Localization and behavior defect modes in 1D photonic crystal with Right and Left handed material

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Abstract: An analytical was direct matching procedure within the Kronig-Penney model by the use of matrix method was applied to analyze the dispersion behavior of the localized defect layer sandwiched within two symmetric semi-infinite one-dimensional (1D) photonic crystals (PCs). In this paper, defect modes of the right and left handed material in 1D photonic crystal were studied by the use of the transfer matrix method. It was shown that the dispersion defect mode for the right handed material was positive while for the left handed material these modes could be negative or nearly zero. In this work, the influence the stack sequences of the two semi-infinite background PCs relative to the defect layer and dependence of the defect modes on physical parameters of the defect layer and field profile defect modes in right and left handed material was also reported.

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1. Introduction

Photonic crystals(PCs) have attracted ever increasing attention in the last decade due to their unique proper and possible applications in future generation of optical and photonic devices like light emitting diodes ,delay lines waveguides and lasers [1,2]. Photonic crystals(PCs), artificial dielectric materials with periodic modulation of refractive index, The propagation behavior of electromagnetic (EM) waves and potential application in ultra small all-optical integrated circuits[3,4]. One-dimensional (1D) PCs with defect layers was used for filters, including single channel filters [5] and multiple channel filters. The defect modes lead to the selective transmission in the 1DPCs and they can be utilized as TE/TM filters and splitters. Therefore, the study of the properties of defect modes in PCs is one of the most attractive subject's since photons are localized [6].

Just as the introduction of defect layers in semiconductor super lattices may result in the electron defect states in the band gaps, with the introduction of a layer different in nature (materials and size) into a PC structure, it is possible to create highly localized defect modes within the photonic band gap. The control of defect modes is of major interest for their application in narrow band filters. However, the design of controllable defect modes in PCs requires predictive formulae for the dependence of the defect mode frequencies on physical parameters of PCs and on the angle of incident light [7-9].

Left handed materials (LHM's), in which the dielectric permittivity and magnetic permeability are simultaneously negative, have received a great

deal of attention during the last few years [10]. This is due to the unusual physical properties of these materials that have raised strong theoretical interest and may lead to potential applications in optical devices. Some peculiar properties of LHM have already been discussed some thirty years ago by Veselago [11], for instance, a Poynting vector directed opposite to the propagation wave vector k. The realization of such media [12, 13] is based on the propositions of Pendry et al. for specific structures [14]. In these calculations, the dielectric permittivity ε and magnetic permeability μ are, in general, assumed to take constant values. Although these parameters in LHM are in general frequency dependent, our results can be used to design specific metamaterials that would lead to a typical behavior around a given frequency[10].

We employ the conventional transfermatrix method (TMM) to calculate the defect modes the transmission spectra in the view point of multiple scattering of Bragg waves from various composite layers and unit cells. The appearance of sharp transmission peaks within the photonic band gaps (PBG) should correspond to the existence of defect states. A conceptually different model, the Blochmode approach has been developed to investigate optical properties of one-dimensional (1D) PC with a single defect [17]. Up to now, to the best of our knowledge, these are few works on the obtaining of analytical dispersion relation of defect modes in 1D symmetric PCs. Properties of defect modes are mainly determined by the geometrical and physical parameters of the defect layer, such as the refractive index and with. In addition to these parameters, the structure of background PCs specially, the neighbor

layers of defect layer play an important role on the defect modes behavior. In this communication, we will introduce a method to derive a dispersion relation for a single- defect sandwiched between two arbitrary semi-infinite 1DPCs [18-20].

In this paper, we have theoretically investigated localized defect in 1D photonic crystal modes with the Right and Left handed media. This paper is arranged as follows. In Section 2, we theoretically applied the direct matching procedure for the solution of localized surface optical modes in 1D PCs. We have followed a similar procedure used by Steslica et al. [10] in study of the localized electronic states in semiconductors super lattices. In Section 3, the behavior of defect layer on the optical modes of the defect modes have been investigated. Finally, Section 4 concludes with a brief comment.

2. The basic equations

In order to obtain the defect modes of dispersion relation 1D PCs, the direct matching procedure within the Kronig–Penney model has been used [10]. We consider a 1D PCs periodic dielectric multilayer structure with a defect layer shown in Fig.1. Suppose each unit cell is composed of two layers, which are stacked along the z -axis direction with d_i , ε_i and μ_i are the thickness, the dielectric constant and the magnetic permeability of the *i* th layer, respectively ε_d , d_d are the physical parameters of defect layer. Where $d = d_1 + d_2$ is the period of the structure.



Figure 1. Schematic picture of a defect layer sandwiched between two Symmetric semi-infinite 1D photonic crystals

The defect layer with width d_d and dielectric constant ε_d are sandwiched between two symmetric semi-infinite1D PCs. We assumed that the EM wave propagates in the xz plane. Due to translational invariance in the xoy plane, the parallel wave vector k_{\parallel} is a conservative quantity in all domains of the crystal. For the *TE* polarization, the electric field is parallel to interfaces referred to as x direction in this paper.

$$E = e^{ik\|y}\varphi(z)\hat{x} \tag{1}$$

Solving Maxwell's equations for photonic crystals leads to the following differential equation. The

dispersion relation of the PC has the well known form:

$$\frac{d^2\varphi(z)}{dz^2} + (\varepsilon\mu\frac{\omega^2}{c^2} - k_{\parallel}^2)\varphi(z) = 0$$
(2)

Therefore, φ the first and second layers respectively are written as follows.

$$\varphi_{1}(z) = C_{1} \sin(k_{1}z) + D_{1} \cos(k_{1}z) \qquad (3)
\varphi_{2}(z) = C_{2} \sin(k_{2}(z-d_{1})) + D_{2} \cos(k_{2}(z-d_{1})) \qquad (4)$$

Where $c_{1,2}$, $D_{1,2}$ Coefficients are constant and are obtained by applying the boundary conditions at the interface layers. Where the perpendicular wave vector component in each medium is given by $k_i = \sqrt{\varepsilon_i \mu_i k_0^2 - k_{\parallel}^2}$, (i = 1,2), $k_{\parallel}^2 = k_x^2 + k_y^2$ and ω is the angular frequency, and c is the light speed in vacuum. To obtain photonic band structure for the *TE* polarization of the following four boundary conditions we use the above relations i.e. continuity of the tangential components of the

electric
$$\varphi(z)$$
 and magnetic fields $\frac{1}{\mu} \frac{d\varphi(z)}{dz}$

$$\begin{cases} \varphi_{1}(z = d_{1}) = \varphi_{2}(z = d_{1}) \\ \frac{1}{\mu_{1}} \frac{d\varphi_{A}(z)}{dz} \Big|_{z=d_{1}} = \frac{1}{\mu_{2}} \frac{d\varphi_{2}(z)}{dz} \Big|_{z=d_{1}} \\ \varphi_{1}(z = 0) = e^{-ikd} \varphi_{2}(z = d) \\ \frac{1}{\mu_{1}} \frac{d\varphi_{1}(z)}{dz} \Big|_{z=0} = e^{-ikd} \frac{1}{\mu_{2}} \frac{d\varphi_{2}(z)}{dz} \Big|_{z=d} \end{cases}$$
(5)

Using these relations, the transfer matrix can be expressed as follows:

$$S = N_1^{-1} M_2 N_2^{-1} M_1 \tag{6}$$

Where matrix element is:

$$\begin{split} M_{i} &= \begin{bmatrix} e^{ik_{i}d_{1}} & e^{-ik_{i}d_{1}} \\ \frac{ik_{i}}{\varepsilon_{1}} e^{ik_{i}d_{1}} & -\frac{ik_{i}}{\varepsilon_{1}} e^{-ik_{i}d_{1}} \end{bmatrix}, \\ N_{i} &= \begin{bmatrix} 1 & 1 \\ \frac{ik_{i}}{\varepsilon_{i}} & -\frac{ik_{i}}{\varepsilon_{i}} \end{bmatrix}. \end{split}$$

For the *TE* polarization, dispersion relations for perfect photonic crystal can be obtained as follows.

$$\cos(kd) = \cos(k_1d_1)\cos(k_2d_2) - \frac{1}{2}\left(\kappa + \frac{1}{\kappa}\right)\sin(k_1d_1)\sin(k_2d_2) = D(\omega)$$
(7)

That is $(\kappa = \frac{k_1 \mu_2}{k_2 \mu_1})$. It is well known that

when any periodic system is limited, k should be complex [10].

$$k = i\mu + \frac{n\pi}{d}, \mu \rangle 0, n = 0, \pm 1, \pm 2, \dots$$
 (8)

To obtain the defect modes are bounded with the first layer. The explicit form of the electric field in the first layer, $0 < z < d_1$, is rewritten as follows[10].

$$\varphi_{pc}(z) = C_1[\sin(k_1 z) + \lambda_1 \cos(k_1 z)] \tag{9}$$

Here λ_1 is given by

$$\lambda_{1} = \frac{\sin(k_{1}d_{1}) + \kappa e^{ikd}\sin(k_{2}d_{2})}{e^{ikd}\cos(k_{2}d_{2}) - \cos(k_{1}d_{1})}$$
(10)

Since defect modes are localized state. In order to preserve the finite nature of the electric field in defect layer $(-d_{def} < z < 0)$, it has to be taken as

$$\varphi_d(z) = A_1 \sinh(k_d z) + A_2 \cosh(k_d z) \quad (11)$$

Where $k_d = \sqrt{k_{\parallel}^2 - \varepsilon_d \mu_d k_0^2}$, by applying

the boundary conditions at the interface defect layer with the first layer of photonic crystal using a symmetric condition, we can write the following relations.

$$\begin{cases} \varphi_{pc}(z=0^{+}) = \varphi_{d}(z=0^{-}) & (12) \\ \frac{1}{\mu_{pc}} \frac{d\varphi_{pc}(z=0^{+})}{dz} = \frac{1}{\mu_{d}} \frac{d\varphi_{d}(z=0^{-})}{dz} \\ \varphi_{d}(-\frac{d_{d}}{2}) = 0, \frac{d\varphi(-\frac{d_{d}}{2})}{dz} = 0 \end{cases}$$

Defect modes of symmetric case are eigenvalues of parity and the symmetry property of the structure implies. Imposing Eq. (12) yields the following relations:

$$\frac{\lambda_1}{G} \tanh \left(\frac{k_d d_d}{2} \right) = 1$$
 (13)

$$\frac{\lambda_1}{G} \operatorname{coth} \left(\frac{k_d d_d}{2} \right) = 1$$
 (14)

Where
$$G = \frac{k_1 \mu_d}{k_d \mu_1}$$
, for odd and even states,

respectively. These equations are the basic relations, including the frequency of defect modes and parameter μ . By eliminating μ between (7) and (13, 14) and using (8), we get the following relation

$$B(\omega) = D(\omega) \mp [D^{2}(\omega) - 1]^{1/2}$$

$$B(\omega) = (-1)^{n} e^{-\mu d}$$
(15)

Where – and + correspond to n even and n odd, respectively [15]. By doing this procedure, some spurious solutions are introduced. This can be identified and should be rejected by applying Existence condition, $e^{-\mu d} \langle 1$ [16]. Considering that the modes of defect layer, within the defect layer Localized and the in layers photonic crystal is decay oscillatory.

Then, using the obtained relations, the field profile in each layer obtains, and then use the theorem of Bloch Eq. (16) can be related to other periods [16].

$$\varphi(z+d) = e^{ikd}\varphi(z) \tag{16}$$

3. Results and discussion

In this section, we discuss the numerical results obtained for the defect modes of a semiinfinite 1D PC containing defect layers of right and left handed material. Calculated band diagrams of the background PCs and the defect modes supported by a two-constituent symmetric structure defect TEpolarizations arrayed in the for ... $| \varepsilon_2 | \varepsilon_1 | \varepsilon_d | \varepsilon_1 | \varepsilon_2 |$... sequence. The physical in constants this calculation are $\mathcal{E}_1 = 12$, $\mathcal{E}_2 = 1$, $\mu_1 = \mu_2 = 1$ and $d_1 = d_2 = 0.5$. Fig.2 shows TE projected band structure and defect modes for super lattice structure which was displayed in Fig.1. The curves give $\frac{\alpha d}{2\pi c}$ as function of $\frac{k_{\parallel}d}{\pi}$. The dark areas represent the extending Bloch states, while the light areas are the PBG regions. The scales for frequency ω and parallel wave vector k_{\parallel} are all normalized to be dimensionless quantities, from ad and $\frac{k_{\parallel}d}{\pi}$. The circled curves in the band gaps denote the defect modes corresponding to different defect layers. When k_{\parallel} is in the small value region, the curves exhibit approximately parabolic variation. With the value of k_{\parallel} becoming larger, the dispersion curve is close to a straight inclined line.

In Fig. (2) is considered, It is well known that the dispersion relations of the defect modes in a conventional (right handed material) 1DPCs are all positive. To show the distinction dispersion and behavior dispersion defect modes dependent defect lavers are composed of left handed material defect modes For TE polarization is plotted in Fig. (3). Comparison of Figs. (2) and (3) that Unlike the right handed materials, behavior dispersion defect modes in left handed material, is quite different. that the second forbidden gap frequency, dispersion with negative slope and the zero-gap defect mode has emerged. Also, with the suitable choice of physical parameters of defect layer, modes dispersion in the second band gap frequency is almost zero. From the symbol curves in Fig.3, we know that the dispersion relations of the defect modes change from positive to negative types with the increase of the k value.



Figure 2. Calculated bulk and defect modes forte polarization in a 1D photonic crystal and the defect modes. The defect layer has the parameters of $\varepsilon_d = 2$, $\mu_d = 1$ and $d_d = 0.8d$ with *d* being the lattice constant for the two-constituent samples.



Although the defect modes have their field and energy strongly localized in the defect layer and can be greatly affected by the defect layer parameters such as the dielectric constant and width, the stack sequences of the two semi-infinite PCs may also have great effects on the properties of the defect modes.

We also investigated the localization of the defect modes. Therefore, for a state allowable obtained within the first band gap from Fig.(2), calculated the behavior of the field inside the defect

layer and layers photonic crystal, behavior field for frequencies allowable in the Figs. (4) and (5) is calculated, behavior field in both figure (4 and 5) is so well-known that field localized inside the defect layer and within the layers photonic crystal is exponentially is decaying. With the increase of k_{\parallel} , Figure 5a, b display the electric field is highly localized. Whereas, defect mode result of Fig. (3.a) in panel (b) has more concentration in the defect layers than panel (a). We can see that the peak of each mode is in t the inter face of defect layer and first layer of super lattice. Behavior field defect modes result Fig. (3.b) display in Fig. (6), for the different values of k_{\parallel} . Thus, from the figure can be seen that the defect mode in k_{\parallel} more is localized



Fig.4. Field behavior of defect modes in defect layer and layers photonic crystal for each point from Fig.



Figure 5. Field behavior of defect modes in defect layer and layers photonic crystal for each point from

Fig. (3). (a)
$$k_{\parallel} = 1.1396 \frac{\pi}{d}$$
 and $\omega = 0.3532 \frac{2\pi c}{d}$. (b)
 $k_{\parallel} = 1.997 \frac{\pi}{d}$ and $\omega = 0.4106 \frac{2\pi c}{d}$

The zero and negative dispersion phenomena may be useful in the design of large incidence angle filters and narrow frequency and sharp angle filters, respectively.



Fig.6. Field behavior of defect modes in defect layer and layers photonic crystal for each point from Fig.

(3). (b)

$$k_{\parallel} = 0.9858 \frac{\pi}{d}$$
 and $\omega = 0.3318 \frac{2\pi c}{d}$ and (b). $k_{\parallel} = 1.777 \frac{\pi}{d}$ and
 $\omega = 0.3994 \frac{2\pi c}{d}$

4. Conclusion

In summary, the direct matching procedure has been applied to obtain the localized defect modes containing a defect layer with a right and left handed materials. by inserting a layer defects in photonic crystals, frequencies permissible frequency within the band gap is created.We demonstrated that the defect modes are strongly influenced by the stack sequences PCs and the physical parameters of the defect Layer. Also, by inserting a defect layer of one-dimensional photonic crystal containing left handed materials can be negative or near-zero dispersion of defect modes can be achieved. The zero and negative dispersion phenomena may be useful in the design of large incidence angle filters and narrow frequency and sharp angle filters, respectively.

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