Determining a well-dispersed subset of non-dominated vectors of multi-objective integer linear programming problem

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Abstract: Using the L_1 −norm and the concept of the non-dominated vector, this paper presents a method to find a well-dispersed subset of non-dominated vectors of a multi-objective integer linear programming (MOILP) problem.

In each iteration of the proposed algorithm only the right hand side of an integer linear programming problem is modified and then this problem is solved. With this approach, the optimal solutions of these single objective programming problems are the non-dominated vectors of the MOILP problem. The number of constraints and variables of these single objective problems are same, i.e. the iterations of the proposed algorithm do not increase the number of constraints and variables of these single objective problems, while the iterations of the previous approaches increase the number of the constraints and variables. Each iteration of the proposed algorithm finds at least one element of the well-dispersed subset of non-dominated vectors. The proposed algorithm is convergent and its applicability is illustrated by using a numerical example.

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1. Introduction

Numerous algorithms have been designed to solve multi-objective integer linear programming (MOILP) problem. Pasternak and Passy (1973) conducted an earlier study on designing solving methods for Multi-objective integer linear programming (MOILP) problem. They used the concept of implicit enumeration to resolve 0-1 bi-criterion linear programs. Bitran (1979) used relaxation techniques to generate non-dominated vectors. Bitran (1979) also reported some computational results. Deckro and Winkfsky (1983) reported computational results in terms of implicit enumeration compared to Bitran's works. Liu et al. (2000) proposed a method using data envelopment analysis (DEA) technique to generate some non-dominated vectors of 0-1 MOLP. Liu et al. (2000) used BCC DEA model (Banker et al. (1984)) to evaluate the generated vectors. The existence of the convexity constraint in the BCC model may eliminate some non-dominated vectors of the 0-1 MOLP problem. Liu's method does not obtain all non-dominated vectors. Jahanshahloo et al. (2004) proposed a method for generating all non-dominated vectors of 0-1 MOLP. Corresponding to non-dominated vectors which obtain in a iteration, their method adds some constraints and variables to 0-1 single objective problem of next iteration.

If the number of objective functions of 0-1 MOLP problem increase then, the constraints and variables which are added to 0-1 single objective problem will be increased, therefore solving problem needs more computational effort. This deficiency is studied in this paper.

Sylva and Crema (2007) propose a method for finding a well-dispersed subset of non-dominated solutions based on maximizing the infinity norm distance from a set of known solutions. They claim that their approach originally provides a variant of the procedure by Sylva and Crema (2004). The major drawback of this approach is the difficulty of solving the constrained problems due to increasing number of constraints and binary variables.

In the current paper, authors develop a one-stage algorithm which determines at least one non-dominated vector in each iteration. The proposed method solves a single objective integer programming linear program in each iteration and iterations do not increase the number of constraints and variables of these single objective problems. The proposed algorithm reduces computational efforts for solving MOILP problem. For large problems, the improvement can be much more significant.

The organization of this paper is as follows. Section 2 presents a background MOILP problem. Section 3 introduces the proposed method for finding a well-dispersed subset of the non-dominated vectors of an MOILP problem. Section 4 illustrates the procedure using a numerical example. Finally, conclusions are presented in Section 5.

2. Background

Multi-objective programming is an important research area as many practical situations require discrete representations by integer variables and many decision makers have to deal with several objectives. The Multi-Objective programming problem with s-objectives is defined as:

$$\begin{aligned} \max \quad & \left(f_1(W), \dots, f_s(W)\right) \\ \text{s.t.} \quad & W \in X \end{aligned} \qquad (1)$$

where $f_1, \dots, f_k$ are the objective functions and X is a feasible region.

Definition 1: $\overline{W} \in X$ is said to be an non-dominated vector of problem (1) if and only if there does not exist a point $W^o \in X$, such that:

$$(f_1(W^0), \dots, f_s(W^0)) \geq (f_1(\overline{W}), \dots, f_s(\overline{W}))$$

and inequality holds strictly for at least one index. If all variables are restricted to be integer and all objective functions and constraints are linear then, problem (1) is called multi-objective integer linear programming.

When all of the constraints and the objective functions are linear the model is as follows:

$$\begin{aligned} \max \quad & \{C_1 W, \cdots, C_s W\} \\ \text{s.t.} \quad & A_i W \leq b_i, \forall i \\ & W \in Z_+^n \end{aligned} \qquad (2)$$

where, $C_r = (c_{1r}, \dots, c_{nr})(r = 1, \dots, s)$, $A_i = (a_{i1}, \dots, a_{in})(i = 1, \dots, m)$, $Z_+^n = \{(z_1, \dots, z_n) | z_j \in \{0,1,2, \dots\}, j = 1, \dots, n\}$ and $W = (w_1, \dots, w_n)^T$. The set X, which is defined as follows:

$$X = \{W \mid A_i W \leq b_i, W \in Z_+^n, i = 1, \dots, m\},$$

is called the set of feasible solutions of problem (2). Let X be bounded. Corresponding to each $W \in X$ the vector Y is defined as follows (Jahanshahloo et al. (2004)):

$$Y = (y_1, \dots, y_s)^T = (C_1 W, \dots, C_s W)^T.$$

Definition 2: It is said that the vector $Y = (y_1 \dots, y_s)^T$ dominates the vector $Y^o = (y_1^o, \dots, y_s^o)^T$ if for each $r(r = 1, \dots, s)$, $y_r \geq y_r^o$ and there is at least one l such that $y_l > y_l^o$.

Definition 3: The set F, which is defined as

$$F = \{Y \mid Y = (C_1 W, \dots, C_s W)^T, A_i W \leq b_i, i = 1, \dots, m, W \in Z_+^n\},$$

is called the values space of the objective functions in problem (2).

Let $g_r = C_r W_r^* (r = 1, \dots, s)$ where W_r^* is the optimal solution of the $r^{th}(r = 1, \dots, s)$ problem from the following problems:

$$\begin{aligned} g_r = \max \; & C_r W \\ \text{s.t.} \; & A_i W \leq b_i, \forall i \\ & W \in Z_+^n \end{aligned} \qquad (3)$$

Definition 4: The vector g, which is defined as

$$g = (g_1, \dots, g_s)^T = (C_1 W_1^*, \dots, C_s W_s^*)^T,$$

is called the ideal vector (2004).

Theorem 1: *For each $W \in X$, the vector $g = (g_1, \dots, g_s)^T$ dominates the vector $Y = (C_1 W, \dots, C_s W)^T \neq g$.*

Proof: The proof is similar to that of Theorem 2.3 in (Jahanshahloo et al. (2004)) and is not repeated here. □

3. Non-dominated vectors of MOILP problem

As noted by Jahanshahloo et al. (2004), to find the non-dominated vectors of problem (2), we can specify $W \in X$ such that $g - Y = (g_1 - C_1 W, \dots, g_s - C_s W)^T$ is minimized. To this end, the following problem may be solved:

$$\begin{aligned} \min \quad & \{g_1 - C_1 W, g_2 - C_2 W, \dots, g_s - C_s W\} \\ \text{s.t.} \quad & A_i W \leq b_i, \; \forall i \\ & W \in Z_+^n. \end{aligned} \qquad (4)$$

As there is no preference between the objective functions of problem (4) the sum of the absolute value of deviations (that is $\sum_{r=1}^{s} | g_r - C_r W |$) is minimized. Since for each $W \in X, g_r \geq C_r W$ $(r = 1, \dots, s)$, hence:

$$\min_{W \in X} \sum_{r=1}^{s} |g_r - C_r W| = \min_{W \in X} \sum_{r=1}^{s} (g_r - C_r W)$$
$$= \sum_{r=1}^{s} g_r + \min_{W \in X} \sum_{r=1}^{s} (-C_r W)$$
$$= \sum_{r=1}^{s} g_r - \max_{W \in X} \sum_{r=1}^{s} C_r W.$$

Therefore, problem (4) is converted to the following integer linear programming problem:

$$\begin{aligned} \theta_0^* = \max \quad & \sum_{r=1}^{s} C_r W \\ \text{s.t.} \quad & A_i W \leq b_i \forall i \\ & W \in Z_+^n. \end{aligned} \qquad (5)$$

Theorem 2: *Each optimal solution of problem (5) is a non-dominated vector for problem (2).*

Proof: The proof is similar to that of Theorem 1.3 in (Jahanshahloo et al. (2004)) and is not repeated here. □

Let $E_0 = \{W_1^*, \dots, W_k^*\}$ be the set of the optimal solutions of problem (5) and $I_0 = \{1, \dots, k\}$. As can be seen, if $X \neq \phi$, then $E_0 \neq \phi$. Suppose that $Y_j = (y_1^1, \dots, y_1^s) = (C_1 W_j^*, \dots, C_s W_j^*)^T, j \in I_0$. We consider the following set.

$$\overline{Y}_0 = \{y | y \leq \sum_{j \in I_0} \lambda_j y_j, \sum_{j \in I_0} \lambda_j = 1, \lambda_j \in \{0,1\}, j \in I_0\}.$$

It can be seen, for $\overline{X} = X - E_0$ we have,

$$\min_{W \in \overline{X}} \sum_{r=1}^{s} |g_r - C_r W| > \sum_{r=1}^{s} g_r + \theta_0^*$$
$$\Rightarrow \sum_{r=1}^{s} g_r + \min_{W \in \overline{X}} \sum_{r=1}^{s} (-C_r W) > \sum_{r=1}^{s} g_r + \theta_0^*$$
$$\Rightarrow \min_{W \in \overline{X}} \sum_{r=1}^{s} (-C_r W) > \theta_0^*$$

Therefore, we have the following Theorem.

Theorem 3: *There is no non-dominated vector for (2), say* $\widehat{W} \in \overline{X}$, *with* $\sum_{r=1}^{s} C_r \widehat{W} \geq \theta_0^*$.

To find another non-dominated vectors of problem (2), we determine a non-dominated vector of problem (2), say $W_{k+1}^* \in X$, such that W_{k+1}^* is an optimal solution of the model $\min_{W \in X} \sum_{r=1}^{s} g_r - C_r W|$.

Therefore, $Y_{k+1} = (y_{k+1}^1, \dots, y_{k+1}^s) = (C_1 W_{k+1}^*, \dots, C_s W_{k+1}^*)^T \notin \overline{Y}_0$. That is the following inequalities are not satisfied simultaneously.

$$y_{k+1}^1 \leq \sum_{j \in I_0} \lambda_j y_j^1$$
$$y_{k+1}^2 \leq \sum_{j \in I_0} \lambda_j y_j^2$$
$$\vdots$$
$$y_{k+1}^s \leq \sum_{j \in I_0} \lambda_j y_j^s$$

where $\sum_{j \in I_0} \lambda_j = 1$ and $\lambda_j \in \{0,1\}$. In other words, we have

$$\begin{aligned} & y_{k+1}^1 > \sum_{j \in I_0} \lambda_j y_j^1 \text{ or} \\ & y_{k+1}^2 > \sum_{j \in I_0} \lambda_j y_j^2 \text{ or} \\ & \vdots \qquad\qquad \text{or} \\ & y_{k+1}^s > \sum_{j \in I_0} \lambda_j y_j^s. \end{aligned} \qquad (6)$$

where $\sum_{j \in I_0} \lambda_j = 1$ and $\lambda_j \in \{0,1\}$. That is, there exists $i (i \in \{1, \dots, s\})$ such that $y_{k+1}^i > \sum_{j \in I_0} \lambda_j y_j^i$. Therefore, there exists $i (i \in \{1, \dots, s\})$ such that $y_{k+1}^i > \max_{j \in I_0}\{y_j^i\} = y_q^i, \lambda_q = 1, \lambda_j = 0, j \in I_0, j \neq q$. Therefore, using (6) we can consider the following constraints:

$$\begin{aligned} & y_{k+1}^1 > \max_{j \in I_0}\{y_j^1\} = y_q^1 \quad \text{or} \\ & y_{k+1}^2 > \max_{j \in I_0}\{y_j^2\} = y_q^2 \text{ or} \\ & \vdots \\ & y_{k+1}^s > \max_{j \in I_0}\{y_j^s\} = y_q^s. \end{aligned} \qquad (7)$$

In other words, $\exists i \in \{1, \dots, s\}$ such that $y_{k+1}^i > \max_{j \in I_0}\{y_j^i\}$. Let M be a large positive real number and $\delta_i \in \{0,1\}$ for $i \in \{1, \dots, s\}$. Instead of (7) we consider the following constraints which are satisfied simultaneously.

$$\begin{aligned} & y_{k+1}^1 > \max_{j \in I_0}\{y_j^1\} - M\delta_1 \\ & y_{k+1}^2 > \max_{j \in I_0}\{y_j^2\} - M\delta_2 \\ & \vdots \\ & y_{k+1}^s > \max_{j \in I_0}\{y_j^s\} - M\delta_s \\ & \sum_{i=1}^{s} \delta_i \leq s - 1 \\ & \delta_i \in \{0,1\}, i = 1, \dots, s. \end{aligned} \qquad (8)$$

Therefore, to obtain another non-dominated vector of problem (2) we consider the following model:

$$\begin{aligned} \theta_1^* = \\ \max \quad & \sum_{r=1}^{s} C_r W \\ \text{s.t.} \quad & A_i W \leq b_i, \forall i \\ & \sum_{r=1}^{s} C_r W < \theta_0^* \\ & C_r W > \max_{j \in I_0}\{y_j^r\} - M\delta_r, \forall r \\ & \sum_{i=1}^{s} \delta_i \leq s - 1 \\ & \delta_r, W \in Z_+^n, \forall r. \end{aligned} \qquad (9)$$

Note that, if $\delta_r = 1$, then the constraint $C_r W > \max_{j \in I_0}\{y_j^r\} - M\delta_r$ is redundant. The constraint $\sum_{i=1}^{s} \delta_i \leq s - 1$ implies that at least one of the constraints $C_r W > \max_{j \in I_0}\{y_j^r\} - M\delta_r$, $r = 1, \dots, s$ is not redundant (we can choose $\max_{1 \leq r \leq s}\{\sum_{j=1}^{n} |c_{rj}|\}$ as a lower bound for M).

Let $\{W_{k+1}^*, \dots, W_{k+q}^*\}$ be the optimal solutions of problem (9). Using $I_1 = I_o \cup \{k+1, \dots, k+q\} = \{1, \dots, k, k+1, \dots, k+q\}$ we can obtain another non-dominated vector of problem (2). Therefore,

$$\begin{aligned} \theta_2^* = \\ \max \quad & \sum_{r=1}^{s} C_r W \\ \text{s.t.} \quad & A_i W \leq b_i, \forall i \\ & \sum_{r=1}^{s} C_r W < \theta_1^* \\ & C_r W > \max_{j \in I_0}\{y_j^r\} - M\delta_r, \forall r \\ & \sum_{i=1}^{s} \delta_i \leq s - 1 \\ & \delta_r, W \in Z_+^n, \forall r. \end{aligned} \qquad (10)$$

Theorem 4: *The optimal solutions of problem (9) are the non-dominated vectors of problem (2).*

Proof: Let W^* be an optimal solution of problem (9) and by contradiction, suppose that W^* is not non-dominated vector of problem (2). Hence, problem (2) has a feasible solution, say W^o, so that:

$$\begin{aligned} & C_r W^o \geq C_r W^*, r = 1, \dots, s \text{ and} \\ & \exists i, i \in \{1, \dots, s\};\ C_i W^o \geq C_i W^*. \end{aligned} \qquad (11)$$

Since W^* is feasible solution of problem (9),

$$C_r W^* > \max_{j \in I_0}\{y_j^r\} - M\delta_r, r = 1, \dots, s. \qquad (12)$$

From (11) and (12) we conclude that, $C_r W^o > \max_{j \in I_o}\{y_j^r\} - M\delta_r, r = 1, \dots, s$.

Therefore, W^o is a feasible solution of problem (9). By summing inequalities (11), we will have $\sum_{r=1}^{s} C_r W^o > \sum_{r=1}^{s} C_r W^*$ which is a contradiction. □

To illustrate the proposed method, let we consider Figure 1, in which g is ideal vector and $s = 2$.

Using model (5) we have,
$\sum_{r=1}^{2} g_r + min_{W\in X} \sum_{r=1}^{2}(-C_rW\ \ ) =$
$\sum_{r=1}^{2} g_r - \sum_{r=1}^{2} C_rW^* = \sum_{r=1}^{2} g_r + \theta_o^* = AB + By_1$, where W^* is an optimal solution of the model (5), and $Y_1 = (C_1W^*, C_2W^*)$ is its objective vector. Therefore, W^* is a non-dominated vector of the model (2) and in the first iteration, model (9) finds $\overline{W}^*$ as second non-dominated vector with $Y_2 = (C_1\overline{W}^*, C_2\overline{W}^*)$. Because,
$\sum_{r=1}^{2} g_r - \sum_{r=1}^{2}(-C_r\overline{W}^*) = \sum_{r=1}^{2} g_r + \theta_1^* = AD + Dy_2$ & $AD + Dy_2 > AB + By_1$.

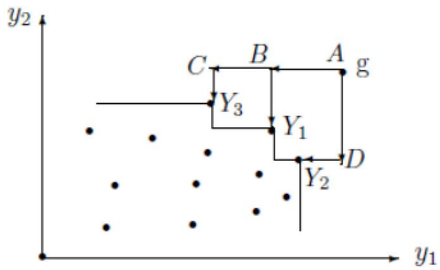

Figure 1: The objectives values space with $s = 2$

3.1 The Proposed Algorithm

Step 0: Solve problem (5) and suppose that I_o is the indices set of its optimal solutions,

Step 1: Solve the following model:

$$\theta_q^* = \begin{array}{ll} \max & \sum_{r=1}^{s} C_rW \\ \text{s.t.} & A_iW \le b_i, \forall i \\ & \sum_{r=1}^{s} C_r W < \theta_{q-1}^* \qquad (13) \\ & C_rW > \max_{j\in I_0}\{y_j^r\} - M\delta_r, \forall r \\ & \sum_{i=1}^{s} \delta_i \le s-1 \\ & \delta_r, W \in Z_+^n, \forall r \end{array}$$

and suppose that F_q is the set indices of the optimal solutions of model (13).

Step 2: If F_q is empty, stop. Otherwise, put $I_q = I_{q-1} \cup F_{q-1}$, where $F_o = \{\}$, and go to step 1.

Theorem 5: *The optimal solutions of problem (13) are the non-dominated vectors of problem (2).*

Proof: The proof is similar to that of Theorem 4 and is omitted. □

Let $E_q = \{W_1^*, \ldots, W_d^*\}$ be the set of the non-dominated vectors of problem (2) which have been generated by iterations 1 trough q of the proposed algorithm.

Theorem 6: *The proposed algorithm is convergent.*

Proof: The feasible region of problem (2) is bounded. Therefore, the number of its feasible solutions and the efficient solutions are finite. On the other hand, the proposed algorithm finds a subset of the efficient solutions. Therefore, the proposed algorithm is convergent. □

4. Numerical Example

Let us consider the following 0-1 MOLP:

$$\begin{array}{lll} \max & 3w_1 + 6w_2 + 5w_3 - 2w_4 + 3w_5 & \\ \max & 6w_1 + 7w_2 + 4w_3 + 3w_4 - 8w_5 & \\ \max & 5w_1 - 3w_2 + 8w_3 - 4w_4 + 3w_5 & \\ \text{s.t.} & -2w_1 + 3w_2 + 8w_3 - w_4 + 5w_5 & \le 13 \\ & 6w_1 + 2w_2 + 4w_3 + 4w_4 - 3w_5 & \le 15 \\ & 4w_1 - 2w_2 + 6w_3 - 2w_4 + w_5 & \le 11 \\ & w_1, w_2, w_3, w_4, w_5 \in \{0,1\}. & \end{array}$$

This problem is an adaptation of an example from Liu et al. (2000).

Step 0: Initially, we solve the following 0-1 linear problem and form the set I_o

$$\begin{array}{lll} \min & -14w_1 - 10w_2 - 17w_3 + 3w_4 + 2w_5 & \\ \text{s.t.} & -2w_1 + 3w_2 + 8w_3 - w_4 + 5w_5 & \le 13 \\ & 6w_1 + 2w_2 + 4w_3 + 4w_4 - 3w_5 & \le 15 \\ & 4w_1 - 2w_2 + 6w_3 - 2w_4 + w_5 & \le 11 \\ & w_1, w_2, w_3, w_4, w_5 \in \{0,1\} & \end{array}$$

where, $\sum_{r=1}^{3}(-C_rW) = -14w_1 - 10w_2 - 17w_3 + 3w_4 + 2w_5$. $W_1^* = (1,1,1,0,0)^T$ is an optimal solution to this problem and $\theta_0^* = -41$ is its optimal value. Therefore, $W_1^* = (1,1,1,0,0)^T$ is a non-dominated vector, $Y_1 = (14,17,10)^T$ and $I_o = I_1 = \{1\}$.

Iteration 1

Step 1: To obtain a new non-dominated vector we solve the corresponding problem to I_o with $M = 100$

$$\begin{array}{lll} \min & -14w_1 - 10w_2 - 17w_3 + 3w_4 + 2w_5 & \\ \text{s.t.} & -2w_1 + 3w_2 + 8w_3 - w_4 + 5w_5 & \le 13 \\ & 6w_1 + 2w_2 + 4w_3 + 4w_4 - 3w_5 & \le 15 \\ & 4w_1 - 2w_2 + 6w_3 - 2w_4 + w_5 & \le 11 \\ & 3w_1 + 6w_2 + 5w_3 - 2w_4 + 3w_5 & > 14 - M\delta_1 \\ & 6w_1 + 7w_2 + 4w_3 + 3w_4 - 8w_5 & > 17 - M\delta_2 \\ & 5w_1 - 3w_2 + 8w_3 - 4w_4 + 3w_5 & > 10 - M\delta_3 \\ & 14w_1 + 10w_2 + 17w_3 - 3w_4 - 2w_5 & < 41 \\ & \delta_1 + \delta_2 + \delta_3 & \le 2 \\ & w_1, w_2, w_3, w_4, w_5, \delta_1, \delta_2, \delta_3 \in \{0,1\}. & \end{array}$$

We have an optimal solution to this problem for $W_2^* = (1,1,1,1,1)^T$, which is a non-dominated vector and $\theta_1^* = -36$.

Step 2: Using $W_2^* = (1,1,1,1,1)^T$, we have

$Y_2 = (15,12,9)^T, F_1 = \{2\} \neq \phi, I_2 = I_1 \cup F_1 = \{1,2\}, \max_{j\in\{1,2\}}\{y_j^1\} = \max\{14,15\} = 15,$ $\max_{j\in\{1,2\}}\{y_j^2\} = \max\{17,12\} = 17,$ and $\max_{j\in\{1,2\}}\{y_j^3\} = \max\{10,9\} = 10.$

Iteration 2

Step 1: Using $I_2 = \{1,2\}$ we have the following 0-1 linear problem:

$$\begin{array}{lll} \min & -14w_1 - 10w_2 - 17w_3 + 3w_4 + 2w_5 & \\ \text{s.t.} & -2w_1 + 3w_2 + 8w_3 - w_4 + 5w_5 & \leq 13 \\ & 6w_1 + 2w_2 + 4w_3 + 4w_4 - 3w_5 & \leq 15 \\ & 4w_1 - 2w_2 + 6w_3 - 2w_4 + w_5 & \leq 11 \\ & 3w_1 + 6w_2 + 5w_3 - 2w_4 + 3w_5 & > 15 - M\delta_1 \\ & 6w_1 + 7w_2 + 4w_3 + 3w_4 - 8w_5 & > 17 - M\delta_2 \\ & 5w_1 - 3w_2 + 8w_3 - 4w_4 + 3w_5 & > 10 - M\delta_3 \\ & 14w_1 + 10w_2 + 17w_3 - 3w_4 - 2w_5 & < 36 \\ & \delta_1 + \delta_2 + \delta_3 & \leq 2 \\ & w_1, w_2, w_3, w_4, w_5, \delta_1, \delta_2, \delta_3 \in \{0,1\}. & \end{array}$$

By solving this problem, we obtain $W_3^* = (1,0,1,0,0)^T$, and $\theta_2^* = -31.$

Step 2: By $W_3^* = (1,0,1,0,0)^T$ We have,

$Y_3 = (8,10,15), F_2 = \{3\} \neq \phi, I_3 = I_2 \cup F_2 = \{1,2,3\}, \max_{j\in\{1,2\}}\{y_j^1\} = \max\{8,15\} = 15,$ $\max_{j\in\{1,2\}}\{y_j^1\} = \max\{17,10\} = 17,$ and $\max_{j\in\{1,2\}}\{y_j^1\} = \max\{10,13\} = 13.$

Iteration 3

Step 1: The corresponding problem to I_3 is as follows:

$$\begin{array}{lll} \min & -14w_1 - 10w_2 - 17w_3 + 3w_4 + 2w_5 & \\ \text{s.t.} & -2w_1 + 3w_2 + 8w_3 - w_4 + 5w_5 & \leq 13 \\ & 6w_1 + 2w_2 + 4w_3 + 4w_4 - 3w_5 & \leq 15 \\ & 4w_1 - 2w_2 + 6w_3 - 2w_4 + w_5 & \leq 11 \\ & 3w_1 + 6w_2 + 5w_3 - 2w_4 + 3w_5 & > 15 - M\delta_1 \\ & 6w_1 + 7w_2 + 4w_3 + 3w_4 - 8w_5 & > 17 - M\delta_2 \\ & 5w_1 - 3w_2 + 8w_3 - 4w_4 + 3w_5 & > 13 - M\delta_3 \\ & 14w_1 + 10w_2 + 17w_3 - 3w_4 - 2w_5 & < 31 \\ & \delta_1 + \delta_2 + \delta_3 & \leq 2 \\ & w_1, w_2, w_3, w_4, w_5, \delta_1, \delta_2, \delta_3 \in \{0,1\}. & \end{array}$$

By solving the above problem, we have $W_4^* = (1,0,1,0,1)^T, \theta_0^* = -29.$

Step 2: Using $W_4^* = (1,0,1,0,1)^T$ we have,
$Y_4 = (11,2,16), F_3 = \{4\}, I_4 = I_3 \cup F_3 = \{1,2,3,4\}, \max_{j\in I_4}\{y_j^1\} = \max\{15,11\} = 15,$
$\max_{j\in I_4}\{y_j^1\} = \max\{17,2\} = 17,$ and
$\max_{j\in I_4}\{y_j^1\} = \max\{13,16\} = 16.$

Iteration 4

Step 1: The corresponding problem to I_4 is as follows:

$$\begin{array}{lll} \min & -14w_1 - 10w_2 - 17w_3 + 3w_4 + 2w_5 & \\ \text{s.t.} & -2w_1 + 3w_2 + 8w_3 - w_4 + 5w_5 & \leq 13 \\ & 6w_1 + 2w_2 + 4w_3 + 4w_4 - 3w_5 & \leq 15 \\ & 4w_1 - 2w_2 + 6w_3 - 2w_4 + w_5 & \leq 11 \\ & 3w_1 + 6w_2 + 5w_3 - 2w_4 + 3w_5 & > 15 - M\delta_1 \\ & 6w_1 + 7w_2 + 4w_3 + 3w_4 - 8w_5 & > 17 - M\delta_2 \\ & 5w_1 - 3w_2 + 8w_3 - 4w_4 + 3w_5 & > 16 - M\delta_3 \\ & 14w_1 + 10w_2 + 17w_3 - 3w_4 - 2w_5 & < 29 \\ & \delta_1 + \delta_2 + \delta_3 & \leq 2 \\ & w_1, w_2, w_3, w_4, w_5, \delta_1, \delta_2, \delta_3 \in \{0,1\}. & \end{array}$$

Step 2: The above problem is infeasible. Hence, the algorithm has terminated and $W_1^* = (1,1,1,0,0)^T$, $W_2^* = (1,0,1,0,1)^T, W_3^* = (1,1,1,1,1)^T$ and $W_4^* = (1,0,1,0,0)^T$ are the elements of the well-dispersed subset of the non-dominated vectors.

5. Conclusion

The major drawback of the previous approaches is the difficulty in solving the constrained problems due to the increasing number of constraints and binary variables with an increase in the number of the non-dominated vectors, and therefore they increase computational efforts to find the non-dominated vectors of an MOILP problem. This paper presented a convergent algorithm to find a well-dispersed subset of the non-dominated vectors of an MOILP problem using L_1 −norm. In each iteration of the proposed algorithm, at least one non-dominated vector is found, and it does not increase the number of constraints and binary variables of the constrained problems. In each iteration, the proposed method modifies only right hand side of some constraints of a single objective. Therefore, proposed algorithm reduces the computational efforts for solving an MOILP problem. A modified version of this algorithm can be used for solving a multi objective mixed integer linear programming problems.

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