A Comparison among Homotopy Perturbation Method and the Parameterized Perturbation Method with the Variational Iteration Method for Vibration of Buckled Beams

M. Hajiamiri [1]*, K. Gholaminejad [2], A.Majidian[3]

[1] Lecturer at Mazandaran Institute of technology,Babol,Iran
[2]M.S student at Islamic Azad University Sari Branch,Sari,Iran
[3]Associate professor at Islamic Azad University,Sari,Iran
* Corresponding author. Tel: +989126447611, E-mail: mechanical_amiri@yahoo.com

Abstract: In this article, we implement a relatively new numerical technique and we present a comparative study among Homotopy perturbation method, Parametrized Perturbation method and the Variational Iterational method. These methods in applied mathematics can be an effective procedure to obtain for approximate solutions. The study outlines the significant features of the three methods. The analysis will be illustrated by investigating nonlinear vibration behavior of a buckled beam subjected to an axial load. This method can easily extend to solve other nonlinear vibration equations of the beams, plates and shells in the future.
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1. Introduction

Vibration analysis of the beams is an important issue in structural engineering applications such as long span bridges, aerospace vehicles, automobiles and many other industrial usages. The dynamics of continuous or distributed parameter systems, such as beams, plates, and shells, are governed by nonlinear partial-differential equations in space and time. These partial-differential equations and associated boundary conditions form an initial boundary-value problem. In general it is hard to find exact or closed-form solutions for this class of problems. Consequently, one seeks approximate solutions of the original problem.

The study of nonlinear vibration equations solution has been applied by many researchers and various methods of solution have been used. In recent years, much attention has been devoted to the new developed methods to construct an analytic solution of nonlinear vibration such as He's Homotopy Perturbation Method (HPM) [1-5],Homotopy Analysis Method (HAM) [6-10], He's Parameter-Expanding Method [11,12],He's Variational Iteration Method (VIM) [13-17] and He's Energy Balance Method (EBM) [18-20] and etc.

There are two classes of approximating solutions for initial boundary-value problems: numerical methods (e.g., finite differences, finite elements, and boundary elements) [21-23] and analytical methods [24-27]. Analytical methods can be divided into two categories: direct and discretization techniques. For weakly nonlinear systems, direct techniques are used. Discretization method used to discretized the partial differential equation into a set of nonlinear ordinary differential equation and then solved analytically in time domain. One of the most commonly used methods for discretization is the Galerkin procedure. Besides all advantages of numerical methods, due to convenience for parametric studies and accounting of the physics of the problems, an analytical solution appears more appealing than the numerical one. Also, analytical solutions give a reference frame for verification and validation of other numerical approaches.

The main propose of this study is to obtain analytical expressions for geometrically nonlinear vibration of the buckled beams. With the Galerkin approach, governing nonlinear partial differential equation is reduced to a single nonlinear ordinary differential equation. The latter equation is solved analytically in time domain using HPM,VIM and PPM.

2. THEORICAL FORMULATION

Consider a straight beam of length L, a cross-section A, a mass per unit length m, moment of inertia I, and modulus of elasticity E, that subjected to an axial force of magnitude $\overline{F}$ as shown in figure 1. The equation of motion including the effects of mid-plane stretching is given by [28,29]:

$$EI\frac{\partial^4\bar{W}}{\partial\bar{X}^4}+m\frac{\partial^2\bar{W}}{\partial\bar{t}^2}+\bar{F}\frac{\partial^2\bar{W}}{\partial\bar{X}^2}+\bar{C}\frac{\partial\bar{W}}{\partial\bar{t}}$$
$$-\frac{EA}{2L}\frac{\partial^2\bar{W}}{\partial\bar{X}^2}\int_0^L\left(\frac{\partial\bar{W}}{\partial\bar{X}}\right)^2 d\bar{X}=0 \quad (1)$$

Where C is viscous damping coefficient.

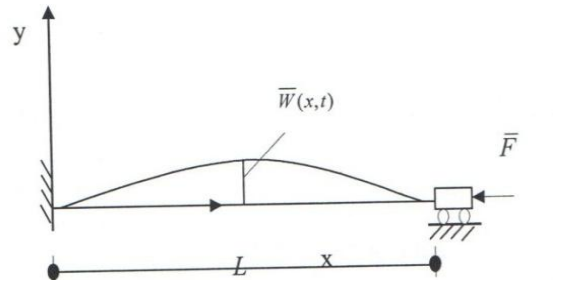


Figure 1. A schematic of a beam subjected to an axial load

For convenience, the following non-dimensional variables are used:

$$\frac{\partial^4 W}{\partial X^4}+\frac{\partial^2 W}{\partial t^2}+F\frac{\partial^2 W}{\partial X^2}+C\frac{\partial W}{\partial t}-\frac{1}{2}\frac{\partial^2 W}{\partial X^2}\int_0^L\left(\frac{\partial W}{\partial X}\right)^2 dX=0$$

$$X=\frac{\bar{X}}{L},\quad W=\frac{\bar{W}}{R},\quad t=\bar{t}\sqrt{\frac{EI}{mL^4}},$$

$$F=\frac{\bar{F}L^2}{EI},\quad C=\frac{\bar{C}L^2}{\sqrt{mEI}}$$

Where $R=\sqrt{\left(\frac{I}{A}\right)}$ is the radius of gyration of the cross-section. As a result Eq. (1) can be written as follows:

(2)

Assuming $W(X,t)=\phi(X)\psi(t)$ where $\phi(X)$ is the first eigenmode of the beam [30] and applying the Galerkin method, the equation of motion is obtained as follows:

$$\ddot{\psi}(t)+C\dot{\psi}(t)+(\alpha+\beta.F)\psi(t)+\gamma\psi^3(t)=0 \quad (3)$$

Where dot denotes differentiation with respect to time and α,β and γ are as follows:

$$\alpha=\left(\int_0^1\phi'''\phi dx\right)/\int_0^1\phi^2dx \quad (4)$$

$$\beta=\left(\int_0^1\phi''\phi dx\right)/\int_0^1\phi^2dx \quad (5)$$

$$\gamma=\left(-0.5\int_0^1\left(\phi''\int_0^1\phi'^2dx\right)\phi dx\right)/\int_0^1\phi^2dx \quad (6)$$

Here prime denotes differentiation with respect to x. The Eq. (3) is the differential equation of motion governing the nonlinear vibration of buckled beam. The center of the beam subjected to the following initial conditions:

$$\psi(0)=A\ \ \dot{\psi}(0)=0 \quad (7)$$

3. Numerical Methods

3-1. Fundamentals of the Homotopy Perturbation Method

To illustrate the basic ideas of this method, we consider the following equation [1-3]:

$$A(u)-f(r)=0,\quad r\in\Omega, \quad (8)$$

with boundary condition

$$B\left(u,\frac{\partial u}{\partial n}\right)=0,\ r\in\Gamma \quad (9)$$

where A is a general differential operator, B a boundary operator, f (r) a known analytical function and Γ is the boundary of the domain Ω.
A can be divided into two parts which are L and N, where L is linear and N is nonlinear.
Eq. (8) can therefore be rewritten as follows:

$$L(u)+N(u)-f(r)=0,\quad r\in\Omega, \quad (10)$$

Homotopy perturbation structure is shown as follows:

$$H(U,p)=(1-p)[L(v)-L(u_0)]+p[A(v)-f(r)]=0,$$
$$p\in[0,1],\quad r\in\Omega \quad (11)$$

where

$$v(r,p):\Omega\times[0,1]\to\Re. \quad (12)$$

In Eq. (11), $p\in[0,1]$ is an embedding parameter and U0 is the first approximation that satisfies the boundary condition. We can assume that the solution of Eq. (11) can be written as a power series in p, as following:

$$\nu = \nu_0 + p\nu_1 + p^2\ \nu_2 + p^3\ \nu_3 + \cdots, \quad (13)$$

and the best approximation for solution is

$$u = \lim_{p\to 1} \nu = \nu_0 + \nu_1 + \nu_2 + \nu_3 + L \quad (14)$$

The above convergence is discussed in [1-3].

3-2. The variational iterational Method

Consider the differential equation

$$L_t\ u + Nu = g(t) \quad (15)$$

Where L is a linear operator, N is a non-linear operator and g(t) is a known and Nonlineer analytical function. Ji Huan He has modified the above method into an iteration method[14]in the following way:

$$u_{n+1} = u_n + \int_0^t \lambda(\ Lu_n(x) + Nu_n(x) - g(x))\ dx \quad (16)$$

where λ is a general Lagrange's multipler, which can be identified optimally via the variational theory, and $\tilde{u}_n$ is a restricted variation which means $\delta\ \tilde{u} = 0$.

It is obvious now that the main steps of He's variational iteration method require first the determination of the Lagrangian multiplier λ that will be identified optimally. Having determined the Lagrangian multiplier, the successive approximations u_{n+1}, $n \geq 0$, of the solution u will be readily obtained upon using any selective function u_0 .Consequently, the solution

$u = \lim un$, for $(n \to \infty)$. (17)

In other words, correction functional (16) will give several approximations, and therefore the exact solution is obtained at the limit of the resulting successive approximations.

3-3.The Parametrized Perturbation Method

The parameterized perturbation method was first proposed in 1999 in [11]. According to [11], an expanding parameter is introduced by a linear trans-formation:

$$\theta = \zeta\nu + b \quad (18)$$

where x is the perturbation parameter, by substituting eq. (18) into an original equation in order to have no secular term in the equation; we can obtain the un-known constant parameter b, then, the solution is expanded in the form:

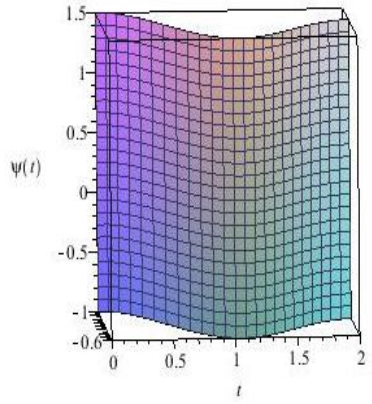
$$\nu = n\sum_{i=0} \zeta^i \nu_i = \nu_0 + \zeta_1\nu_1 + \zeta^2\nu_2 + \zeta^3\nu_3 + ..., \quad (19)$$

Here ζ is an artificial bookkeeping parameter. Unlike traditional perturbation methods, we keep $\nu_0(0) = \nu(0)$ and $\sum_{i=1} \nu_i(0) = 0$.

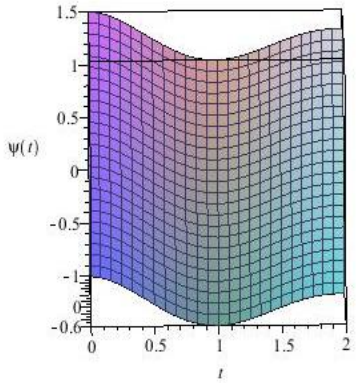
By considering three different type of variation for emissivity versus temperature (no-variation, linear variation and second power variation), three unlike ODEs are solved as:

$$\frac{d(\zeta\nu + b)}{d\tau} + \zeta\nu + \varepsilon(\zeta\nu)^4 = 0$$
$$\frac{d(\zeta\nu)}{d\tau} + \zeta\nu + \varepsilon_a(\zeta\nu)^4 + \varepsilon_a(\zeta\nu)^5 = 0 \quad (20)$$
$$\frac{d(\zeta\nu)}{d\tau} + \zeta\nu + \varepsilon_a(\zeta\nu)^4 + \varepsilon_a\beta(\zeta\nu)^5 + \varepsilon_a\alpha(\zeta\nu)^6 = 0$$

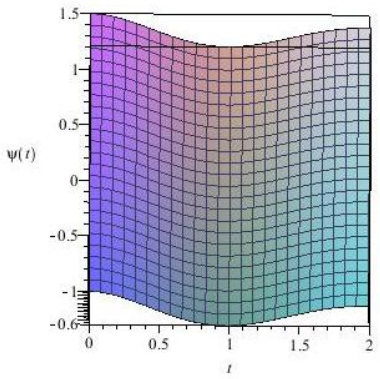
By substituting transformation equation into these equations, Inserting ν terms, rearranging ζ -terms, summation of solutions and subsequently using inverse transformation, the temperature distribution for different conditions will be obtained. All analytical expressions gained by PPM are in very good agreement with numerical results and can be used in many calculations related to industries.



Approximation solution solution via HPM



Approximation solution solution via PPM



Approximation solution solution via VIM

Fig.1.The numerical results for $\psi(t)$ when $0 \le t \le 2$ with initial condition of Eq.(7) by means of HPM,PPM and VIM

4. Comparison among HPM, VIM and PPM

It can be seen from this study, that:

1. Comparison among HPM, VIM and PPM shows that although the results of these methods, HPM does not require specific algorithms and complex calculations, such as PPM or construction of correction functional using general Lagrange multipliers, such as VIM and is much easier and more convenient than PPM and VIM.

2. HPM handles linear and nonlinear problems in a simple manner by deforming a difficult problem into a simple one. But in nonlinear problems, we encounter difficulties to calculate the so-called PPM 's constant(b), when using PPM. Also, optimal identification of Lagrange multipliers via the variational theory can be difficult in VIM.

3. Comparison among HPM, VIM and ADM shows that although the results of these methods,we have the similar answers for $0 \le t \le 2$. (figure 1,2)

Table 1: Comparison among HPM ,PPM ,VIM for $\psi(t)$

t(s)	HPM	VIM	PPM
0	1	1.005476	1
0.2	0.802316	0.801212	0.903486
0.4	0.340247	0.337299	0.507686
0.6	-0.16068	-0.16157	0.005626
0.8	-0.51412	-0.51171	-0.4074
1	-0.6194	-0.61443	-0.60271
1.2	-0.48139	-0.47724	-0.5513
1.4	-0.1953	-0.19594	-0.3159
1.6	0.108204	0.102157	-0.01242
1.8	0.318686	0.309406	0.240408
2	0.378462	0.369151	0.363113
2.2	0.293153	0.287692	0.33626
2.4	0.119539	0.120524	0.196375
2.6	-0.06471	-0.05787	0.012993
2.8	-0.19286	-0.18296	-0.1417
3	-0.22998	-0.22063	-0.21867
3.2	-0.1794	-0.17408	-0.20502
3.4	-0.07486	-0.07538	-0.12196
3.6	0.037021	0.031361	-0.01119
3.8	0.11564	0.107236	0.083422
4	0.139425	0.131433	0.131634

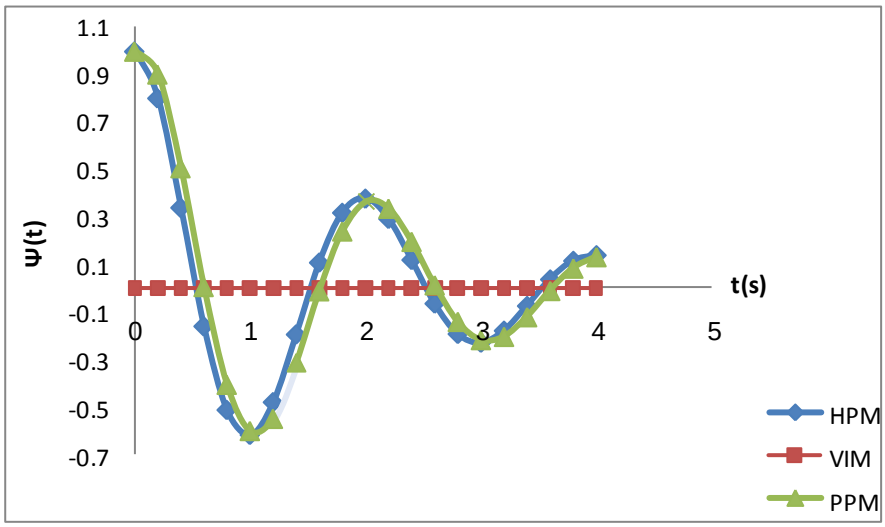


Fig2:The comparison of answers obtained by HPM,VIM,PPM

5. Conclusion

In this letter, we have successfully developed HPM , PPM and VIM to obtain the analytical expression for the nonlinear deflection of Buckled beams. It is apparently seen that these methods are very powerful and efficient techniques for solving different kinds of problems arising in various fields of science and engineering and present a rapid convergence for the solutions. The solutions obtained show that the results of these methods are in agreement but HPM is an easy and convenient one.

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