\alpha_{ij}^{t+1}x_{ij}^{t+1}\leq \alpha_{io}^{t}x_{io}^{t}$$

$$i=1,2,\ldots,m$$

$$\lambda_{jo}^{t+1}\beta_{kj}^{t+1}y_{kj}^{t+1}\geq y_{k}^{t}$$

$$k=1,2,\ldots,s \quad (34)$$

$$\lambda_{jo}^{t+1}\geq 0 \qquad j=1,2,\ldots,n$$

$$y_{k}^{t}\geq 0 \qquad k=1,2,\ldots,s$$

Therefore, the revenue efficiency for DMU_o in period t and frontier period t + 1 is:

$$\theta_{R(t)}^{t+1(FDH-CRS)}=\frac{\sum_{k=1}^{s}p_{ko}^{t}\beta_{ko}^{t}y_{ko}^{t}}{PY_{t}^{t+1(FDH-CRS)}} \quad (35)$$

DEA model with FDH-CRS technology and input orientation .
Frontier period = t and DMU_o in period t + 1.

$$PY_{t+1}^{t(FDH-CRS)}=\text{Max}\ \theta_{j}^{ot+1}=\sum_{k=1}^{s}p_{ko}^{t+1}\beta_{ko}^{t+1}y_{k}^{t+1}$$

$$j=1,2,\ldots,n$$

$$\text{S.t} \qquad \lambda_{jo}^{t}\alpha_{ij}^{t}x_{ij}^{t}\leq \alpha_{io}^{t+1}x_{io}^{t+1}$$

$$i=1,2,\ldots,m$$

$$\lambda_{jo}^{t}\beta_{kj}^{t}y_{kj}^{t}\geq y_{k}^{t+1}$$

$$k=1,2,\ldots,s \quad (36)$$

$$\lambda_{jo}^{t}\geq 0 \qquad j=1,2,\ldots,n$$

$$y_{k}^{t+1}\geq 0 \qquad k=1,2,\ldots,s$$

Hence, the revenue efficiency for DMU_o in period t + 1 and frontier period t is:

$$\theta_{R(t+1)}^{t(CRS)}=\frac{\sum_{k=1}^{s}p_{ko}^{t+1}\beta_{ko}^{t+1}y_{ko}^{t+1}}{PY_{t+1}^{t(FDH-CRS)}} \quad (37)$$

DEA model with Trade Offs technology and input orientation with using FDH models of DEA.
Frontier period = t + 1 and DMUo in period t .

$$PY_{t}^{t+1(FDH-TO)}=\text{Max}\ \theta_{j}^{ot}=\sum_{k=1}^{s}p_{ko}^{t}\beta_{ko}^{t}y_{k}^{t}$$

$$\text{S.t} \quad \lambda_{jo}^{t+1}$$

$$\alpha_{ij}^{t+1}x_{ij}^{t+1}+\sum_{f=1}^{l}\pi_{ft+1}\alpha_{if}^{t+1}d_{if}^{t+1}\leq \alpha_{io}^{t}x_{io}^{t}$$

$$i=1,2,\ldots,m$$

$$\lambda_{jo}^{t+1}\beta_{kj}^{t+1}y_{kj}^{t+1}+\sum_{f=1}^{l}\pi_{ft+1}\beta_{kf}^{t+1}q_{kf}^{t+1}\geq y_{k}^{t}$$

$$k=1,2,\ldots,s \quad (38)$$

$$\lambda_{jo}^{t+1}\geq 0 \qquad j=1,2,\ldots,n$$

$$\pi_{ft+1}\geq 0 \qquad f=1,2,\ldots,l$$

$$y_{k}^{t}\geq 0 \qquad k=1,2,\ldots,s$$

Therefore, the Revenue Efficiency for DMU_o in period t and frontier period=t + 1 is:

$$\theta_{R(t)}^{t+1(FDH-TO)}=\frac{\sum_{k=1}^{s}p_{ko}^{t}\beta_{ko}^{t}y_{ko}^{t}}{PY_{t}^{t+1(FDH-TO)}} \quad (39)$$

DEA model with Trade Offs technology and input orientation by using FDH models of DEA.
Frontier period=t and DMU_o in period t + 1 .

$$PY_{t+1}^{t(FDH-TO)}=\text{Max}\ \theta_{j}^{ot+1}=\sum_{k=1}^{s}p_{ko}^{t+1}\beta_{ko}^{t+1}y_{k}^{t+1}$$

$$\text{S.t} \quad \lambda_{jo}^{t}\alpha_{ij}^{t}x_{ij}^{t}+\sum_{f=1}^{l}\pi_{ft}\alpha_{if}^{t}d_{if}^{t}\leq \alpha_{io}^{t+1}x_{io}^{t+1}$$

$$i=1,2,\ldots,m$$

$$\lambda_{jo}^{t}\beta_{kj}^{t}y_{kj}^{t}+\sum_{f=1}^{l}\pi_{ft}\beta_{kf}^{t}q_{kf}^{t}\geq y_{k}^{t+1}$$

$$k=1,2,\ldots,s \quad (40)$$

$$\lambda_{jo}^{t}\geq 0 \qquad j=1,2,\ldots,n$$

$$\pi_{ft}\geq 0 \qquad f=1,2,\ldots,l$$

$$y_{k}^{t+1}\geq 0$$

So, the revenue efficiency for DMU_o in period t + 1 and frontier period=t is:

$$\theta_{R(t+1)}^{t(FDH-TO)}=\frac{\sum_{k=1}^{s}p_{ko}^{t+1}\beta_{ko}^{t+1}y_{ko}^{t+1}}{PY_{t+1}^{t(FDH-TO)}} \quad (41)$$

Consider the following equations:

$$REC_{\theta}=\frac{\theta_{R(t+1)}^{t+1(FDH-CRS)}}{\theta_{R(t)}^{t(FDH-CRS)}} \quad (42)$$

$$PREC_\theta = \frac{\theta_{R(t+1)}^{t+1(FDH-VRS)}}{\theta_{R(t)}^{t(FDH-VRS)}} \quad (43)$$

$$RTC_\theta = [\frac{\theta_{R(t)}^{t(FDH-CRS)}}{\theta_{R(t)}^{t+1(FDH-CRS)}} \times \frac{\theta_{R(t+1)}^{t(FDH-CRS)}}{\theta_{R(t+1)}^{t+1(FDH-CRS)}}]^{\frac{1}{2}} \quad (44)$$

$$SREC_\theta = [\frac{\theta_{R(t)}^{t(FDH-VRS)}}{\theta_{R(t)}^{t(FDH-CRS)}} \times \frac{\theta_{R(t+1)}^{t+1(FDH-VRS)}}{\theta_{R(t+1)}^{t+1(FDH-VRS)}}] \quad (45)$$

$$\text{Revenue Malmquist Index } (RMI_\theta) = REC_\theta \times RTC_\theta \quad (46)$$

$$\text{Revenue Malmquist Index } (RMI_\theta) = PREC_\theta \times SREC_\theta \times RTC_\theta \quad (47)$$

Therefore

$$EREC_\theta = \frac{\theta_{R(t+1)}^{t+1(FDH-TO)}}{\theta_{R(t)}^{t(FDH-TO)}} \quad (48)$$

$$ERTC_\theta = [\frac{\theta_{R(t)}^{t(FDH-TO)}}{\theta_{R(t)}^{t+1(FDH-TO)}} \times \frac{\theta_{R(t+1)}^{t(FDH-TO)}}{\theta_{R(t+1)}^{t+1(FDH-TO)}}]^{\frac{1}{2}} \quad (49)$$

$$RREC_\theta = [\frac{\theta_{R(t)}^{t(FDH-CRS)}}{\theta_{R(t)}^{t(FDH-TO)}} \times \frac{\theta_{R(t+1)}^{t+1(FDH-TO)}}{\theta_{R(t+1)}^{t+1(FDH-CRS)}}] \quad (50)$$

$$\text{Explanded Revenue Malmquist Index}(ERMI_\theta) = EREC_\theta \times ERTC_\theta \quad (51)$$

Or

$$\text{Explanded Revenue Malmquist Index}(ERMI_\theta) = REC_\theta \times RREC_\theta \times ERTC_\theta \quad (52)$$

$$\text{Explanded Revenue Malmquist Index}(ERMI_\theta) = PREC_\theta \times SREC_\theta \times RREC_\theta \times ERTC_\theta \quad (53)$$

If $RMI_\theta > 1$ or $ERMI_\theta > 1$, it shows DMU had progress.

If $RMI_\theta < 1$ or $ERMI_\theta < 1$, it shows DMU had regress.

If $RMI_\theta = 1$ or $ERMI_\theta = 1$, it shows DMU had not changing.

We define Revenue Malmquist Index Disparity and Expanded Revenue Malmquist Index Disparity

$$RMID - \frac{RMI - RMI_\theta}{RMI} \times 100 \quad (54)$$

$$ERMID = \frac{ERMI - ERMI_\theta}{ERMI} \times 100 \quad (55)$$

5. Conclusion

Considering the variation of relative importance and incorporation them as multipliers in the models shows that, the results for real data have superiority to the other models, the reason is that the cost of inputs and outputs in some data that can be cast in money very with inflation, should be consider seriously, and this should be taking into account in evaluating the Revenue Malmquist index, in different period of results shows in fact . The main reason using FDH Models in DEA for computing Revenue Malmquist Index is that, many of natural agents are not convex.

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