Assessment of Convergent-Divergent Fins Performance In Natural Convection
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Abstract: Convergent-divergent fins are suggested to be used as a heat sink because of their higher surface area and the ability of air natural draught due to their shape. The performance of convergent-divergent fins is compared with those of other types of fins. To carry out this comparison, natural convection heat transfer in air from different type surface is investigated experimentally with consideration of the effects of radiant heat transfer. Plate-fins (Parallelogram fins), cylindrical solid/hollow pin fins and convergent-divergent fins are tested. From now, the plate-fins will be termed as straight fins to distinguish from plain plate which is the array base plate. The solid/hollow pin fins and convergent-divergent fins are arranged in staggered and inline arrangements. The experiments have been performed for different values of heat flux. The results show that, the solid pin fins in an inline arrangement increases the rate of heat transfer considerably when compared to the straight and convergent-divergent fins at $Ra > 2 \times 10^7$ while straight fins increase the rate of heat transfer at $Ra < 2 \times 10^7$. The solid pin fins enhance more the average temperature compared to the other fin types at heat fluxes higher than 800 W/m$^2$. The comparison shows that among the three cylindrical fins, the solid pin fins have the highest heat transfer performance, hollow pin fins have the lowest, while the convergent-divergent fins locate somewhere in between. The heat transfer performance for heat sinks with an array of inline fins was better than that of a staggered arrangement.

Keywords: Natural convection, Pin fins, Convergent-divergent fins, Surface augmentation, Experimental heat transfer

Nomenclature

- $A_{TOT}$: total area of the base plate and fins, m$^2$
- $F(\varepsilon)$: emissivity function
- $F_{1-2}$: view factor
- $g$: gravity, m/s$^2$
- $h$: convective heat transfer coefficient, W/m$^2$K
- $k$: thermal conductivity, W/mK
- $L$: base plate length, m
- $Nu$: Nusselt number
- $Pr$: Prandtl number
- $Q_{CONV}$: convection heat transfer, W
- $Q_{RAD}$: radiation heat transfer, W
- $Q_{TOT}$: electrical power of the heaters, W
- $Ra$: Rayleigh number
- $T_\infty$: ambient temperature, K
- $T_b$: base plate temperature, K
- $T_f$: film temperature, K

Greek symbols

- $\beta$: coefficient of thermal expansion, K$^{-1}$
- $\varepsilon$: emissivity
- $\rho$: density, kg/m$^3$
- $\nu$: kinematic viscosity, m$^2$/s
- $\sigma$: Stefan-Boltzmann constant (=5.67 × 10$^{-8}$ W/m$^2$K$^4$)

1. Introduction

Fins are commonly employed to augment the rate of heat transfer from the primary surface to the surrounding air by increasing the surface area. Different types of fins are used in accordance with the shape of the primary surface in heat exchange applications. Generally, for the primary flat surface, longitudinal or pin fins are used, and for the closed envelop primary surface, annular fins are considered. In the evaporator of refrigeration and air conditioning systems, refrigerant flows through circular pipes where the annular fins are the common choice to attach with its outer periphery to enhance the rate of heat transfer from the surrounding air to the cold fluid.

Heat transfer can be improved by increasing the heat transfer surfaces, increasing the temperature difference between the two media of interest or increasing the coefficients of heat transfer. In many applications, increasing the temperature differences or increasing the heat transfer coefficients have many restrictions; therefore, increasing the heat transfer surface areas becomes an applicable solution in order to improve the heat transfer and subsequently it is one of the methods on which most research is conducted.

The performance of a heat sink in natural convection mode depends on the orientation of the fins relative to the gravitational direction; Karagiozis et al. [1] studied natural convection heat transfer from arrays of isothermal triangular fins. They found that for fins running horizontally on a vertical base plate, heat transfer is significantly lower than when the fins are vertical. Huang et al. [2] studied the orientation effects on natural convection performance of square
pin fin heat sinks. A flat plate and seven square pin fin heat sinks with various arrangements were tested under a controlled environment. The results show that the optimal heat sink porosity is around 83% for the upward arrangement and is around 91% for the sideward arrangement. Heat transfer coefficients for upward and sideward facing orientations are of comparable magnitude while downward facing orientation yields the lowest heat transfer coefficients. The addition of surface area is comparatively more effective for the downward arrangement whereas it is less effective for the sideward arrangement.

Natural convection heat transfer in air from a pin-finned surface investigated experimentally by Sertkaya et al. [3] and considering the effect of radiation heat transfer. The plate oriented as the pin arrays facing either downwards or upwards from the vertical axis with different angles and the experiments performed for different values of heater power input. The results show that the pin fins increase the heat transfer considerably when compared to the unpinned surface. The up facing pins more enhance heat transfer than the down facing pins and the enhancement is decreasing with increasing orientation angle from the vertical axis.

There have been investigations on various aspects of natural convection heat transfer from pin-finned surfaces. Alhara et al. [4] studied free convective/radiative heat transfer from pin fin arrays with a vertical base plate. An empirical correlation for predicting the heat transfer performance of round pin fin arrays was established. Yu and Joshi [5] studied combined natural convection, conduction, and radiation heat transfer of a pin-fin heat sink in a confined space through both experimental measurement and numerical simulation; they concluded that the confined space plays an important role in the heat transfer from the heat sink and thermal radiation contributes significantly to the heat transfer.

In an investigation of pin-fin density, Azar and Mandrone [6] investigated the effect of pin-fin density on thermal performance of shrouded pin-fin heat sinks. Also, Chu et al. [7] studied the effect of fin density with pins having circular, elliptic and square cross sections. They found an optimal number of pin fins beyond which thermal resistance actually increased. They also found that thermal resistance was a function of the approach velocity and the governing flow pattern.

Besides experiments, a number of numerical studies have been conducted to simulate the natural convection heat transfer from fin array heat sinks. Rao and Venkateshan [8] conducted experiments on horizontal fin arrays. They studied the effects of fin height, fin spacing, fin array base temperature and fin emissivity on heat transfer rates. They showed that much larger heat fluxes are in short fins than long fins. Further, the convective heat transfer increased linearly with fin spacing, while the increase in radiation heat transfer showed a non-linear trend. Kobus and Oshio [9] conducted experiments for the influence of radiation on the thermal performance of fin array heat sinks. They also developed a theoretical model to predict the magnitude of this influence on effective thermal resistance.

Rao et al. [10] analyzed heat transfer from a horizontal fin array by natural convection and radiation. They performed experimental and numerical parametric studies on the effects of fin spacing, number of fins, fin height and fin base temperature on the total heat transfer from the fin array. Numerical results were obtained to study the effectiveness for different values of fin heights, emissivities, number of fins in a fixed base, fin base temperature and fin spacing. These results were subjected to non-linear regression and equations were obtained for heat fluxes from the two-fin enclosure and single fin as functions of Rayleigh number, aspect ratio and fin emissivity. In addition, regression equations were obtained to readily calculate the average Nusselt number, heat transfer rate and effectiveness for a fin array. Elshafei [11] experimentally investigated the heat transfer characteristics of round hollow/perforated pin fin heat sinks under natural convection. He also compared the results to those of round solid pin fin heat sinks by considering the orientation effect. The heat transfer performance for heat sinks with hollow/perforated pin fins was better than that of solid pins. The temperature difference between the base plate and surrounding air of these heat sinks was less than that of solid pin one and improved with increasing inside-outside diameter ratio.

The CFD (Computational Fluid Dynamics) software is used as a useful tool in determination of the performance of the convective heat transfer sets. Dvinsky et al. [12] studied numerically the flow and heat transfer behavior for in-line and staggered heat sinks for the approach velocities of 1, 3 and 5 m/s. They found that the in-line design was thermally superior to the staggered design for all the fully shrouded heat sinks. They also found that in a given geometry the non-dimensional pressure drop over a heat sink was almost constant which indicates small viscous drag. Yu et al. [13] compared plate-fin radial heat sinks by modeling natural convection and proposed a correlation to predict the Nusselt number. They found that geometries with long and middle (LM) fins had the best cooling performance. Yu et al. [14] compared numerically analyzing the plate-fin heat sink and a pin-fin heat sink base on cooling
performance and heat sink mass. The results show that impossible to optimize both thermal performance and heat sink mass at the same time. In addition, optimum values of the geometric parameters maximizing heat transfer performance, i.e., minimizing thermal resistance were shown to exist.

Some investigations of the optimum design parameters and the selection of heat sink module have been proposed in order to offer a high-performance heat removal characteristic. Sahray et al. [15] studied optimization of horizontal-base pin-fin heat sinks in natural convection with radiation. The effects of fin height and fin population density were studied experimentally and numerically. They suggested a correlation between the Nusselt number and the Rayleigh number by decoupling convection from radiation. The results show that heat-transfer enhancement due to the fins is not monotonic. The differences between sparsely and densely populated sinks were assessed quantitatively and analyzed for various fin heights. Jang et al. [16] optimized the cooling performance and mass of a pin-fin radial heat sink. Both natural convection and radiation heat transfer were considered in a numerical model and experiments were performed for the validation of their model. The effects of the geometric parameters on cooling performance and system mass were also investigated. They found that the radiation heat transfer improved the thermal resistance by 21.9% compared to cases that only considered natural convection. The pin-fin array showed uniform cooling performance because the middle-fin length increased by 53% on average compared to that of the LM plate-fin array. The mass of the pin-fin radial heat sink was 35% less than that of the LM plate-fin array for a given cooling performance.

Kim [17] studied thermal performance of a vertical plate-fin heat sink under natural convection and optimized for the case in which the fin thickness varied in the direction normal to the fluid flow. The results show that, in the case of an air-cooled heat sink and the fin thickness allowed increasing in the direction normal to the fluid flow the thermal resistance decreases by up to 10%. In addition, the difference between the thermal resistances of heat sinks with uniform thickness and the heat sinks with variable thickness decreases as the height decreases and as the heat flux decreases.

However, no significant study was found in the existing literature that deals with heat transfer with convergent-divergent cylindrical fins. Wang et al. [18] presented a study of flow and heat transfer characteristics in ribbed convergent and divergent square ducts. The heat transfer performance of the divergent/convergent ducts compared with the ribbed straight duct fewer than three constraints; identical mass flow rate, identical pumping power and identical pressure drop. The results show that the heat transfer performance was the highest when using a divergent duct, the lowest is when the convergent duct is used, while the straight duct locates somewhere between.

The above literature indicates that many studies have been carried out for different types of pin arrays, but still there is lack of knowledge of the natural convection heat transfer from a surface with convergent-divergent cylindrical fins with staggered or inline arrangements. The present work deals with the heat transfer performance by natural convection in air from different types of heat sink surfaces which are investigated by considering the radiation heat transfer from these surfaces. The different types of heat sinks are straight, solid/hollow pin in array staggered and inline and convergent-divergent fins in array staggered and inline. The effects of input heat flux and Rayleigh number variation on the heat sink performance are investigated for different fins. The performance comparison of different fin types is presented.

2. Experimental set-up

The experimental test bed is illustrated schematically in Fig.1 and the examined fins are simply illustrated in Fig. 2. The experimental test rig is mainly consisted of an aluminum base plate (the heat sink), electrical power source (heat source), temperature measuring system and a data logger besides to the different types of target fins. The electric heating element has dimensions of 200×200 mm (as the heat sink plate) is attached to the heat sink plate and fixed at the bottom of this plate. The power supplied to the heater was controlled by a variable regulator to obtain constant heat flux along the base plate, and was measured by inline multi-meter (an ammeter and voltmeter). Subsequently, the supplied power to the heater can be determined.

Surface temperature of the base plate was measured by nine pre-calibrated mineral insulated T-type thermocouples located at different positions on the plate. The ambient air temperature was also measured by mineral insulated T-type thermocouple. The heat sink is formed as a square (200×200 mm and 10 mm thick) aluminum plate with thermal conductivity of 237 W/m K. Heat sink where lay down on a wooden box filled in side with an insulation of 60 mm thick glass wool blankets (0.04 W/m K). The whole assembly base plate, heater with associated thermal insulation, was located in a well-fitting open-topped wooden box of 25 mm thick as shown in Fig 1a. Through out this study, two arrangements have been considered for cylindrical fins, inline arrangement (Fig. 1b) and staggered arrangement (Fig. 1c) where straight fins are located parallel in a slotted base plate. The type, dimensions
and number of all considered fins are listed in table (1), where simple drawings of these fins are illustrated in fig. 2 to show their dimensions. The plane straight fins have been used to compare their performance with those of cylindrical fins and their dimensions, as shown in Fig. 2a (200×40×2 mm), have been selected to have a convective area approximately equal to that of solid pin fins and to have a mass approximately equal to that of hollow pin fins.

The base plates and the fins are fabricated from an aluminum alloy (ρ=2770 kg/m³) and all the surfaces, base plates and fins, are black anodized to increase their emissivity. As seen in Figs. 2c and 2d, two small windows (2×3 mm) were perforated at the lower section of both of hollow pin fins and con.-div. fins to allow the air to flow axially through the fins. The room in which the experiments were performed was isolated from the external disturbances and the power supply and measuring devices were located in a neighboring room.

3. Experimental procedures

After the preparation processes are carried out, the heat sink is ready for operation. To attain safe operation for the heat sink, the operation temperature must not exceed more than 120°C in order to avoid any complications or burning out of the device as a result of excessive heat. Each test run took nearly 2.5 hours to reach equilibrium when the power is turned on. The power source has a capacity range of 10-140 W.
Fig. 2 Dimensions of tested fins
a) Straight fin, b) Solid pin fin,
b) Hollow pin fin, c) Convergent-divergent fin

Table (1) Type, dimensions and number of examined fins

<table>
<thead>
<tr>
<th>Type</th>
<th>Dimensions, mm</th>
<th>No. of fins</th>
<th>convection area, m²</th>
<th>Conduction area*, m²</th>
<th>Mass, kg</th>
<th>A_{cond} / A_{plate}, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight fin</td>
<td>W_f=200, r=2, H_f=40</td>
<td>10 inline</td>
<td>0.162</td>
<td>4.00×10⁻³</td>
<td>0.443</td>
<td>10.0</td>
</tr>
<tr>
<td>Solid pin fin</td>
<td>D_o=10, H_f=60</td>
<td>100 inline</td>
<td>0.188</td>
<td>7.85×10⁻³</td>
<td>1.305</td>
<td>19.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>95 staggered</td>
<td>0.179</td>
<td>7.46×10⁻³</td>
<td>1.238</td>
<td>18.64</td>
</tr>
<tr>
<td>Hollow pin fin</td>
<td>D_o=10, D_i=7.5, H_f=60</td>
<td>100 inline</td>
<td>0.329</td>
<td>3.44×10⁻³</td>
<td>0.570</td>
<td>8.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>95 staggered</td>
<td>0.313</td>
<td>3.27×10⁻³</td>
<td>0.543</td>
<td>8.17</td>
</tr>
<tr>
<td>Convergent – divergent fin</td>
<td>D_o=10, D_i=5, H_f=60</td>
<td>100 inline</td>
<td>0.276</td>
<td>3.44×10⁻³</td>
<td>0.474</td>
<td>8.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>95 staggered</td>
<td>0.262</td>
<td>3.27×10⁻³</td>
<td>0.448</td>
<td>8.17</td>
</tr>
</tbody>
</table>

* The conduction area represents the contact area between the fins and base plate

4. Data reduction

For comparison, the experiments were done for straight, solid/hollow pin and convergent-
divergent fins in array staggered and inline. In all experiments, the measured values were the input power and temperatures of the base plate and the surrounding air. For better comparison, the heat transfer characteristics are defined for base plate surface temperature and dimensions for different type fin surfaces. The heat transfer coefficient was calculated from the convection part of the total heat transfer from the surfaces and the temperature difference between the base plate surface mean temperatures and the ambient temperatures, Sertkaya et al. [3] and Sparrow and Vemuri [19];

\[ h = \frac{Q_{\text{CONF}}}{A_{\text{TOT}}(T_b - T_\infty)} \]  

(1)

Where \( Q_{\text{CONF}} \) is the convection part of the total heat transfer from the surfaces, \( A_{\text{TOT}} \) is the total surface area of the base plate and fins, and \( T_b \) and \( T_\infty \) are the base plate surface and ambient mean temperature, respectively. The Nusselt number is defined as;

\[ Nu = \frac{hL}{k} \]  

(2)

Where \( L \) is the length of the base plate equal 200 mm and \( k \) is the thermal conductivity of the fluid. The independent variable of the problem is the temperature difference \( (T_b - T_\infty) \), it is represented via the Rayleigh number as;

\[ Ra = \frac{g\beta(T_b - T_\infty)L^2 \text{ Pr}}{\nu^2} \]  

(3)

All the thermo-physical properties of air are evaluated at the film temperature, which is defined as Sparrow and Vemuri [20];

\[ T_f = \frac{T_b + T_\infty}{2} \]  

(4)

Here, the coefficient of thermal expansion can be found as:

\[ \beta = \frac{1}{T_f} \]  

(5)

Assuming that the conduction heat loss is negligible through the insulated sides of the base
plates, the convection part of the total heat transfer from the surfaces may be written as:

\[ Q_{\text{CONV}} = Q_{\text{TOT}} - Q_{\text{RAD}} \]  

(6)

Where: \( Q_{\text{TOT}} \) is equal to the electrical power of the heaters. Radiation heat transfer from a surface located in an ambient of temperature \( T_\infty \) can be calculated from:

\[ Q_{\text{RAD}} = F_{1-2} \sigma A_{\text{TOT}} (T_b^4 - T_\infty^4) \]  

(7)

Where; and \( F_{1-2} \) is the view factor equal about 0.5133. The percentage enhancement gain of average temperature for different type of fins can be calculated as:

\[ \text{Enhancement } = 100\% \left( T_{i} - T_{j} \right) / T_{i} \]  

(8)

Where: \( i \) refers to plain plate (without fins) or straight fin and \( j \) refers to the fin type in interest.

5. Uncertainty analysis

To estimate the uncertainties in the results presented in this work, the approach described by Barford [21] was applied. The uncertainty in the measurements is defined as the root sum square of the fixed errors of the instrumentation and the random errors observed during different measurements. Since \( Nu = f(Q_{\text{CONV}}, L, A, T_b, T_\infty) \), the measured uncertainties summed into the Nusselt number are from the heater power supply, dimensions of the base plates, base plate surface temperature and the ambient air temperature. The measured dimensions were accurate to ±0.1 mm and the temperatures were measured using calibrated T-type thermocouples with a resolution of ±0.1 K. The power supplied was measured by an accuracy of ±0.1W. An additional 5% uncertainty was estimated together for the view factor and the emissivity function in radiation heat transfer analysis. The highest uncertainty calculated is 8% for the lowest input power and this value is rather smaller for increased input power.

6. Results and Discussion

In the present study, the effect of the heat flux and Rayleigh number variation on the heat sink performance is investigated for different fins in interest. Throughout this study, the heat flux varied from 250 to 3250 W/m². Fig. 3 shows the variation of the average temperature for both the inline and the staggered arrangements of cylindrical fins (solid/hollow pin fins and convergent-divergent fins) as well as the straight fins. For the inline arrangement, as seen in Fig. 3a, the best performance is due to the solid pin fins where the worst one is due to the straight fins. In the case of staggered arrangement, as seen in Fig. 3b, the best performance is due to straight fins where the worst one is due to the hollow pin fins. For both of inline and staggered arrangements and at low heat fluxes, the fin type has a very slight effect on average temperature of the heat sink where at high heat fluxes the fin type plays a pronounced effect on the average temperature. The enhancement in average temperature is more pronounced for the inline array than for the staggered array.

Figure 4 shows the percentage enhancement in the fin performance of all tested fin types for both of the inline and the staggered arrangements. This enhancement based on the plain plate (without fins) performance and calculated by the aid of Eqn. 8. From Fig. 4a, it is observed that the solid pin fin has the highest performance for heat flux > 750 W/m² where the hollow pin fin has the lowest one for heat flux < 1750 W/m² and after this value; the straight fin has the lowest one. From Fig. 4b, it is clear that the straight fin has the highest performance where the hollow pin fin has the lowest one and this is valid for all values of heat flux. In both cases of inline and staggered arrangements, the convergent-divergent fins are taking place between the solid and hollow pin fins. The highest performance for all fin types occurred at a heat flux of value of 1500W/m² for both the inline and staggered arrangements. After this value, the enhancement of all fins comes to decrease due to the limitation of the thermal capacity of the heat sinks of all tested fins and due to the effect of the thermal boundary layers that are playing a pronounced effect on this enhancement at high heat fluxes. Instead of the less convective area and less mass of convergent-divergent fins compared with those that of pin hollow fins (see Table 1), the performance of convergent-divergent fins is higher than that of pin hollow fins. This means that the air draft due to convergent-divergent form is better than that due to pin hollow form.
Fig. 3 The average surface temperature for different tested fins;
(a) Inline arrangement (b) Staggered arrangement

Fig. 4 The percentage enhancement of the performance of tested fins based on the plain plate performance;
(a) Inline arrangement (b) Staggered arrangement
Based on the straight fin performance, Fig. 5 shows the percentage enhancement in the fin performance of the cylindrical types. From Fig. 5a for the inline arrangement, it is clear that the solid pin fin has the highest performance for heat flux > 750 W/m² where for a heat flux < 750 W/m², the straight fin has the highest one. In the case of staggered arrangement as shown in Fig. 5b, it is observed that the straight fin has the highest performance for all values of heat fluxes. That is due to the symmetric hydraulic boundary layers in the case of the straight fins. For both cases of inline and staggered arrangements, the performance of convergent divergent fins lies between the solid and hollow pin fins.

To study the heat transfer characteristics, Fig. 6 has been illustrated. This figure presents the variation of the average Nusselt number against the Rayleigh number for all examined fins. From Fig. 6a it can be seen that the enhancement in the average Nusselt number for solid pin fins is approximately 130% when the Rayleigh number is increased from 3.7 \times 10^6 to 25.75 \times 10^6. This higher enhancement is due to high convection area as shown in table 1 and a small temperature difference as...
shown in Fig. 3a. For both types of arrangement, as shown in Figs 5a and 5b, it is observed that the average Nusselt number increases with increasing Rayleigh number. The highest heat transfer is recorded when using straight fins for Rayleigh number less than $20 \times 10^6$ while, for Rayleigh number higher than $20 \times 10^6$, the highest heat transfer is recorded when using solid pin fins.

7. Conclusions

In the present study, natural convection heat transfer in air from different types of surfaces is investigated experimentally with consideration of the effect of radiation heat transfer. The effects of heat fluxes on the fin performance for straight, solid/hollow pin and convergent-divergent fins in an array are analyzed. Based on the obtained results, the following conclusions can be drawn:

- The fin type plays a pronounced effect on the system performance at high heat flux rates where this effect is very slight at low heat flux rates. Therefore, the selection of the fin type is very important for high heat flux systems.
- The system performance with convergent-divergent fins is less than that with solid pin fins and higher than that with hollow pin fins.
- The heat transfer performance for inline heat sink fin array was better than that of staggered array.
- The heat transfer performance due to the straight fins is higher than that due to the cylindrical fins regardless of the array arrangement.
- For cylindrical fins, the effect of Rayleigh number on the fin performance in the case of inline array is greater than that in the case of staggered array.
- Due to the less volume of the convergent-divergent fins (subsequently less weight) compared with pin fins (solid/hollow), the convergent-divergent fins could be more suitable for compact heat sinks especially at low heat fluxes.

References


